Anuário do Instituto de Geociências
Universidade Federal do Rio de Janeiro
https://revistas.ufrj.br/index.php/aigeo/

ISSN 0101-9759
e-ISSN 1982-3908

Strategy for Connecting to the IHRF: Case Study for the Tide Gauge of Cananeia–SP

Estratégia para Conexão ao IHRF: Estudo de Caso para o Marégrafo de Cananeia – SP

Fabio Luiz Albarici[1], Gabriel do Nascimento Guimarães[2],
Marcelo Carvalho Santos[3] & Jorge Luiz Alves Trabanco[4]

[1]Instituto Federal do Sul de Minas Gerais, Campus Inconfidentes, Setor de Agrimensura e Cartografia, Inconfidentes, MG, Brasil
[2]Universidade Federal de Uberlândia, Campus Monte Carmelo, Instituto de Geografia, Monte Carmelo, MG, Brasil
[3]University of New Brunswick, Department of Geodesy and Geomatics Engineering, New Brunswick, Canada
[4]Universidade Estadual de Campinas, Faculdade de Engenharia Civil Arquitetura e Urbanismo, Departamento de Transportes, Campinas, SP, Brasil
E-mails: fabio.albarici@ifsuldeminas.edu.br; gabriel@ufu.br; msantos@unb.ca; trabanco@fec.unicamp.br

Abstract

In July 2018, IBGE launched the new heights of the Brazilian Geodetic System (BGS), the normal height, which has associated gravity. These new heights are replacing the old normal-orthometric ones, in which there was only the non-parallelism correction. The IBGE informs that the values farther from the origin, have less accuracy. This lower accuracy may interfere in the future, the connection of the local tide gauges to IHRF (International Reference Frame Height). Thus, this paper proposes the integration of the local tide gauge of Cananeia-SP to the IHRF. In order to validate the methodology, the normal, Helmert, and rigorous orthometric heights using two distinct references: the Imbituba-SC tide gauge, as the origin of the BGS and the Cananeia-SP tide gauge, as a local tide gauge to be integrated into the IHRF. Calculating the three heights through these two origins, we analyzed the discrepancies in comparison to the heights calculated by IBGE. Numerical tests indicate that there was an improvement in terms of a mean and standard deviation when using the Cananeia gauge as origin in the calculation of normal, Helmert, and rigorous heights. In the congruence analysis, the calculations indicate that the highest standard deviation is presented when using IBGE normal heights. Thus, we have a new origin that is reliable and functional, can be integrated with the IHRF, where the Helmert and rigorous orthometric heights have the best statistical results.

Keywords: Helmert orthometric height; Normal height; Local reference system

Resumo

Em julho de 2018, o IBGE lançou as novas altitudes do Sistema Geodésico Brasileiro (SGB), a altitude normal, tendo gravidade associada. Estas novas altitudes estão substituindo as antigas altitudes normal-ortométrica, onde apenas havia a correção de não-paralelismo. Em seu relatório o IBGE informa que os valores mais distantes da origem, possuem menor acurácia. Essa menor acurácia pode interferir, no futuro, a conexão dos marégrafos locais ao IHRF (*International Height Reference Frame*). Dessa forma, este artigo propõem a integração do marégrafo local de Cananeia-SP ao IHRF. Para validar a metodologia calculou-se as altitudes normal, ortométrica de Helmert e rigorosa utilizando-se de duas origens distintas: o marégrafo de Imbituba-SC, como origem do SGB e o marégrafo de Cananeia-SP, como um marégrafo local a ser integrado ao IHRF. Calculadas as três altitudes através destas duas origens, analisa-se as discrepâncias confrontando-se com as altitudes calculadas pelo IBGE. Testes numéricos indicam que houve uma melhora, em termos de média e desvio-padrão quando utiliza-se o marégrafo de Cananeia como origem para o cálculo das altitudes normal, Helmert e rigorosa. Na análise de congruência os cálculos indicam que se tem o maior desvio-padrão quando se utiliza da altitude normal do IBGE. Assim, tem-se um marégrafo local confiável e funcional, podendo ser integrado ao IHRF, onde as altitudes ortométricas de Helmert e Rigorosa possuem os melhores resultados estatísticos.

Palavras-chave: Altitude ortométrica de Helmert; Altitude normal; Sistema de referência local

1 Introduction

A question normally discussed in physical geodesy surveys and related areas is: A purely physical height system can be accurately performed? This question arises since a purely physical height system must necessarily be related to the Earth's gravitational field (Kingdon, 2012). In the last 40 years, many researchers have tried to answer this question by working on the accuracy of the heights, about 1 cm or better (Ellmann & Vaníček 2007; Roman & Smith 2002; Sansò & Rummel 1997; Vaníček & Martinec 1994).

The most intuitive system is the orthometric height, which takes the geoid surface as its *datum* and defines the height of a point as the length of the plumb line extending from that point to the surface (Heiskanen & Moritz 1967). It is known that the geoidal surface is a reference only for the orthometric, dynamics, and geopotential heights (Vaníček & Krakiwsky 1986).

However, the normal heights (Molodensky 1945), referring to the physical surface, offer the distance from the point of interest to the quasi-geoid (Molodensky et al. 1962), which is a surface similar to the geoid with a geometrical meaning, and not an equipotential surface of the Earth's real gravity field without any physical significance (Heiskanen & Moritz 1967).

Recently, Brazil updated its height system, moving from normal-orthometric to normal height (IBGE 2018). However, despite being the first step in the update of the Brazilian altimetry system, what was the technical/theoretical motivation for choosing the normal height? One possible answer is in the official SIRGAS recommendation on the physical heights where normal height should be used; however, if some countries wish to adopt orthometric heights, the new vertical reference system for SIRGAS is defined in terms of potential quantities. In this way, in the realization of the system, each country can introduce the type of physical height preferred (SIRGAS 2020). Once the geopotential number was calculated, the IBGE could be tested for the performance of a classical height system (Helmert orthometric) which is the calculated measure of the difference between the geoid heights (N_{Grav}), derived from a gravimetric geoidal model, concerning the geoid heights (N_{Lev}), derived from the level reference leveling (GNSS/leveling). These measures are also called congruence and are a metric for assessing the quality of the classic height system. A second height, the rigorous orthometric (see Albarici et al. 2019; Santos et al. 2006; Tenzer et al. 2005) could also be tested for its congruence. Thus, we would have the analysis of which height best adapts to the Brazilian territory.

The objective of this paper is to integrate the tide gauge of Cananeia-SP into the IHRF. For this, it was calculated the normal (H^N), Helmert (H^O) and Rigorous Orthometric (H^{RO}) heights using two distinct reference points, one being the origin of the Brazilian height system, belonging to the Brazilian Geodetic System (BGS), the Imbituba – SC tide gauge, and as a local tide gauge to be future integrated into the IHRF, the Cananeia tide gauge. It should be noted that the Santana tide gauge (in the Amapá state) is also an origin point of the height system, but in this work, it was not used. To do so, we obtained information about several level references (RRNN) distributed in eight Brazilian states, São Paulo, Minas Gerais, Rio de Janeiro, Paraná, Rio Grande do Sul, Santa Catarina, Mato Grosso do Sul, and Espírito Santo. For the validation and analysis of the heights obtained, the separations between normal heights, Helmert, and rigorous orthometric calculated by the two references and the normal heights available in the Geodetic Database (BDG) of IBGE (Brazilian Institute of Geography and Statistics) will be verified.

2 Area, Materials and Methods

2.1 Theory Review

The primary component of any vertical reference system for physical heights is an equipotential surface of the Earth's gravity field, which represents what is commonly called the vertical *datum*. Regardless of the particular type of physical height inserted into a vertical reference system, the underlying vertical reference point defines an unequivocal level of zero height about which vertical land positions can be obtained by geodesic leveling techniques (Heiskanen & Moritz 1967). However, it should be borne in mind that a geometric interpretation of such vertical positions may not always be feasible (that is, dynamic heights) or may be associated not with the vertical *datum* but with other non-equipotential auxiliary reference surfaces (that is, normal heights). Only the use of orthometric heights theoretically allows a simple geometric relationship between vertical physical positions and their inherent vertical *datum* (Heiskanen & Moritz 1967). However, the role of a vertical *datum* is equally important for all types of physical heights that quantify absolute vertical positions in terms of geopotential differences relative to a conventional zero height level (Kotsakis et al. 2012).

In July 2015, the International Geodesy Association launched Resolution n° 1 for the definition and realization of an International Height Reference System (IHRS). According to this resolution, the vertical coordinates (C)

are potential differences referring to the equipotential surface of the Earth's gravity field, performed by the conventional value $W_0 = 62636853.4\ m^2/s^2$ and the value of the potential of severity at the point of interest (W_P) (IAG, 2015). The main objective of the realization of the IHRS is the integration of the height systems existing in the globe; this means that the existing vertical coordinates must refer to the same surface realized by the conventional W_O (Sánchez & Sideris 2017).

The main aspects to be considered in relation to the establishment of the IHRF refer to the need to integrate the existing Brazilian Geodetic System (BSG) into the new concept brought by the IHRS. These new concepts involving the modernization of the BGS meet some protocols (Dalazona & Freitas 2020): 1) Definition of strategies for the realization of existing networks through physical heights; 2) Integration in geopotential space to form the Vertical Reference; 3) Approaches to referencing the Vertical Reference to the IHRS W_O value. The relationship between the IHRF and the Vertical Reference can be established when the national network is realized based on geopotential numbers.

The geopotential number is given as the difference of the potential of real gravity (W_O) in the geoid and of the relative gravity potential (W_P) and terrestrial surface, so that (Kingdon et al. 2005; Sánchez 2013):

$$W_o - W_P = C \cong \sum_0^P g\,\delta_n \quad \textbf{(1)}$$

where: δ_n is the level difference between two points. If the continuous points are observed between two points, then the sum can be replaced by an integral:

$$W_0 - W_P = C \cong \int_0^P g\,\delta_n \quad \textbf{(2)}$$

The fundamental equation for the definition of Helmert orthometric height (H^O) can be described by equation 3 (Heiskanen & Moritz 1967):

$$H^o = \frac{C}{g_m} \quad \textbf{(3)}$$

Mathematically, the orthometric height (H^O) defined by the geopotential number (C) divided by the mean gravity (g_m) along the plumb line between the point of interest on the land surface and the geoidal surface (Kingdon et al. 2005).

The mean gravity can be approximated by the average normal gravity that results in the normal height; when the *Poincaré-Pray* model is used, the Helmert orthometric height is obtained. According to the *Poincaré-Pray* theory, the mean value is caused by the Bouguer Shell correction and the free-air gravity gradient at the point of interest, assuming that the mass density of the plate is constant and equal to $\rho_0 = 2670\,kg\,m^{-3}$ (Foroughi et al. 2017; Santos et al. 2006).

$$g_m = g\left(r_t\Omega\right) - \left(\frac{1}{2}\frac{\delta\gamma}{\delta h} + 2\pi G\rho_0\right)H^o \quad \textbf{(4)}$$

which $\delta\gamma / \delta h$ is the vertical gradient of the normal gravity at the earth's surface G is the Newtonian gravitational constant, Ω represents the geocentric spherical coordinates (λ: longitude φ: latitude) of the point of interest, and r_t is the point of the beam on the ground surface ($r_t \approx R + H^o$).

For normal height systems, the mean real gravity inside the masses is replaced by the average normal gravity between the reference ellipsoid and the teluroid (Heiskanen & Moritz 1967; Tenzer et al. 2005). The normal gravitational field is defined by an ellipsoid that best fits the Earth and contains the Earth's total mass (including its atmosphere), as well as an equivalent constant angular velocity (Moritz 1980). The normal height (H^N), replaces g_m in equation (3) (which was measured along the plumb line) by normal gravity, γ, measured along the normal of the reference ellipsoid (Jekeli 2000).

Normal height is an approach to orthometric heights, describing the heights on a fictitious surface, the quasi-geoid. Normal height requires the use of an amount known as height anomaly (Vaníček et al. 2003). The normal height of Molodensky can be determined from equation 5 (Hofmann-Wellenhof & Moritz 2006):

$$H^N = \frac{C}{\gamma}\left[1 + \left(1 + f + m - 2fsin^2_{\varphi}\right)\frac{C}{a\gamma} + \left(\frac{C}{a\gamma}\right)^2\right] \quad \textbf{(5)}$$

where γ is normal gravity, f is the geometrical flattening of the ellipsoid, m is the geodesic parameter (ratio of gravitational and centrifugal forces in the equator) and φ is the geodesic latitude at the point.

The corrections of conversion of Helmert orthometric heights at rigorous orthometric heights are verified in Tenzer et al. (2005), Santos et al. (2006), Foroughi et al. (2017) and Albarici et al. (2018). For the calculation of the rigorous orthometric height, there is a need to apply some corrections, whose difficulty is in the calculation of the average gravity. The most rigorous gravity equation (equation 6) is used for the calculation of gravity:

$$\varepsilon_{\bar{g}}(\Omega) = \bar{g}(\Omega) - \bar{g}^H(\Omega) = \left[\bar{\gamma}(\Omega) - \gamma(r_t,\Omega) + {}^1/_2\frac{\delta\gamma}{\delta h}H^o(\Omega)\right] + \left[\bar{g}_B^T(\Omega) - g_B^T(r_t,\Omega) + 2\pi G\rho_0 H^o(\Omega)\right] + \left[\overline{\delta g}^{NT}(\Omega) - \delta_g^{NT}(r_t,\Omega)\right] + \left[\bar{g}_R^T(\Omega) - g_R^T(r_t,\Omega)\right] + \left[\bar{g}^{\delta\rho}(\Omega) - g^{\delta\rho}(r_t,\Omega)\right] \quad \textbf{(6)}$$

where: G is Newton's gravitational constant, Ω represents the geocentric spherical coordinates (ϕ,λ), $r_t(\Omega)$ is the geocentric radius of the Earth, R is the inner radius of the shell, $H^o(\Omega)$ is the orthometric height, $\overline{\delta g}^{NT}$ is the mean geoid-generated gravity disturbance. $\delta_g^{NT}(r_t,\Omega)$ is the gravity generated by the masses within the geoid, $\bar{g}_R^T(\Omega)$ is the mean gravitation value generated by the roughness of the terrain, $g_R^T(r_t,\Omega)$ is the gravitation generated by the terrain roughness, $g^{\delta_\rho}(r_t,\Omega)$ is the effect on gravitation due to lateral mass and density variations within the topography regarding the reference value of $\rho_0 = 2670\,kg\,m^{-3}$, $\varepsilon_{\bar{g}}(\Omega)$ correction to Helmert's mean gravity.

In equation 6, in the right part of the equality we have: in the first term, the correction for the Second-order correction for normal gravity (ε_H^{γ}); the second term, the Bouguer Shell effects (ε_H^{B}); the third, fourth and fifth terms have, respectively, the correction of non-topography (ε_H^{NT}), the correction of the terrain/rugosity (ε_H^{R}) and the lateral correction of variable topographic density ($\varepsilon_H^{\delta_\rho}$). Thus, Helmert orthometric heights are converted into rigorous orthometric height through equation 7 (Foroughi et al. 2017; Santos et al. 2006):

$$\varepsilon_{H^o} = -\frac{H^o(\Omega)}{\bar{g}(\Omega)}\left(\varepsilon_H^{\gamma} + \varepsilon_H^{B} + \varepsilon_H^{NT} + \varepsilon_H^{R} + \varepsilon_H^{\delta_\rho}\right) \quad \textbf{(7)}$$

where, $\varepsilon_{H^o}(\Omega)$ is the correction of the Helmert orthometric height to convert it to the rigorous orthometric height.

It is known that the geoidal (N) and quasi-geoidal heights (ζ) (or height anomaly) can be compared through the difference between heights (H^o and H^N) and ellipsoidal (h) elevations obtained from GNSS observations. Geoid and quasi-geoid heights are calculated from gravimetric observations. The normal (H^N) and orthometric (H^o) height, in turn, are calculated from GNSS/leveling observations. The classical conception (due to Stokes) uses geoidal heights and orthometric height or rigorous orthometric height (equation 3 and equation 7), whereas the modern conception (due to Molodensky) uses the quasi-geoidal height and normal height (Foroughi et al. 2017; Heiskanen & Moritz 1967).

From the difference between the heights one can evaluate the separation between geoid and quasi-geoid heights:

$$N - \zeta = H^o - H^N \quad \textbf{(8)}$$

The most common method of assessing the congruence of a geoidal model (N_{Grav}) and orthometric heights are to compare the geoidal heights (N_{Lev}) with the difference between the orthometric heights provided by the leveled and ellipsoidal heights h provided by GNSS.

2.2 Study Area

In this experiment we used all the available points of the state of São Paulo, plus the points of the neighboring states - Minas Gerais, Rio de Janeiro, Paraná, Rio Grande do Sul, Santa Catarina, Mato Grosso do Sul and Espírito Santo - obtained in the Brazilian National Spatial Data Infrastructure (INDE) (available at https://inde.gov.br/) portal and the leveling data of the state of São Paulo made available by IBGE (http://www.bdg.ibge.gov.br/appbdg/), where only the points containing the necessary information (latitude, longitude, geometric height from GNSS observations, height from leveling and ground gravimetric observations) to perform the normal, Helmert orthometric and orthometric rigorous height calculations. Figure 1 shows the distribution of the points (1130) used in the calculations:

The geopotential number was calculated by equation 1, so the normal heights (H^N) were obtained by equation 5 and Helmert orthometric (H^o) was obtained by equation 3. Thus, these two heights were calculated using two different origins, the first being the official Brazilian vertical *datum*, the Imbituba tide gauge, and the other origin was the Cananeia tide gauge, where it is the new local vertical reference point. Therefore, we have the normal and Helmert heights obtained by two different references. The last height to be calculated is the rigorous orthometric, where the previously neglected corrections are applied to the Helmert orthometric heights. The terms of each correction contained in equation 6 were calculated, and equation 7 was used to obtain the total correction.

Considering that IBGE has the best determined normal height (even having values interpolated for gravity in some points, which can introduce some error because of the truncation in the interpolation), the comparative analyzes were carried out.

These heights were analyzed and compared to the discrepancies/differences in five parts: (1) Discrepancy between the normal height of IBGE and the normal height calculated with origin in two tide gauges; (2) Geoid/quasi-geoid separation between normal heights of IBGE, Helmert orthometric and rigorous orthometric calculated in Imbituba and Cananeia origin; (3) Geoid/quasi-geoid separation between normal, Helmert orthometric and rigorous orthometric heights, calculated in Imbituba and Cananeia; (4) Comparison of geopotential numbers calculated by IBGE and the ones calculated with origin in two tide gauges; (5) Difference between the geoid heights (N_{Grav}) derived from a gravimetric geoid model to the ones (N_{Niv}) derived from the leveling of the references (GNSS/leveling).

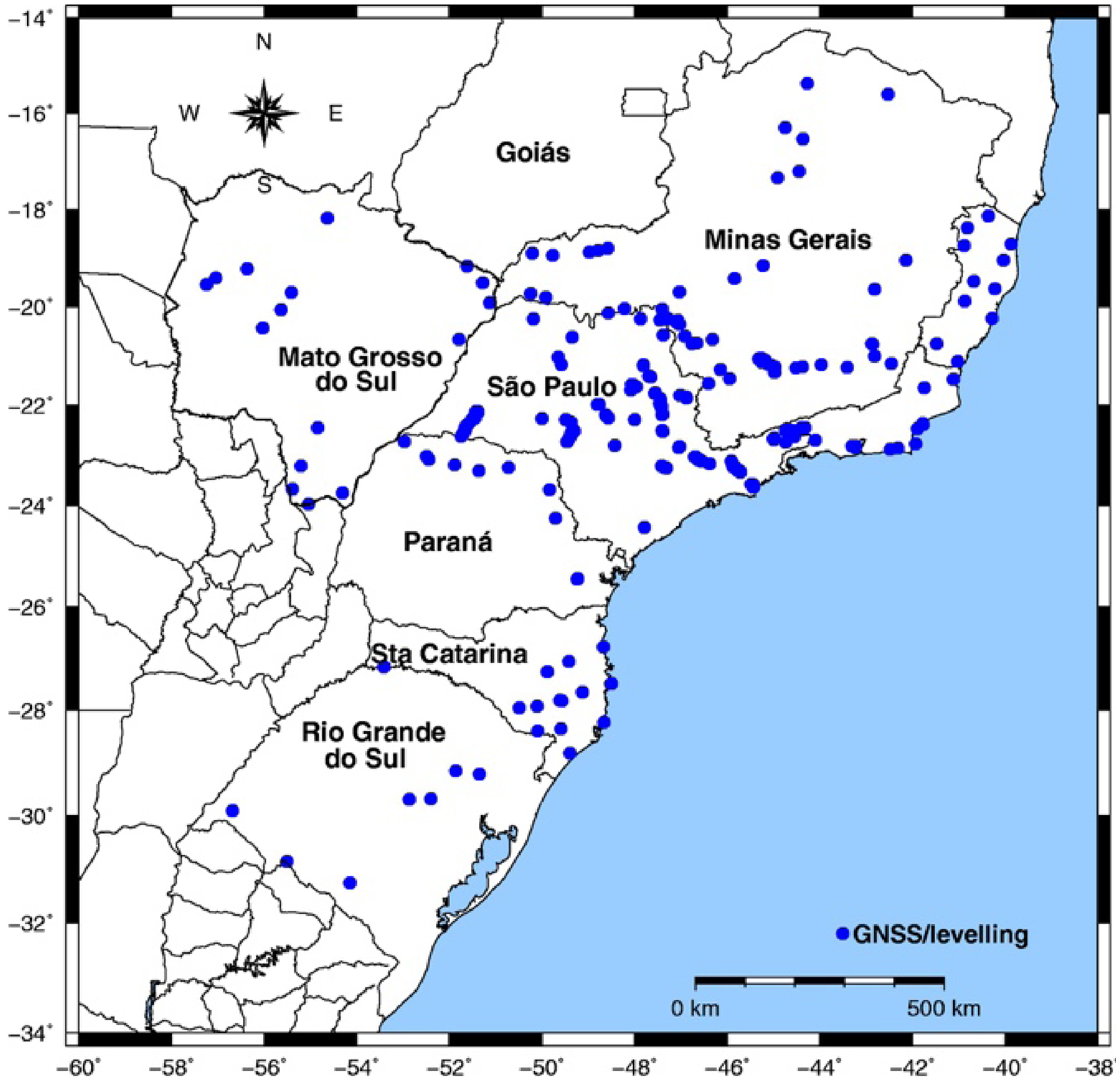

Figure 1 Geographical distribution of points

Since the Brazilian geoidal model is MAPGEO2015, and there is not a model for height anomaly, the best way to evaluate congruence was to use the global EIGEN-6C4 geopotential model (*European Improved Gravity Model the Earth by New Techniques*). For further details of these models see Förste et al. (2014) and Gilardoni et al. (2016). This model calculates the geoid (*N*) and height anomaly (ζ) and the results are made available by the ICGEM (*International Center for Global Earth Models*) by the website http://icgem.gfz-potsdam.de/calc. To calculate the values of this model of geoid and height anomaly, the following parameters were used: GRS80 reference system, *tide free* correction, and the terms for the coefficients of the spherical harmonic functions up to degree and order of 2190.

3 Results and Discussion

IBGE has updated its geodetic database to the normal heights, but for many points, the gravity values are not presented, because they are interpolated values; this can be a problem when comparing the values of the normal and Helmert heights calculated with the values of IBGE's BDG, perhaps having very different values in some points. Thus, it was chosen to filter the *outlier* very distant from the average. After filtration, approximately 4% of the total points were eliminated. Statistical analyses were performed between the normal, Helmert, and rigorous heights obtained by the Imbituba and Cananeia tide gauges and the normal heights available in the BDG by IBGE.

(1) The first analysis was to verify the discrepancy between the normal heights of IBGE and those calculated from Imbituba and Cananeia. Table 1 contemplates all comparative values, and Figure 2 shows the values obtained point to point.

It should be noted that the difference values for Imbituba and Cananeia are: minimum ~26 cm and ~ -28 cm and maximum ~ 30 cm and ~ 28 cm, respectively. What is noticeable is that the mean and standard deviation are lower for the heights calculated by the Cananeia tide gauge.

(2) The second analysis involves the normal heights of IBGE and the Helmert and rigorous orthometric heights calculated from the Imbituba and Cananeia tide; therefore, the geoid/quasi-geoid separation will be analyzed. The statistics applied for the minimum, maximum, mean, and standard deviation values also indicate an improvement in the mean and standard deviation when using the Cananeia tide gauge as the origin of the system according to Table 2. Figures 3 and 4 present these values, where it can be verified that point-to-point behavior is similar since the Helmert and rigorous orthometric height are close.

Table 1 Discrepancy between the normal height of IBGE and the normal height calculated from the two tide gauges.

Statistics	Min (cm)	Max (cm)	Mean (cm)	Std (±) (cm)
$\mathbf{H}^{N}_{IBGE} - \mathbf{H}^{N}_{IMBITUBA}$	-26.352	30.734	13.855	7.693
$\mathbf{H}^{N}_{IBGE} - \mathbf{H}^{N}_{CANANEIA}$	-28.252	28.836	13.576	7.159

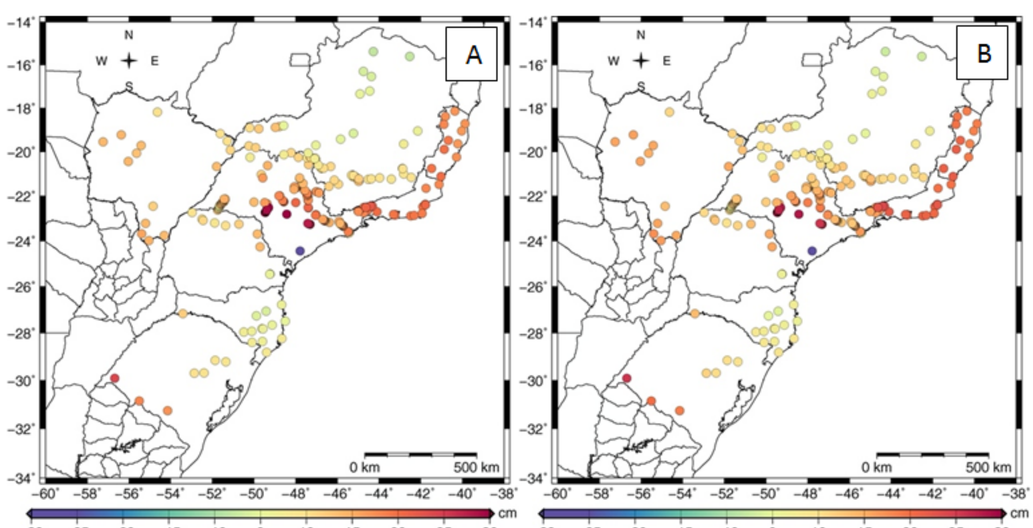

Figure 2 Behavior of the differences: A. $H^{N}_{IBGE} - H^{N}_{IMBITUBA}$; B. $H^{N}_{IBGE} - H^{N}_{CANANEIA}$

Table 2 Geoid/quasi-geoid separation between Normal of IBGE, Helmert and Rigorous heights calculated from Imbituba and Cananeia.

Statistics	Min (cm)	Max (cm)	Mean (cm)	Std $(\pm)$ (cm)
$\mathbf{H^{o}_{IMBITUBA} - H^{N}_{IBGE}}$	-25.284	26.561	-8.646	8.420
$\mathbf{H^{o}_{CANANEIA} - H^{N}_{IBGE}}$	-25.500	28.460	-8.367	8.094
$\mathbf{H^{Ro}_{IMBITUBA} - H^{N}_{IBGE}}$	-26.835	26.534	-10.464	8.136
$\mathbf{H^{Ro}_{CANANEIA} - H^{N}_{IBGE}}$	-25.528	28.433	-10.171	7.874

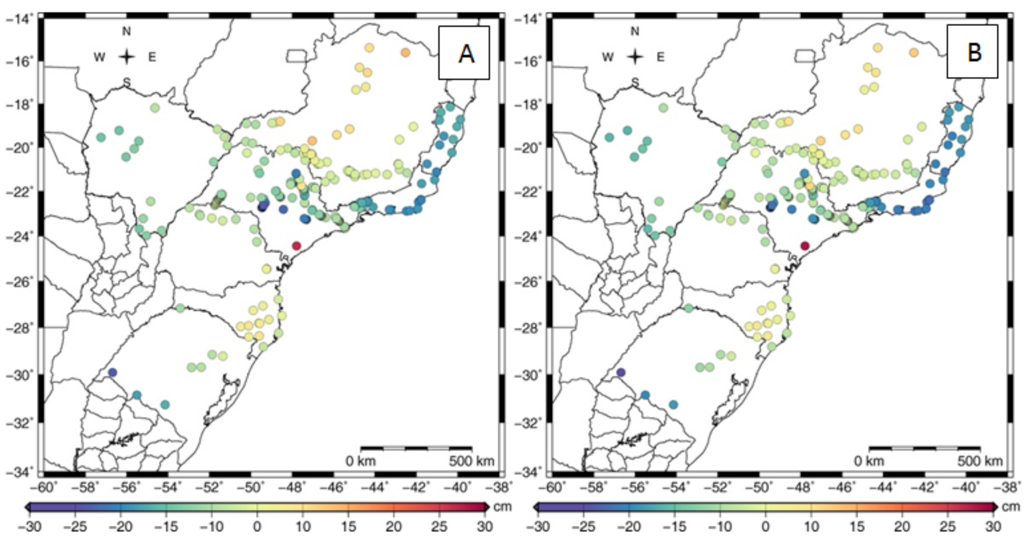

Figure 3 Separation between geoid and quasi-geoid: A. $H^{o}_{IMBITUBA} - H^{N}_{IBGE}$; B $H^{o}_{CANANEIA} - H^{N}_{IBGE}$.

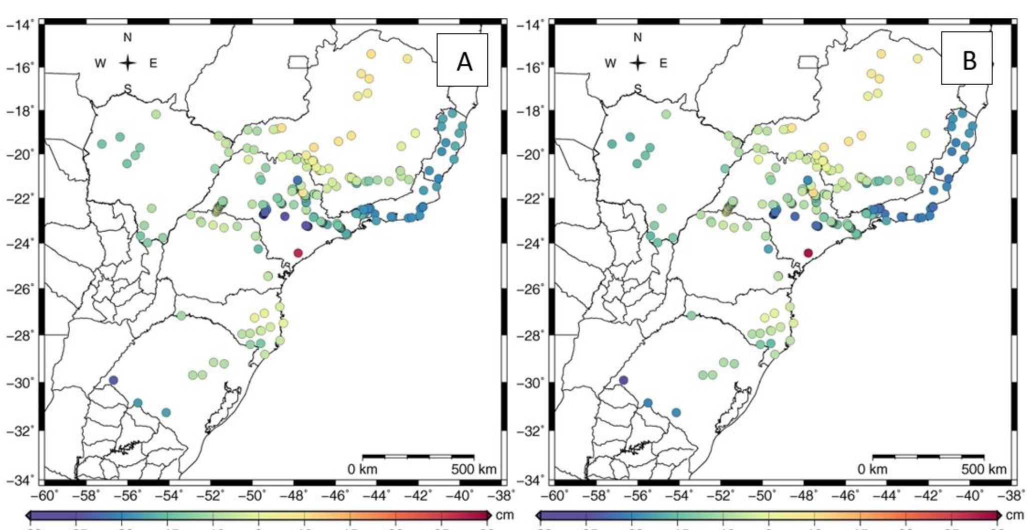

Figure 4 Separation between geoid and quasi-geoid: A. $H^{Ro}_{IMBITUBA} - H^{N}_{IBGE}$; B $H^{Ro}_{CANANEIA} - H^{N}_{IBGE}$.

(3) The third analysis made the geoid/quasi-geoid separation, but only between the calculated heights. In this case, it is verified that the statistical values of this separation are much smaller concerning the values of the analysis in Table 2, with values in the mean and standard deviation of ~ 5.2 cm and ~ 3.6 cm for Helmert height and ~ 3.3 cm and ~ 4.0 cm for the rigorous height, respectively. Table 3 shows the values obtained. Note that the statistical values are close when using the Helmert and rigorous orthometric heights, with a difference in the mean of ~ 1.7 cm and in the standard deviation of ~ 0.3 cm, so it is reinforced that the methodology applied in the tide gauge of Cananeia can be replicated in other places, having good results. Especially in places where the accuracy of the system is lower, that is, more distant from the SGB origin. Figure 5 illustrates the specific differences of the geoid/quasi-geoid separation between the normal and Helmert orthometric height, the figure of the geoid/quasi-geoid separation was not applied to the rigorous orthometric height, since the values are close, which cannot be noticed in the figure.

Table 3 Geoid/quasi-geoid separation between Normal, Helmert, and Rigorous heights calculated from Imbituba and Cananeia.

Statistics	Min (cm)	Max (cm)	Mean (cm)	Std $(\pm)$ (cm)
$H^{o}_{IMBITUBA} - H^{N}_{IMBITUBA}$	-6.914	25.293	5.208	3.647
$H^{o}_{CANANEIA} - H^{N}_{CANANEIA}$	-6.913	25.291	5.208	3.649
$H^{Ro}_{IMBITUBA} - H^{N}_{IMBITUBA}$	-10.182	25.378	3.391	4.050
$H^{Ro}_{CANANEIA} - H^{N}_{CANANEIA}$	-7.612	25.376	3.424	3.998

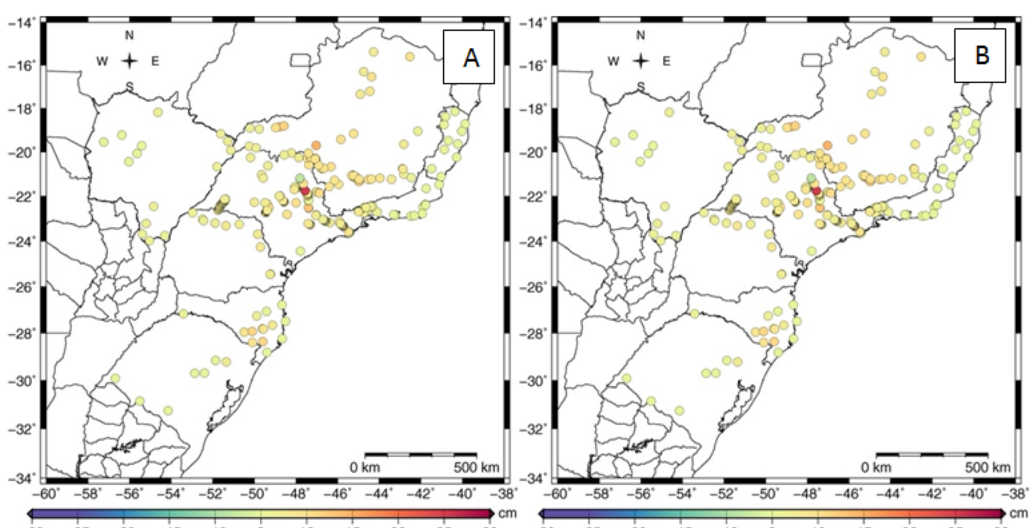

Figure 5 Separation between geoid and quasi-geoid: A. $H^{o}_{IMBITUBA} - H^{N}_{IMBITUBA}$; B $H^{o}_{CANANEIA} - H^{N}_{CANANEIA}$.

When performing a numerical evaluation between the results obtained in Table 2 and 3, we can verify that the geoid/quasi-geoid separation values are lower when using the heights calculations performed by the authors with the IBGE heights. The differences between the values of the mean and standard deviation between Tables 2 and 3 are ~ 3.43 cm and 4.78 cm ~ for the Helmert height and the rigorous height there is a difference in the average of ~6.78 cm and standard deviation of ~ 3.88 cm.

According to the report IBGE (2018), the new normal heights calculated in approximately 76% of the RRNN have a discrepancy of +20 to +30 cm about normal orthometric heights (in force until July 2018) in the midwest and southeast regions; in the south, the values differ between -5 to +20 centimeters. Albarici et al. (2018), in a numerical test, with rigorous orthometric height, using a leveling line of 400 km between the cities of Caraguatatuba and Ribeirão Preto (north coast and center of the state of São Paulo, respectively), values were 0 to + 15 centimeters, which validates the methodology applied since the values are lower than those calculated by IBGE.

(4) Again, by trying to validate the proposed methodology, a comparison was made between the values of the geopotential numbers made available by IBGE in its database and those calculated by the authors. The differences were made with the geopotential numbers obtained by the two tide gauges. Table 4 shows the statistical values obtained.

Table 4 Comparison of the geopotential numbers calculated by the IBGE and those calculated in this research project for the two tide gauges.

Statistics	Min (cm)	Max (cm)	Mean (cm)	Std (±) (cm)
$C_{IMBITUBA}$	-26.379	87.180	14.166	8.722
$C_{CANANEIA}$	-21.101	91.089	16.060	8.302

According to IBGE (2018), the quality of REALT-2018 results (Altimetric Network Readjustment Report with Geopotential Numbers) can be performed based on the standard deviations of the geopotential numbers, where the values are between 6 and 10 centimeters. When analyzing Table 4, we see that the standard deviations calculated using the two tide gauges as reference have values of ~ 8.3 to 8.7 centimeters. What we can say is that the calculated values fall within the range obtained by IBGE.

(5) Finally, to determine the congruence between the heights provided by IBGE and those calculated in the two tide gauges, the values of the global model EIGEN-6C4 were used, noting that the reason for not using MAPGEO2015 is that it does not make available the values for the height anomaly.

The residual values for the classical system (Helmert) and the modern design (Molodensky), are presented in Table 5 and Table 6, respectively. The analyzes of the congruence using Helmert orthometric heights were performed only with the heights calculated by the two origins since there is no availability of Helmert orthometric heights in the IBGE's BDG. It can be seen that the values in Table 5 are similar, with a small improvement in the standard deviation when using Imbituba as a reference. In Table 6, the worst result is when the congruences with IBGE's heights are analyzed, and, again, the values between Imbituba and Cananeia are close, with a slight improvement in Imbituba.

An attempt to compute the potential values was carried out at the Cananeia tide gauge. The computation was based on Hayden et al. 2012 as follows:

$$W_P = W_0 - \left[\left(h - N - CD + Z_0\right)\gamma_P\right] \quad \textbf{(9)}$$

where h is the ellipsoidal height, CD is the Chart Datum, and Z_0 is the height of the local MSL above the chart datum. W_0 refers to an equipotential surface of the Earth's gravity field provided by the conventional value of W_0 = 62,636,853.4 $m^2\ s^{-2}$ (Sánchez et al. 2016).

Following IAG resolution No. 1, in which data should be related to mean tidal system, the computation was carried out at zero tide system and the results were transformed to mean tide system according to transformation parameters provided by Heikkinen (1978), Ekman (1989) and Rapp (1989). The W_P calculation was performed at an IBGE geodesic station (SAT 91723) with precise coordinates and gravity value available. This point is close to the tide gauge and far from 74,00 m from GNSS permanent station (RBMC station – "NEIA station") as shown in Figure 6.

Table 5 Statistics of the residuals between calculated geoid height (N_{Lev}) and those estimated from the global geoid model EIGEN-6C4 (N_{Grav}).

Statistics	Min (cm)	Max (cm)	Mean (cm)	Std (±) (cm)
$N^{RO}_{IMBITUBA}$	-1.050	0.531	-0.471	0.161
$N^{RO}_{CANANEIA}$	-1.060	0.514	-0.467	0.162

Table 6 Statistics of the residues between calculated height anomalies and those estimated from the global quasi-geoid model EIGEN-6C4.

Statistics	Min (cm)	Max (cm)	Mean (cm)	Std (±) (cm)
$\zeta^{N}_{IMBITUBA}$	-1.046	0.533	-0.469	0.165
$\zeta^{N}_{CANANEIA}$	-1.123	0.455	-0.526	0.166
ζ^{N}_{IBGE}	-0.959	0.593	-0.331	0.179

Figure 6 Quantities at the research station in Cananeia (Source: http://www.io.usp.br/).

The ellipsoidal height (h) value was derived from SAT 91723 station, while the geoid undulation (N) was obtained at the Brazilian official geoid model (MAPGE2015). We considered in this study N equal to ζ, once the point is on the coast. The *CD* is available at (https://www.marinha.mil.br/chm/) and the value used in this computation (0.933 m) was measured in 199 and corresponds to the statistical and harmonic analysis of 709 days of observing the tide. Z_0 was estimated by the authors, according to the Permanent Service for Mean Sea Level (PSMSL) and the value obtained was 0.920 m. The W_P value computed at this point is 62,636,835.34 m^2s^{-2}. From this value, we computed the geopotential number using expression 2 and consequently the normal height from expression 5. We carried out the leveling from the zero at tide gauge to SAT 91723 and NEIA station. Thereby, we compared the heights based on leveling from local mean sea level (tide gauge measurements) with the heights computed using expression (9), which are presented in Table 7.

Table 7 shows the heights computed at SAT 91723 and NEIA GNSS station. Regarding the point SAT 91723 the difference between the value estimated from leveling and the one computed according to expression (9) is 0.118 m. In terms of NEIA station, the difference is 0.124 cm. At this point, we also compared the height with the IBGE value. In this case, the difference is 0.133 m. This computation was the first attempt to calculate the potential value from W_0 and some geometric available functionals. We understand that the most current value of the Chart Datum is desirable to do a more reliable estimation.

Table 7 Computed heights at the research station in Cananeia.

Station	*H* from leveling (m)	*H* from W_p (m)	*H* from IBGE (m)
SAT 91723	1.694	1.812	–
NEIA Station	7.773	7.897	7.764

4 Conclusions

To ascertain the potentiality of the methodology applied in this paper, integrating the tide gauge of Cananeia-SP to the IHRF, the normal, Helmert orthometric, and rigorous orthometric heights calculated by two distinct origins, Imbituba and Cananeia, and the normal heights calculated by IBGE were analyzed. Thus, the geoid/quasi-geoid separation was analyzed by the classical method (equation 8).

When comparing the normal heights calculated by the authors in the two origins with the normal heights calculated by IBGE, we can see that the values have differences (Table 1), with a minimum, maximum, mean, and standard deviation values of ~ -26 cm, ~ 30 cm, ~ 13 cm and ~ 7 cm, respectively. There is no explanation and/or a specific cause for this great difference, only remaining the expectation that IBGE presents the methodology used in the calculations made so that all researchers can help improve the system, or even, if necessary, exchange it for actions that are better suited to the Brazilian territory. Even so, the values obtained through the Cananeia tide gauge have an improvement in the value of the mean and standard deviation, indicating that the methodology worked.

Analyzing the values shown in Table 2, where the geoid/quasi-geoid separation is presented using the normal height calculated by IBGE, the values are equivalent, considered high with ~8 cm of standard deviation, even with the same analysis using the normal, Helmert, and rigorous heights, calculated in Imbituba and Cananeia tide gauges; note that the values (Table 3) are much smaller, with a standard deviation of Helmert and Rigorous height of 3.6 cm and 4 cm, respectively, which indicates that the separation between geoid and quasi-geoid is lower in the rigorous orthometric height system. Thereby, it can be concluded that this system is more accurate since the understanding of the separation between the geoid and the quasi-geoid is important considering the modernization of any system.

The differences, which can be called residuals, between Helmert and rigorous orthometric heights and those obtained from the geoid model, give us the means to measure the congruence we want to assess. In an ideal world, these residues would all be zero, however, in the real world, they are different from zero; their magnitudes thus can be measured in a statistical sense. The smaller the statistical measure of the residuals (standard deviation), the better the congruence. The same can be said when using the normal heights and height anomaly obtained from models. These differences also give us a tool to measure how much congruence improves when different corrective measures are taken.

Tables 5 and 6 show that the standard deviation residuals values of the rigorous orthometric heights are lower than those of the normal height, and the highest standard deviation value occurs when using the heights provided by IBGE.

Despite the difficulty in obtaining the data on the two tide gauges, and the observations (GNSS/leveling and gravity) for the calculation of the heights, the results of this work indicate that the Helmert or rigorous orthometric height have better statistical results than the normal height. However, the Cananeia tide gauge is ready to be integrated into the IHRF in the future, and any of the heights shown here can be used.

5 Acknowledgments

The authors are grateful to FEC/Unicamp, GGE – the University of New Brunswick, and UFU – Monte Carmelo Campus for their support during the Sandwich Course period, and to IFSULDEMINAS for the first author's license in the development of the doctorate. This study was carried out with the support of the Coordination of Higher Education Improvement Personnel – Brazil.

6 References

Albarici, F.L., Guimarães, G.N., Foroughi, I., Santos, M. & Trabanco, J.A. 2018, 'Separação Entre Geoide e Quase-Geoide: Análise das Diferenças Entre as Altitudes Normal-Ortométrica e Ortométrica Rigorosa', *Anuário do Instituto de Geociências*, vol. 41, no. 3, pp. 71-81. https://doi.org/10.11137/2018_3_71_81.

Albarici, F.L., Foroughi, I., Guimarães, G.N., Santos, M. & Trabanco, J.A. 2019, 'New Perspective for Physical Heights in Brazil', *Bulletin of Geodetic Sciences*, vol. 25, no 1, pp. 1-20. https://doi.org/10.1590/s1982-21702019000100001.

Dalazoana, R. & de Freitas, S.R.C. 2020, 'Sistemas Geodésicos de Referência: Rumo ao GGRS/GGRF', *Revista Brasileira De Cartografia*, vol. 72. no. Special, pp. 962-82. https://doi.org/10.14393/revbrascartogr.

Ekman, M. 1989, 'Impacts of Geodynamic Phenomena', *Bulletin Géodésique*, vol. 63, pp 281-96. https://doi.org/10.1007/BF02520477.

Ellmann, A. and Vaníček, P. 2007, UNB application of Stokes-Helmert's approach to geoid computation, *Journal of Geodynamics*, vol. 43, no. 2, pp. 200–13. https://doi.org/10.1016/j.jog.2006.09.019.

Foroughi, I., Vaníček, P., Sheng, M., Kingdon, R.W. & Santos, M.C. 2017, 'In defense of the classical height system', *Geophysical Journal International*, vol. 211, no. 2, pp 1176-83. https://doi.org/10.1093/gji/ggx366.

Förste, C., Bruinsma, S.L., Abrykosov, O., Lemoine, J.M., Marty, J.C., Flechtner, F., Balmino, G., Barthelmes, F. & Biancale, R. 2014, 'EIGEN-6C4 The latest combined global gravity field model including GOCE data up to degree and order 2190 of GFZ Potsdam and GRGS Toulouse', *5th GOCE USER WORKSHOP*, *GFZ Data Services, Paris, GFZ*, p. 25–8. https://doi.org/10.5880/icgem.2015.1.

Gilardoni, M., Reguzzoni, M. & Sampietro, D. 2016, 'GECO: a global gravity model by locally combining GOCE data and EGM2008', *Studia Geophysica et Geodaetica*, vol. 60, pp. 228-47. https://doi.org/10.1007/s11200-015-1114-4.

Heiskanen, W.A. & Moritz, H. 1967, *Physical Geodesy*, Freeman and Company, San Francisco.

Heikkinen, M. 1978, *On the tide-generating forces*. Finnish Geodetic Institute, Helsinki, Finland.

Hofmann-Wellenhof, B. & Moritz, H. 2006, *Physical Geodesy*, Springer-Verlag, Berlin.

IBGE - see Instituto Brasileiro de Geografia e Estatística.

Instituto Brasileiro de Geografia e Estatística 2018, *Relatório Reajustamento da Rede Altimétrica com Números Geopotenciais REALT-2018*, Rio de Janeiro, viewed 3 January 2019,<ftp://geoftp.ibge.gov.br/informacoes_sobre_posicionamento_geodesico/rede_altimetrica/relatorio/relatorio_REALT_2018.pdf>.

International Association of Geodesy 2015, *International Height Reference System (IHRS)*. viewed 15 January 2021, <https://iag.dgfi.tum.de/fileadmin/IAG- docs/IAG_Resolutions _2015.pdf>.

Jekeli, C. 2000, *Heights, the Geopotential, and Vertical Datums. Report 45*9. Ohio, Department of Civil and Environmental Engineering and Geodetic Science, viewed 04 January 2021, <https://kb.osu.edu/bitstream/handle/1811/78667/SES_GeodeticScience_Report_459.pdf?sequence=1> .

Kingdon, R., Vaníček, P., Santos, M.C., Ellmann, A., Tenzer. R. 2005, 'Toward an improved orthometric height system for Canada', *Geomatica*, vol. 59, no. 3, pp. 241–50 (Errata: Figure 4 on Geomatica, vol. 60, no, 1, pp. 101).

Kingdon, R. 2012, 'Advances in Gravity based Height Systems', Ph.D. Thesis, Department of Geodesy and Geomatics Engineering, University of New Brunswick. http://www2.unb.ca/gge/Pubs/TR292.pdf.

Kotsakis, C., Katsambalos, K. & Ampatzidis, D. 2012, 'Estimation of the zero-height geopotential level W0LVD in a local vertical datum from inversion of co-located GPS, leveling and geoid heights: a case study in the Hellenic islands', *Journal of Geodesy*, vol. 86, no. 6, pp. 423-39. https://doi.org/10.1007/s00190-011-0530-7.

Molodensky, M. 1945, 'Fundamental Problems of Geodetic Gravimetry', TRUDY Technical Report No. 42, *Geodezizdat Novosibirskiy Institut Inzhenerov Geodezii*, Aerofotos yemki i Kartografii (NIIGAiK), Moscow.

Molodensky, M., Eremeev, V. & Yurkina, M. 1962, *Methods for the study of the external gravitational field and figure of the Earth*', Israel Program for Scientific Translations, Jerusalem.

Moritz, H. 1980, 'Geodetic reference system 1980', *Bull. Géodésique*, vol. 54, no. 3, pp. 395–405. https://doi.org/10.1007/BF02521480.

Rapp, R. 1989, 'The treatment of permanent tidal effects in the analysis of satellite altimetry data for sea surface topography', *Manuscripta geodaetica*, vol. 14, pp. 368–72.

Roman, D. & Smith, D. 2002, 'Recent investigations toward achieving a one-centimeter geoid. Gravity, Geoid and Geodynamics 2000', *International Association of Geodesy Symposia*, vol. 123, Springer, New York, pp. 285–90.

Santos, M.C., Vaníček, P., Featherstone, W.E., Kingdon, R., Ellmann, A., Martin, B. -A., Kuhn, M. & Tenzer. R. 2006, 'The relation between Rigorous and Helmert's definition of orthometric heights', *Journal Geodesy*, vol. 80, pp. 691–704. https://doi.org/10.1007/s00190-006-0086-0.

Sánchez, L. 2013, 'Towards a vertical datum standardisation under the umbrella of Global Geodetic Observing System', *Journal of Geodetic Science*, vol. 2, no. 4, pp. 325-342. https://doi.org/10.2478/v10156-012-0002-x.

Sánchez, L., Čunderlík, R., Dayoub, N., Mikula, K., Minarechová, Z., Šíma, Z., Vatrt, V. & Vojtíšková, M. 2016, 'A conventional value for the geoid reference potential W0', *Journal Geodesy*, vol. 90, no. 9, pp. 815–35. https://doi.org/10.1007/s00190-016-0913-x.

Sánchez, L. & Sideris, M.G. 2017, 'Vertical datum unification for the International Height Reference System (IHRS)', *Geophysical Journal International*, vol. 209, no. 2, pp 570–86. https://doi.org/10.1093/gji/ggx025.

Sansò, F. & Rummel, R. 1997, '*Geodetic Boundary Value Problems in View of the One Centimeter Geoid'. Springer*, New York, U.S.A.

Sistema de Referência Geocêntrico para as Américas 2020, *Geocentric Reference System for the Americas*, viewed 28 December 2020, <http://www.sirgas.org/en/sirgas-definition/>.

Tenzer, R., Vaníček, P., Santos, M., Featherstone, W.E. & Kuhn, M. 2005, 'The rigorous determination of orthometric heights', *Journal of Geodesy*, vol. 79, pp. 82-92. https://doi.org/10.1007/s00190-005-0445-2.

Vaníček, P. & Krakiwsky, E. 1986, *Geodesy: The Concepts*. Amsterdam, Elsevier Science Publishers.

Vaníček, P. & Martinec, Z. 1994, 'Stokes-Helmert scheme for the evaluation of a precise geoid', *Manuscripta Geodaetica*, vol. 19, no. 2, pp. 119–28.

Vaníček, P., Santos, M., Tenzer, R. & Hernández-Navarro, H. 2003, 'Algunos aspectos sobre alturas ortométricas y normales', *Revista Cartográfica*, vol. 76/77, pp. 79-86.

Received: 25 January 2021

Accepted: 17 June 2021

How to cite:

Albarici, F.L., Guimarães, G.N., Santos, M.C. & Trabanco, J.L.A. 2021, 'Strategy for Connecting to the IHRF: Case Study for the Tide Gauge of Cananeia–SP', *Anuário do Instituto de Geociências,* vol. *44*: 41104. https://doi.org/10.11137/1982-3908_2021_44_41104