

ANAIS DE FILOSOFIA CLÁSSICA

Geometrical premises in Aristotle's *Incessu Animalium* and kind-crossing

Lucas Angioni
University of Campinas

ABSTRACT: At some point in the *Incessu Animalium*, Aristotle appeals to some geometrical claims in order to explain why animal progression necessarily involves the bending (of the limbs), and this appeal to geometrical claims might be taking as violating the recommendation to avoid “kind-crossing” (as found in the *Posterior Analytic*). But a very unclear notion of kind-crossing has been assumed in most debates. I will argue that kind-crossing in the *Posterior Analytic* does not mean any employment of premises from a discipline other than that to which the *explanandum* belongs. Kind-crossing was meant to cover a specific sort of employment of premises from a different discipline, namely, the case in which premises from a discipline X are taken as the most important explanatory factor that delivers the fullest appropriate explanation of an *explanandum* within discipline Y. If this is so, the employment of geometrical premises in the *Incessu Animalium* is not an instance of the prohibited kind-crossing, but something that is in line with the theory of the *Posterior Analytic*.

KEY-WORDS: Aristotle; explanation; causality; kind-crossing; locomotion; essencialism.

RESUMO: Em certa passagem do *Incessu Animalium*, Aristóteles recorre a algumas afirmações geométricas para explicar por que a progressão animal necessariamente envolve a flexão (dos membros), e esse apelo a afirmações geométricas pode ser considerado uma violação da recomendação de evitar “*kind-crossing*” (tal como exposto nos *Segundos Analíticos*). No entanto, uma noção pouco clara de “*kind-crossing*” tem sido assumida no debate. Argumentarei que “*kind-crossing*” nos *Segundos Analíticos* não significa qualquer emprego de premissas de uma disciplina diferente daquela à qual o *explanandum* pertence. “*Kind-crossing*” se refere a um tipo específico de emprego de premissas provenientes de uma disciplina diferente, a saber, o caso em que premissas de uma disciplina X são tomadas como o fator explicativo mais importante que fornece a explicação mais apropriada de um *explanandum* na disciplina Y. Se isso é assim, o emprego de premissas geométricas no *Incessu Animalium* não é um exemplo de “*transposição*” proibida, mas algo que está de acordo com a teoria da *Segundos Analíticos*.

PALAVRAS-CHAVE: Aristóteles; explicação; causalidade; transposição; locomoção; essencialismo

1. Introduction:

At some juncture in the *Incessu Animalium*, Aristotle appeals to some geometrical claims in order to explain why animal progression necessarily involves the

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bending – either of the limbs (for animals that have limbs) or of the whole body (for limbless animals, as the snakes). But appealing to geometrical claims within a branch of zoology seems to violate one recommendation found in the *Posterior Analytics*, namely, the recommendation to avoid “kind-crossing”. Most scholars would be inclined to say something stronger: since kind-crossing was prohibited by Aristotle, the *Incessu Animalium* seems to depart from the methodology found in the *Posterior Analytics*. Yet it is quite unclear what kind-crossing exactly means. I will argue that kind-crossing in the *Posterior Analytics* does not mean *any employment of premises* from a discipline other than that to which the explanandum belongs. Kind-crossing was meant to cover a specific sort of employment of premises from a different discipline, namely, the case in which premises from a discipline *X* are taken as the most important explanatory factor that delivers the fullest appropriate explanation of an explanandum within discipline *Y*. Within this picture, what is going on in the *Incessu Animalium* is not an instance of the prohibited kind-crossing. Explanation of animal progression in the *Incessu Animalium* is in agreement with the theory of the *Posterior Analytics*.

2. The kind-crossing issue (*metabasis eis allo genos*):

My aim is not to offer a full-fledged discussion of the problem of kind-crossing in Aristotle's theory of science in the *Posterior Analytics*.¹ I will only give a sketchy characterisation of what I take to be some decisive issues involving that problem, then I will discuss one passage from Aristotle's *Incessu Animalium* which I take to confirm the view I have sketched.

As the problem is usually put, Aristotle seems to forbid the “crossing” from one scientific domain to another – for instance, the “crossing” from arithmetic to cosmology (cf. *Metaphysics* 986a3-12, *de Caelo* 300a14-19), or the “crossing” from geometry to physics in general (cf. *de Caelo* 299a25-30ff.). So far so good, but this is too much generic. We need a more fine-grained understanding of what kind-crossing consists in.

¹ For a full-fledged discussion, see Steinkrüger 2018. I am very sympathetic to his views, but disagree in many points. I have no room here to engage in a careful debate, but I mention that one of his merits is to avoid taking “*genos*” in *Posterior Analytics* I.7 as *discipline* – he takes it as the particular kind involved in a particular demonstration (p. 111) and thereby avoids the conflation of kind-crossing with what I label Question 2.

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Such a more fine-grained understanding can arise from answers to the following questions:

Question 1: Did Aristotle claim that the fullest appropriate explanation of a given explanandum within a particular domain could never have its most important explanatory factor transported from a different scientific domain?

Question 2: Did Aristotle claim that a given scientific domain could never employ premises which properly belongs to a different scientific domain?

Both questions are still quite general and ask for more specifications – and I will offer some further specifications below – but they are sufficiently distinct from each other.

An example of Question 1 will be the following: “Can the fully appropriate explanation of why the progression of animals requires the bending of their limbs be given in explanatory terms transported from geometry?”.

An example of Question 2 will be the following: “Is the expert who studies why the progression of animals requires the bending of their limbs allowed to employ in her explanatory story some propositions transported from geometry?”

Another example of Question 1 will be the following: “Can the fully appropriate explanation of why some bodies are heavier than others be given in explanatory terms transported from geometry?”.

Another example of Question 2 will be the following: “Is the expert who studies why some bodies are heavier than others allowed to employ in her explanatory story some propositions transported from geometry?”

Before proceeding to further specifications of these questions, let me stress that they are indeed different and that conflation between them can mar the debate about kind-crossing. This conflation stems from a major one, namely, a conflation between two different issues concerning what is going on in the *Posterior Analytics* (I will label them Issue 1 and Issue 2 in order to make reference easier and avoid confusion with Question 1 etc.):

Issue 1: Is Aristotle (in the *Posterior Analytics*) describing what it is for an expert to have the fullest appropriate explanation of a given explanandum?

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Issue 2: Is Aristotle (in the *Posterior Analytics*) describing what it is for an expert to have full mastery over a domain?

The tasks involved in Issue 1 and Issue 2 are indeed inter-related, but even so they are different. Take 2R's belonging to triangles as an explanandum. Attaining the fullest appropriate explanation of why 2R belongs to triangles requires a series of conditions: surely, one must know what a straight line is, one must know that the triangle is a rectilinear plane figure enclosed by three sides, one must know that the alternate angles of a line crossing two parallel lines are equivalent (Euclid's proposition I.29), one should be able to draw a parallel line to one of the sides of the triangle, one must know that angles around a straight line are equal to two right angles (cf. *Metaphysics* 1051a24-25). Now, knowing all that requires a specific level of expertise – but it is still possible for a learner to grasp the full appropriate explanation of why 2R belongs to triangles without having full mastery over the domain. The learner might have no knowledge at all about many other things within the domain of geometry – things about other figures such as hexagons, rhombuses etc., other sort of propositions involving angles and lines etc. Even so, the teacher might stress that the fullest appropriate explanation of why 2R belongs to triangles depends on specific factors of such and such a sort, and stress that the learner must look for factors of the same sort when attempting to solve a different problem or to attain the fullest appropriate explanation of a different explanandum.²

I need not dwell on the nature of these specific factors that make an explanation the fullest appropriate explanation of its explanandum. But some features of them will be important to stress the difference between Issue 1 and Issue 2. Thus, I add – and highlight – that the fullest appropriate explanation of each explanandum has two prominent features in Aristotle's theory of science, namely: the fullest appropriate explanation should grasp the explanatory factor that allows us to tell *what the explanandum is* or, in other terms, the essence of the explanandum (see 90a14-15, 31-34); and the fullest appropriate explanation should stick to what the explanandum *as*

² I have taken the learner as an example in which the distinction between Issue 1 and Issue 2 is most conspicuous. But the distinction does not depend on any diachronic development of the expertise. For any expert, it is a different thing, for her, (i) to have the fullest appropriate explanation of a given explanandum and (ii) to have full mastery over the domain. (I stress again that *distinction* does not imply *mutual independence*).

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such is instead of sticking to a generic, concomitant factor that the explanandum happen to have in common with other explananda (cf. 75b37-76a9). Now, both these features point to a sort of relation between explanans and explanandum that can be stated in terms of “reciprocation” (mutual entailment, if they are treated as propositions, or coextensiveness, if they are treated as terms in a demonstration). In the case of 2R’s belonging to triangles, Aristotle has paid attention to it when characterising its demonstration in terms of the strict notion of “*katholou*” as found in I.4-5. In extensional terms, the appropriate subject to which 2R is attributed must be coextensive with it; in intensional terms, the appropriate explanatory factor explaining why 2R is attributed to the triangle must focus on what exactly the triangle is as a triangle (but not as a particular isosceles, or equilateral, or scalene which might happen to be employed in a particular diagram).³

I have stressed these features of the fullest appropriate explanation of a given explanandum because they might seem strange (and too stringent) when we move to Issue 2 and ask what it is for an expert to have full mastery over a domain. Of course, Aristotle will never recommend the following points to any expert: “when you are selecting the propositions that properly belong to your field, stick to the *reciprocating ones*; when practising the procedures that properly belong to your field, stick to those which involve reciprocating”. Having mastery over a domain includes many other things which do not require the strict conditions that apply to the fullest appropriate explanation of a given explanandum. Having mastery over a scientific domain includes, for instance, being able to collect facts in an organised way,⁴ being able to classify items under general headings and to divide classes,⁵ being able to spot common factors which allow the reduction of several problems to a single one⁶ etc.

Let me go back to the distinction between Issue 1 and Issue 2. It is clear that Aristotle (in the *Posterior Analytics*) is not exclusively dedicated to one of the tasks involved in Issue 1 or Issue 2 with exclusion of the other. Aristotle is concerned with

³ About the “*katholou*” in I.5, see Hasper 2005.

⁴ See for instance *Prior Analytics* 43b1-11.

⁵ See 98a13-19, 96b25ff.

⁶ See for instance 98a20-29.

both of them within the *Posterior Analytics* as a whole.⁷ But, when we come to examine a specific passage, a pressing methodological question is for us to ascertain which of those issues (Issue 1 or Issue 2) is the main focus of Aristotle *exactly in that passage*.

Kind-crossing is discussed mainly in three passages in the *Posterior Analytics*: I.7 (the whole chapter), part of I.9 (76a9-15, 22-25) and the end of I.13 (78b34-79a16). We can see how important the distinction between Issue 1 and Issue 2 is for a correct assessment (and a correct answer) of Question 1 and Question 2. Several views conflate Issue 1 and Issue 2 and so give positive answers to both Question 1 and Question 2. Thus, according to them: (i) Aristotle has claimed that the fullest appropriate explanation of a given explanandum within a particular domain could never have its most important explanatory factor transported from a different scientific domain, and (ii) Aristotle has also endorsed the stronger claim that a given scientific domain could never employ premises which properly belong to a different scientific domain (with the few exceptions recognised as 'subalternate sciences').⁸

However, I believe that Aristotle's answers to Question 1 and Question 2 are different: while (a) he indeed has claimed that the fullest appropriate explanation of a given explanandum within a particular domain could never have its most important explanatory factor transported from a different scientific domain, (b) he has never endorsed the stronger claim that a given scientific domain could never employ premises which properly belongs to a different scientific domain. And the reason why I believe that this is so is the following: the key passages in which Aristotle is discussing kind-crossing are part of what is involved in Issue 1, but not of what is involved in Issue 2. In other words, when discussing kind-crossing, Aristotle is describing what it is for an expert to have the fullest appropriate explanation of a given explanandum (Issue 1) – but he is not concerned with the more general issue of what it is for an expert to have full mastery over a domain (Issue 2) and, consequently, he is not addressing the more

⁷ Taking only uncontroversial cases: Issue 1 is tackled (e.g.) in I.5 as a whole, as well as in I.9, whereas I.10 can be taken as mostly concerned with Issue 2. I believe most scholars will agree on that. Furthermore, as I have argued elsewhere, the definition of scientific knowledge in 71b9-12 belongs to Issue 1, as well as the six requirements for the premises (71b20-32) and the discussion of necessity in I.6 – but this is highly controversial.

⁸ See, for instance, Nussbaum 1978, p. 109-113; McKirahan 1992, p. 50-2; Barnes 1993, p. 131; Hankinson 2005, p. 38-40; Mignucci 2007, p. 175.

specific question – Question 2 – of whether an expert is allowed or not to employ in her explanatory story a premise which properly belongs to a different domain.⁹

Thus, the picture which results from what I have defended so far is this:

The expert studying why (e.g.) the progression of animals requires the bending of their limbs *is indeed allowed* to employ in her explanatory story some terms transported from geometry (Question 2 answered by “no”). But that same expert will never be able to attain the most appropriate explanation of a given explanandum within her domain through an explanatory factor transported from a different scientific domain (Question 1 answered by “yes”). Thus, the expert studying the progression of animals can employ geometrical premises. *But those geometrical premises alone will never deliver by themselves the most appropriate explanation of the explanandum at stake.* The geometrical premises would give some important contribution, and even a contribution that could never be eliminated – which is equivalent to saying that, *without those geometrical premises*, the most appropriate explanation will not be attained. However, this contribution – in terms of *sine quibus non* conditions – is different from being the most important explanatory factor for explaining the explanandum at stake.

Now, the kind-crossing issue is itself more complicated and Question 1 deserves further specifications, as I have already mentioned. It is helpful to assess the complications having the triadic framework of explanations in mind.¹⁰ In *any* domain, an explanandum should be parsed as a relation between an attribute and a subject and expressed in terms of a predicative sentence, which will occur as the conclusion of a demonstration. This parsing and this predicative expression are a reasoned part of Aristotle's philosophy of scientific investigation – and has nothing to do with naïve beliefs about the transparency of natural language or about the straightforward metaphysical structure of the world. The attribute in the formulation of the explanandum must spot what it is exactly that must be explained, and the subject must describe its appropriate subject in a way that is relevant to its bearing the attribute to be

⁹ There is no room to offer a full-fledged argument in favour of this point, which depends on a careful analysis of what is going on in *Posterior Analytics* I.2, I.4-10. I have provided an analysis of I.5-6 and I.9 elsewhere.

¹⁰ I mean: explanations for Aristotle are framed within a triadic structure in which the explanandum is a predicative relation, whereas the explanatory factor is a middle term: see 90a5-15, 99a4-5, 16-18. In what follows, my employment of the expressions “major terms”, “minor terms” and “middle terms” is only meant to capture this triadic structure of explanations without implying *anything* concerning the syllogistic formulation of demonstrations.

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explained. Thus, when it comes to explain thunder, the attribute to be explained must be described in a convenient way: the researcher must present the right description of the sort of noise that happens to the clouds; similarly, the researcher must present a relevant description of its subject, the clouds.¹¹

This is important for the classical examples involving the kind-crossing discussion. When it comes to explain (e.g.) an explanandum in the domain of optics, the explanandum has a complex nature: the subject itself (i.e., the minor term of the demonstrative syllogism) belongs to a given domain of things, namely, the natural things. Yet optics is concerned with only some attributes of those things – the attributes that relate to the way they reflect the light and are seen by us etc. Now, it happens that the specific sorts of attributes optics is concerned with can be appropriately explained by geometrical propositions. But this is so because the natural lines are considered by optics not *as natural* (or at least not with the whole set of features that make them natural) but only *qua extended* (or something like that). Thus, the *genus* of that explanandum is (so to speak) different from the *genus* to which its subject itself belong (taking *genus* as equivalent to *domain*). Natural lines belong to the domain of natural items. Yet the selected attributes which optics are concerned with belong rather to a different domain – the domain of items that depend on the relations between the primary items (points, lines, surfaces) studied in geometry¹².

Thus, it happens that, in the case of optics as well as in the case of other subordinated sciences, Aristotle's answer to Question 1 will be more complex. One sort of answer might be tempted to focus on the *domain* to which the basic items of the discipline belong: those basic items (the minor terms of the conclusions), namely, the natural lines, are natural items after all. But another answer would focus on the *nature of the attributes to be explained* (the major terms of the conclusions), which are the attributes that occur to the natural lines only *qua extended lines*.¹³ Aristotle favours the

¹¹ This is the gist of *Posterior Analytics* II.2, confirmed in II.8 (93a21ff.) and, more importantly, in II.16 (98b32-39) and II.17 (99a2-6). For discussion of the latter passages, see Zuppolini 2018.

¹² See *Posterior Analytics* 77b1-2, 79a3-13; *Metaphysics* 1078a5-21; *Physics* 194a7-12 (which asks for a more careful discussion – see next note). See Lennox 2008, p. 159-162; Frey (forth.), p. 17.

¹³ See previous note for references. The passage in *Metaphysics* 1078a14-16 is especially important for my point, for Aristotle explicitly says that optics is concerned with its subject not *qua sight* (ἢ ὄψις) but *qua lines* (ἢ γραμμαί). This claim does not clash with the *Physics* passage (194a-12). The context of each passage is different. In the *Physics* passage, Aristotle is stressing that optics takes mathematical lines not as mathematical but as applied to natural bodies, for otherwise (within the framework presupposed in the

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latter sort of answer: the *genus* of the object of optics (namely, the *genus* of the attributes of natural lines *qua extended*) is after all (and perhaps under some conditions) the same as the genus of the extended lines studied by the geometer.¹⁴

Thus, Aristotle's answer to Question 1 will still be negative: the fullest appropriate explanation of a given explanandum within the domain of optics could never have its most important explanatory factor transported from a different scientific domain.

However, this answer will only be correct because the genus of optics and the genus of geometry will be the same in some specific way – so that geometry will not qualify as a “different scientific domain” in the context of Question 1, even if geometry does qualify as a “different scientific domain” in the context of Question 2 (for the overall set of propositions, basic procedures, basic minor terms etc., are quite different in optics and in geometry). The genus of optics and the genus of geometry turn out to be the same *in a way* because the nature of the attributes to be explained in optics is basically the same as the nature of the attributes to be explained in geometry – and their appropriate explanatory factors are also the same. But, of course, the content of most propositions involved in optics will be different from the content of the propositions employed in geometry, for in optics the same attributes will be predicated of subjects that belong to a different kind, the natural lines, described as limits of natural bodies.¹⁵ But the difference in content of the predicative propositions involved in each science does not affect the *suggeneia* of the major and middle terms: the attributes to be explained (the major terms) in optics are the same as (a subset of) those in geometry, as well as the explanatory factors (the middle terms) in optics are the same as (a subset of) those in geometry. And *suggeneia* between major and middle terms is what really

passage) it would collapse into geometry after all. Aristotle is focusing on the minor terms of demonstrations, which make optics and geometry qualify as different disciplines within the scope of Question 2. However, in the *Metaphysics* passage, Aristotle's focus is on the major terms of demonstrations, as well as on the fact that they are demonstrated from the same principles as the major terms in geometry (for this language about “minor/major terms”, see note 10).

¹⁴ I take this to be the gist of 75b8-9: the genus of optics and the genus of geometry are the same *in a way* (πῆ). This answer also explains why Aristotle is most inclined to include optics within the branches of *mathematical* sciences, see *Physics* 194a7-8 (taking “μαθημάτων” as partitive), *Posterior Analytics* 79a18-20.

¹⁵ See *Physics* II.2 194a7-12 (cf. 193b31-33 to complete the picture; *APo* I.13, 79a8-10, I.27, 87a33-34).

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matters in the discussion of kind-crossing, which thus turns out to be within the scope of Question 1, not Question 2.¹⁶

3. Examining the passage from *Incessu Animalium*:

In chapter 9 of *Incessu Animalium*, Aristotle addresses the question about the bending of limbs in animal progression. There are several details which I cannot discuss in this paper – for instance, the question is originally framed within the domain of animals that make the progression *by means of specific parts*, namely, the limbs (cf. 711a20-21), but Aristotle then extends the same solution to the domain of animals that make the progression with no limbs at all (since they have no limbs – snakes, leeches, worms etc.) – without discussing or even spotting some potential difficulties of this extended solution.¹⁷ Besides, it is hard to believe that Aristotle has not spotted any difficulty in applying his paradigm explanation to other cases, such as flying and swimming (cf. 709b9ff.), specially because one of the points about the necessity of bending the limbs is to avoid falling down in the ground (709a16-17). However, I will not examine any of these difficulties. I will focus only on the specific passages in which Aristotle appeals to some geometrical stuff in order to explain why bending the limbs is necessary for the animal which makes the progression.

The specific thesis for which Aristotle is arguing is this:

T1 “now, if there were no bending, there would be no progression (or swimming or flying)” (708b26-27, my translation).¹⁸

The relevant sort of bending was previously defined in general terms in 708b22-24.¹⁹ Aristotle starts his argument in favour of the thesis by saying this:

¹⁶ *Suggeneia* between the major and the middle terms is at stake not only in 76a8-9, but also in 75a10-11, where I take “*ta akra*” to refer only to the major terms (the plural being distributive: *for every demonstration, its major term* etc.).

¹⁷ For discussion of these issues, see Frey (forth.), Section 4.

¹⁸ As Farquharson’s translation is problematic in the most important junctures (and Forster’s too), I will not use it systematically, and will add several modifications to it.

¹⁹ For discussion, see Frey (forth.), Section 2, p. 2-4.

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T2 “For since limbed creatures stand and take their weight alternately on one or other of the opposite legs, if one be thrust forward the other must of necessity be bent.” (708b27-30, Farquharson’s translation).

The animal makes the progression by means of limbs – or, even more particularly, legs (for Aristotle seems to have taken the human being as a paradigm to explain the general point²⁰). Besides, having said that the limbs are always even (never odd) in number (708a21-22, 708b20), Aristotle adds that the animal must distribute its weight alternately in each of its limbs while progressing. But, while the animal does this, the limb which stays at rest (and supports the weight) must bend. Why is this so? (Note that Aristotle has not exactly said that *only the resting limb* should bend while the projecting limb must move itself compactly etc. Aristotle is probably aware that both limbs do some bending. This is important because it suggests that some simplifications are allowed in the *description of particular cases* in order to focus on the core theoretical issue at stake, which is the strict necessity of the bending)²¹. In order to explain why the animal must bend its limbs while progressing, Aristotle states this (which is one of my key passages):

T3 “For the corresponding limbs are naturally of equal length, and the one which is under the weight must be a kind of perpendicular at right angles to the ground. But when a leg advances, there becomes an hypotenuse, equivalent to the size of the resting leg plus the line between [the moving leg and the ground]. Since the legs are equal, the leg at rest must bend (either at the knee or at some articulation)” (708b30-709a3, my translation, adapted from Farquharson’s and Forster’s).

Aristotle’s language is very compact here, but the employment of geometrical terminology makes his claim clear. Following Michael of Ephesus (153.24-32, 154.10-11; 155.28-30), both Farquharson and Forster have translated 709a1-2 as conveying

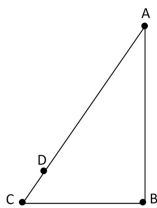
²⁰ See Michael of Ephesus 153.8-10.

²¹ For discussion of this issue, see Frey (forth.), Section 2.2.

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some reference to the Pythagorean theorem, although nothing like that is needed.²² The expression “δυναμένη” (709a1) is employed in the ordinary sense of “being equivalent to”.²³ The relevant issue hinges on the expression “καὶ τὴν μεταξύ”, which I take (with Louis, cf. 158, n2) to be referring to the line *DC* in *Figure* below, that is, the line covering the interval between the point where the moving leg ends (*D*) and the point which that leg must reach (*C*). This might be reasonably argued from the context: Aristotle is talking about *progression*, which is the change from one position to another. When one leg is projected forward and the other stays in rest, they suggest two sides of a right-angled triangle, of which the remaining side is the ground. Let *A* be the point from where both legs hang.²⁴ Let *B* be the point where the resting leg touches the ground. Let *C* be the point which the moving leg is going to reach. Now, it is clear that the hypotenuse of this triangle – line *AC* – is greater than the moving leg of the animal. Let *D* be the point where the moving leg ends, so that line *AD* is equal in length to line *AB* (for both legs are naturally equal in length). Since the moving leg must reach point *C* but its length only allows it to reach point *D*, something is needed: there is some help coming from the bending of the resting leg.



²² My point does not imply denying that the Pythagorean theorem is true about the leg's triangle. My point is that the truth of that theorem does not play any explanatory role in the story. For a different view, see Frey (forth.), p. 8, 18.

²³ Actually, it is puzzling to see Farquharson and Forster preferring the authority of Michael of Ephesus than Liddell & Scott's authority, who have given 709a19 as an example of the ordinary employment of “δύναμαι” in sense II.2: “to be equivalent to” (Liddell&Scott, p. 452). Now, it is true that “δυναμένη” in 709a1 and “δυνήσεται” 709a19 could have different forces, but this is not the case. Taking “δυναμένη” in 709a1 as conveying some relevant reference to the Pythagorean theorem has misled many readers.

²⁴ Or, as Michael of Ephesus has put it (153.17): “the place from where the dry excrement goes out”. However, it is hard to believe that, some lines before, Michael of Ephesus has said something so wrong as this (153.15-16): “when Socrates is standing still, there comes to be an isosceles triangle”. The comparison with an isosceles triangle, which appears in 709a22-23, refers to the moment when Socrates is walking and his projecting leg reaches the ground – like the Beatles in Abbey Road.

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A more sophisticated geometrical story might be fleshed out,²⁵ but the gist of Aristotle's argument is the following: given that (i) the animal must reach point C (since progression is moving from B to C), but (ii) the length of the moving limb (AD) is smaller than needed to reach point C , (iii) the animal must bend the resting limb (AB) in order to make (or help) the projecting limb to reach C . And, of course, the animal must bend the resting limb *forward*, namely, in the direction of point C (711a27-29). By bending its resting limb, the animal allows its projecting limb (AD) to become greater than the resting limb (AB) while being equal to it – in other words, the animal allows its projecting limb (AD) to 'become equal' to the hypotenuse (AC) and to reach point C . This is what Aristotle explains more carefully in this pleasantful passage:

T4 "For if one leg is at right angles to the ground and the other is advanced, the latter will be greater while being equal. For it will be equivalent to the stationary leg and also equivalent to the hypotenuse" (709a17-20, adapted from Farquharson's and Forster's).

Let me spell this out. When saying that "the latter will be greater", Aristotle means that the advanced leg will be greater than the resting leg – he means that, as the advanced leg will reach point C , it will somehow be equal to AC , which is greater than AB . When he adds the remark "while being equal", Aristotle means that, although it reaches point C and becomes somehow equivalent to AC , the advanced leg (AD) is still equal in length to the resting leg (AB). When Aristotle proceeds and says that the advanced leg "will be equivalent to the stationary leg", he is remarking that both legs are still equal in length, i.e., AD is equal in length to AB (and this answers to "while being equal" in the previous sentence). But in adding that the advanced leg "will be also equivalent to the hypotenuse" (which answers to "will be greater" in the previous sentence), Aristotle is stressing that, while remaining the same in length as AB , the advanced leg has become equivalent to the hypotenuse AC and has reached point C – which means that the animal has made a progressing step.²⁶ And the "trick" – to make AD greater than AB while remaining equal to it – has been made by the bending of the

²⁵ See Frey (forth.), Section 2.1.

²⁶ Michael of Ephesus (155.29-30) wrongly says that "ὑποτείνουσιν" in 709a20 changes its referent: it designates the basis of the right-angled triangle. But this makes no sense.

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resting limb.²⁷ (Actually, as Aristotle will make clearer further on, the “trick” is the result of the bending of the resting limb together with other factors, such as the animal leaning forward as a whole etc., cf. 709a21).

4. The explanatory role of the several factors in the explanation:

What this sort of explanation amounts to? Premise (ii) in the argument sketched above can be explained with the aid of the geometry. That the lines referred to by *AD* and *AB* are equal to each other is something that comes from the zoologist's consideration that corresponding limbs are normally of the same length (cf. 708b30-31). But that line *AD* is smaller than *AC* is something that only depends from the specific principles of geometry.

Thus, we see principles from geometry doing *some* explanatory work here. This fact raises the important exegetical question: did Aristotle in *Incessu Animalium* depart from his *Posterior Analytics* theory about kind-crossing? Is Aristotle now allowing the zoologist to perform a once forbidden kind-crossing by employing geometrical premises? But this is Question 2. There is also Question 1: do the geometrical propositions involved in Aristotle's reasoning deliver the most important explanatory factor to explain the explanandum at stake, namely, why animals bend their limbs in progressing?

My answer to Question 2 is straightforward: the discussion of kind-crossing in the *Posterior Analytics* has never forbidden the situation found in the *Incessu Animalium*, namely, the employment of premises from a given discipline (geometry) as an *auxiliary tool* for explaining an explanandum in a different discipline (zoology).²⁸

As it comes to Question 1, I remark that the geometrical propositions involved in the *Incessu Animalium* passage are *not* playing the most important role in explaining the explanandum at stake – why animals bend their limbs in progressing. The proposition

²⁷ The commentary of Michael of Ephesus is convoluted, but at 153.32-34 he seems to have grasped the essential: “the right leg was equal to the left one before moving: *it is clear, therefore, that the left leg has bent in the knee*”.

²⁸ Thus, medicine is not exactly treated as a subordinate science (such as optics is taken to be subordinated to geometry), but nevertheless can employ a geometrical premise when dealing with the explanation of some explananda (cf. 79a13-16). It is no surprise, therefore, to find Aristotle developing what Hussey (1991) has called mathematical physics: what he has said in *Posterior Analytics* has never forbidden it. See also Frey (forth.)

that line AD is smaller than AC is applied to describe an important feature of reality to which even animals and their movements are constrained. However, there is more in the explanation. Let us consider it again (with some rephrasements), having the figure in mind:

- (i) the animal must reach point C ;
- (ii) the length of the projecting limb (AD) is smaller than needed to reach point C (for AD is smaller than AC);
- (iii) the length of the projecting limb (AD) must become greater than AD (and AB) to reach point C ;
- (iv) the animal must bend the resting limb (AB) in order to make (or help) the projecting limb to reach C (which is for it to become greater than AB while remaining equal to it).

There is geometry – explicit or underlying – only in steps (ii) and (iii). But two remarks are in order here. Firstly, step (i) introduces important information about the explanandum: its subject is an animal – i.e., a living being with such and such features and an articulated structure that is suited to the performance of its activities. Secondly, there are still some gaps in the passage from (i)-(ii)-(iii) to the conclusion (iv). Steps (ii) and (iii) stems from geometry and capture important conditions of reality that constrain (besides other things) the progression of the animals: the fact that lines AD and AB are equal to each other but smaller than AC ; the fact that there is a distance between C and D . But it does not follow automatically from these geometrical facts that the animal must bend its limbs, nor that different animals must bend their limbs in the way they actually bend. Further on, Aristotle says that, if there were no bending, “the animal would either fall or not advance at all when its position forms an angle smaller than the right one” (709a16-17). Now, maybe the falling at stake is equivalent to loosing its erect position and collapsing on the floor (for Aristotle seems to have taken the human being as his paradigm). However, it is plausible to take the falling as equivalent to “falling (in a clumsy way) on its foot”. In that case, animals will clumsily progress as some grotesque robots in science fiction movies – each step, a sort of falling down in the

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ground.²⁹ But why things are not this way? And why different kinds of animals bend their limbs in different ways?

These two questions are related. Progression is not a clumsy falling on its feet (from *D* to *C*) because nature does nothing in vain – which is one of the ultimate principles evoked by Aristotle at the beginning of the *Incessu Animalium*:

T5: “Nature does nothing in vain, but always the best possible in each kind of living being by reference to its essential constitution.” (704b15-18, Farquharason’s translation, modified).³⁰

This principle has nothing to do with a presumed Nature governing the world providentially etc. The word “nature” (φύσις), in several occurrences in the biological treatises, refers to something concrete: it is the structure of the animal as a whole, which corresponds to its essence (cf. 712b23, 714a8) – an articulated structure in which several parts have grown together and became suited to each other to perform the living activities that characterize the animal.³¹ This force of “*phusis*” is closely connected with the employment of the verb “*phuo*” and its cognates to describe how a living being grows articulatedly – in such a way that different parts become suited to each other. This employment of “*phusis*” is on the root of Aristotle’s teleological principle. The proposition that “nature does nothing in vain”, far from appealing to some Mother Nature outside natural things, is a generalisation of observed facts: living beings develop themselves as articulated structures in which different parts constrain each other so that the whole results in a well-functioning system.

Thus, why do animals bend their limbs in progressing, instead of falling clumsy on their feet in each step? The geometrical premises involved in steps (ii)-(iii) above play an important part in the fullest appropriate explanation of this fact. But, ultimately, the most important explanatory factor to explain why animals bend their limbs does not come from geometry: it comes instead from one of the most important principles of

²⁹ Remember the first scene in which the robot CASE appears in Nolan’s *Interstellar* – just CASE, which later will prove a perfection in progression.

³⁰ See also *Physics* 198b8-9, where teleology is evoked with the same flavour: nature aims at what is the best for the animal, within a given range of possibilities; for a careful discussion, see Lennox 2001b, p. 206-7, 216-8; Judson 2015, p. 183.

³¹ For careful discussion about this issue, see Lennox 2001a, especially p. 188-9, 195.

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natural science, namely, that the nature of each animal is a complex articulated structure in which several parts have grown together and became suited to each other to perform the living activities in a successful way. This has nothing to do with geometry (the geometer would probably be more happy with the sci-fi robotwise progressing). The geometrical propositions involved in steps (ii) and (iii) of the explanatory story are only setting the general conditions that constrain any progression (or actually any locomotion). But the most important explanatory factor in that explanation stems from the nature of the animal or, more precisely, from the fact that the animal is a natural living being, which is structured to perform its activities in the way that is the best possible within its essential constitution. Falling down on its feet is not as good as bending its limbs and progressing smoothly.

5. Conclusion:

Thus, *Incessu Animalium* is in agreement with the *Posterior Analytics*:

(1) Aristotle still maintains the claim that the fullest appropriate explanation of a given explanandum within a particular domain (e.g., the bending of the limbs in animal progression) could never have its most important explanatory factor transported from a different scientific domain (e.g., geometry).

(2) But Aristotle has never claimed that a given scientific domain could never employ premises which properly belongs to a different scientific domain. What is forbidden is to employ such premises as if they were the most important explanatory factor to attain the fullest appropriate explanation of a given explanandum.

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