

## Numerical Analysis of Soil Deformation in Earth Dams Using a Stiffness Variation Technique

*Análise Numérica de Deformações no Solo em Barragens de Terra Através de Técnica de Variação de Rigidez*

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### Abstract

Small earth dams have been used since ancient times and are often the main source of water in semi-arid regions. In this particular type of engineering work, soils saturated by water present changes in their mechanical properties (especially in the first filling), which usually cause a simultaneous loss of stiffness and shear strength. In this context, the software UNSTRUCT presents itself as a very useful tool, as it allows calculating the final state of stress  $\times$  strain for a plane strain state, using the Finite Element Method, both for saturated and unsaturated embankments, considering an elastic model and the effect of suction (and its variation), through a stiffness variation technique. The present research used UNSTRUCT to assess the behavior of the embankment of a hypothetical earth dam, varying the features of its typical cross-section: homogeneity; internal drainage structures; geometry along a longitudinal profile; and the influence of the compaction level. The results showed that the soil responded better to strains for the scenario of better compaction along the dry branch of the compaction curve, as well as with the use of a vertical drainage element. It was also observed that the strains were higher when greater sections considered, which, in a real situation, would cause distortions along the longitudinal profile of the dam. The software UNSTRUCT proved to be efficient in obtaining and visualizing the modeling results, also making it possible to analyze the development of failure over time after the dam was filled. With UNSTRUCT, a more realistic analysis of the behavior of the hypothetical dam – considering factors that are specific to unsaturated soils – was possible. Worth mentioning also that the software allowed the appraisal of multiple changes in the dam's geometry and parameters.

**Keywords:** UNSTRUCT; Unsaturated soils; Dam collapse

### Resumo

Pequenas barragens de terra têm sido usadas desde os tempos remotos e, muitas vezes, são a principal fonte de água em regiões semiáridas. Neste tipo particular de obra de engenharia, solos saturados por água apresentam mudanças em suas propriedades mecânicas (principalmente no primeiro enchimento), o que, em geral, provoca perda simultânea de rigidez e resistência ao cisalhamento. Neste contexto, o *software* UNSTRUCT se apresenta como ferramenta bastante útil, pois permite determinar, usando o Método de Elementos Finitos, o estado final de tensão  $\times$  deformação para um estado plano de deformações, tanto em maciços saturados quanto não-saturados, considerando um modelo elástico e o efeito da sucção (e de sua variação) através da técnica de variação de rigidez. A presente pesquisa utilizou o UNSTRUCT para avaliar o comportamento do corpo de uma barragem de terra hipotética, variando as características da seção-tipo: homogeneidade; estruturas internas de drenagem; geometria ao longo de um perfil longitudinal; e influência do nível de compactação. Os resultados mostraram que o solo respondeu melhor às deformações para a cenário de melhor compactação ao longo do ramo seco da curva de compactação, bem como com a utilização de um elemento drenante vertical. Observou-se, também, que as deformações foram tão maiores quanto maiores foram as seções consideradas, o que, numa situação real, ocasionaria distorções ao longo do perfil longitudinal da barragem. O programa UNSTRUCT mostrou-se eficiente na obtenção e visualização dos resultados da modelagem, possibilitando, também, analisar o desenvolvimento do colapso através do tempo após o enchimento da barragem. Com o UNSTRUCT, uma análise mais realista do comportamento da barragem hipotética – considerando fatores que são específicos dos solos não-saturados – foi possível. Vale destacar também que o software permitiu avaliar múltiplas variações na geometria e parâmetros da barragem.

**Palavras-chave:** UNSTRUCT; Solos não-saturados; Colapso em barragem

## 1 Introduction

Unsaturated circumstances are probably the most frequent condition of nearly all geotechnical structures ever built. Fortunately, in recent decades, the broader knowledge on unsaturated soils has greatly improved the modeling process of these types of civil works.

Fredlund and Rahardjo (1993) pointed out that over one third of the planet correspond to arid and semi-arid regions, where upper soil layers remain unsaturated most of the year. In those areas, earth dams have been for centuries the main source of water for different uses.

Earth dams are compacted structures that rely on their weight to resist sliding and overturning. During the first filling, the changes in the stress state can cause the embankment to present deformations and even cracks. Modeling this scenario can be difficult sometimes due to the coexisting saturated and unsaturated conditions which are diffuse throughout the embankment.

In this context, the software UNSTRUCT (UNsaturated STRUCTure analysis), initially developed by Miranda (1988), can be a very useful tool to determine the final stress  $\times$  strain state for a plane strain state in saturated and unsaturated embankments. The program uses the Finite Element Method and an elastic model to assess the effect of suction and its variation along time.

Silva Filho (1998) expanded the program's ability to model nonlinear behavior. Nowadays, UNSTRUCT is capable of assessing the stiffness variation of the embankment material through an interpolation of results of double edometric tests carried out using saturated and unsaturated samples.

This study used the software UNSTRUCT to evaluate different scenarios of a hypothetical dam and the influence of suction (and its variation) on the performance of the embankment. Changes in suction were evaluated through pore pressure values in the embankment by analyzing results from double edometric tests that used saturated and unsaturated specimens of a typical Brazilian soil.

## 2 Compacted Unsaturated Soils and Cracks in Earth Dams

Classical soil mechanics describes with good precision the hydro-mechanical behavior for completely dry or completely saturated soils. However, applying the effective stress approach to unsaturated soils violates the core assumptions from the classical continuum mechanics. Hence, its concepts cannot correctly model the behavior of partially saturated soils, very common in tropical countries such as Brazil (Teixeira et al. 2000).

The constitutive models for unsaturated soils aim to express the mechanical and hydraulic behavior of the soil when it is subjected to changes in the stress state and degree of saturation. They represent the variations in soil strength and deformability when suction varies (Cordão Neto 2005).

A lot of engineering works (e.g., dams, retaining walls, roads) employ compacted soils. Compacting a soil means making it denser through mechanical processes and reducing its void index through the expulsion (or compression) of the air in the voids of the soil. This process clearly differs from consolidation, where densification occurs due to the slow expulsion of water from the voids.

Compaction seeks to obtain a better behavior of the soil for the purposes it was intended: improving engineering properties, increasing shear strength and erosion resistance, decreasing settlements (deformability) and/or hydraulic conductivity.

In terms of compressibility, if the same dry unit weight and the same compaction energy are considered, a soil compacted in the dry branch is less compressible than in the wet branch (Massad 2010).

When it comes to earth dams, the first filling is an extremely critical stage – the embankment is rapidly loaded almost to the maximum load that it will support during its life span. Thus, the first filling and the first years of operation can be used to evaluate the hypotheses and assumptions made during the basic and/or detailed design stages. Interaction effects (e.g., generation of traction zones, development of plastification zones, formation of cracks due to hydraulic fracturing) will have consequences that may compromise the entire safety of the work (Pereira 1996).

Sandroni (2021) defines fissure as a gap in the interior of the embankment that may or may not manifest externally and that can occur even in well-compacted soil masses due to: differential settlements (differences in the deformability of materials in the foundation); sudden variation in the topography of soil layers supporting the embankment; decrease in soil volume in the upstream zone after the reservoir is filled (collapse by submersion, which is the focus of the present study); contraction due to soil drying; and redistribution of stresses.

An inherent feature the unsaturated soils is their tendency to present variations in volume when the moisture content is changed under practically constant stresses.

In low density soils, an increase in the moisture content will reduce soil strength at the inter-particle contacts, commonly due to a reduction in suction. This phenomenon is called *collapse*. When the soil experiences abrupt volume reduction due to wetting but solely under self-weight, it is designated as a truly collapsible soil. If

the volume reduction takes place under any additional overload, the soil is said to be conditionally collapsible (Reginatto & Ferrero 1973).

The collapse potential of a soil and the magnitude of an eventual collapse can be assessed through laboratory or field tests, such as the edometric tests (also known as consolidation tests), which can be the simple edometric test (one-dimensional consolidation) or the double edometric test (double consolidation).

Multiple criteria can be used to determine the susceptibility of a soil to collapse. However, as a general rule, unsaturated soils will most likely experience a reduction in stiffness if suction (i.e., moisture content) changes, causing them to experience different deformations, even under similar loads. This is precisely what the software UNSTRUCT investigates and helps modeling.

### 3 Materials and Methods

For the numerical analysis of the hypothetical dam, it was necessary to associate two softwares that use the finite element method (FEM): (i) RocScience Slide 6.0, in order to model the transient unsaturated flow and (ii) UNSTRUCT, to simulate the development of stresses at the end of construction, as well as the stress  $\times$  strain behavior related to the transient flow, also considering the effect of the variation of stiffness imposed on the soils when suction changes.

Slide 6.0 is a commercial software used to calculate slope stability using 2D equilibrium limit. It has a two-dimensional percolation analysis tool that uses FEM, where the mesh is automatically generated according to pre-established preferences. Slide also allows the user to choose between steady or transient flow analysis, considering saturated or unsaturated conditions.

The software requires the insertion of data regarding embankment geometry, water loads, as well as permeability and water retention curves of the modeled soils. These variables directly influence the flow nets and the pore pressures developed during the defined time stages.

When using UNSTRUCT, the input variables comprise: Poisson's ratio for the saturated soil ( $\nu_{sat}$ ); Poisson's ratio for the unsaturated soil ( $\nu_{nsat}$ ); dry unit weight ( $\gamma_d$ ); porosity ( $n$ ); results of double consolidation tests (loads, strains, and suction). At the end of the analysis, the software will yield the output data regarding each discretized element:  $x$ ,  $y$ ,  $\sigma_x$ ,  $\sigma_y$ ,  $\sigma_z$ ,  $\sigma_1$ ,  $\sigma_3$ ,  $\theta$ , pore pressure, and displacements of each node in both directions ( $u_x$  and  $u_y$ ).

In order to provide the necessary soil parameters to both softwares, laboratory tests were carried out using a deformed sample collected from Piau Dam (State of Piauí, Brazil) by the staff of the Laboratory of Soil Mechanics and Paving (LMSP) of the Federal University of Ceará (UFC).

The sample was compacted at the desired moisture content and specific weight. Characterization, compaction, and double consolidation tests were carried out, using saturated and unsaturated samples, as well as a convenient methodology to obtain the necessary input parameters for UNSTRUCT.

The laboratory tests carried out followed the recommendations of the Brazilian technical standards and included: (i) grain-size analysis – NBR 7181 (ABNT 2016c); (ii) determination of Atterberg limits – NBR 6459 (ABNT 2016a), NBR 7180 (ABNT 2016b); (iii) determination of specific gravity – ME 093 (DNER 1994); and (iv) compaction (Standard Proctor) – NBR 7182 (ABNT 2016d).

Figure 1 shows the grain-size distribution of the tested soil. Figure 2 shows the compaction curve obtained with Proctor standard energy. Table 1 presents the geotechnical parameters obtained from characterization tests and the USCS classification for the tested soil samples.

The laboratory series comprised 8 edometric tests, performed at 4 different moisture contents, chosen among points on the dry branch of the compaction curve for Proctor standard energy. Two tests were carried out for each of the 4 assessed moisture contents: the first considering the moisture content that was determined while the sample was being molded (unsaturated condition) and the second one using a flooded sample (saturated condition).

The abovementioned pairs (i.e., the unsaturated and saturated soil samples) were labeled as Point 01, Point 02, Point 03, and Point 04, as shown in Figure 3, which displays the 4 moisture contents and the respective dry unit weights chosen for the sample pairs. Table 2 details the different parameters obtained while the sample pairs Point 01, Point 02, Point 03, and Point 04 were compacted: maximum dry unit weight, void index ( $e$ ), porosity ( $n$ ), and moisture content below the optimum moisture content (OMC). Important to highlight that Point 04 corresponded to OMC and maximum dry unit weight, as determined through the Proctor Standard compaction test.

Figures 4, 5, 6, and 7 present the curves “stress  $\times$  strain” for the performed double edometric tests (Points 01-04).

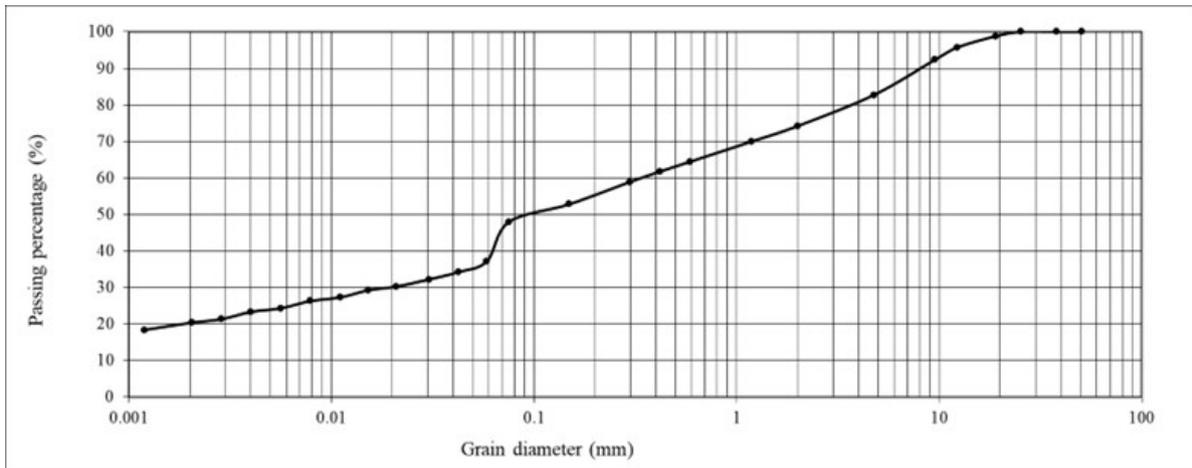


Figure 1 Grain-size distribution.

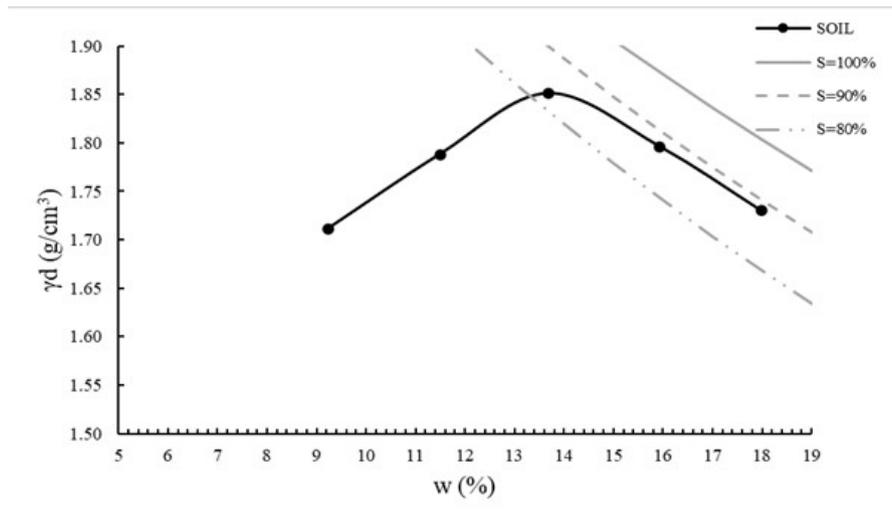


Figure 2 Soil compaction curve.

Table 1 Soil parameters and USCS Classification.

Parameter	
Liquid Limit	29%
Plastic Limit	16%
Plasticity Index	13%
Specific Gravity	2.67g/cm³
Maximum Dry Apparent Specific Gravity	1.85
Optimum Moisture Content	13.68%
USCS Classification	SC



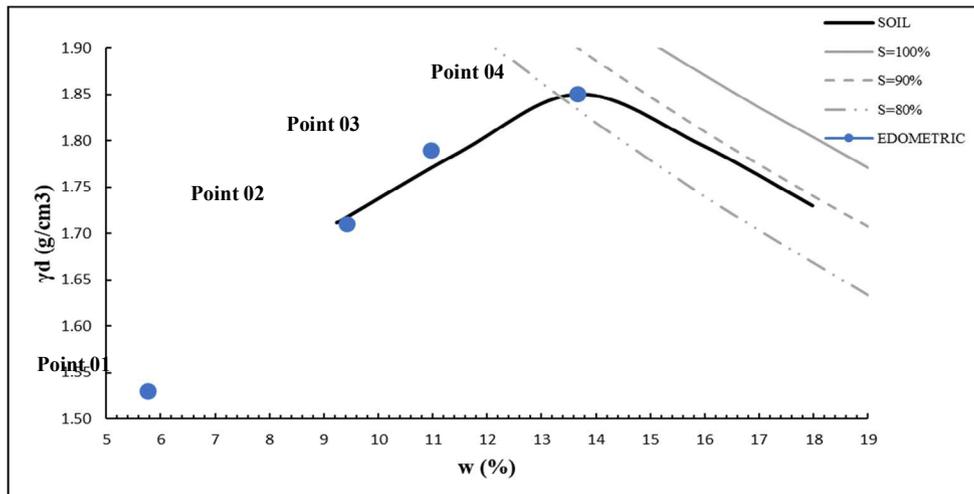


Figure 3 Testing scenarios chosen from the dry branch of the compaction curve with Proctor standard energy.

Table 2 Conditions of tested soil samples (Points 01-04).

Test pair	Moisture content at compaction (%)	Dry unit weight (g/cm <sup>3</sup> )	$e$	$n$	Moisture content below OMC (%)
Point 01	5.76	1.53	0.75	0.43	7.9
Point 02	9.43	1.71	0.56	0.36	4.23
Point 03	10.98	1.79	0.49	0.33	2.68
Point 04 (OMC)	13.66	1.85	0.44	0.31	N/A

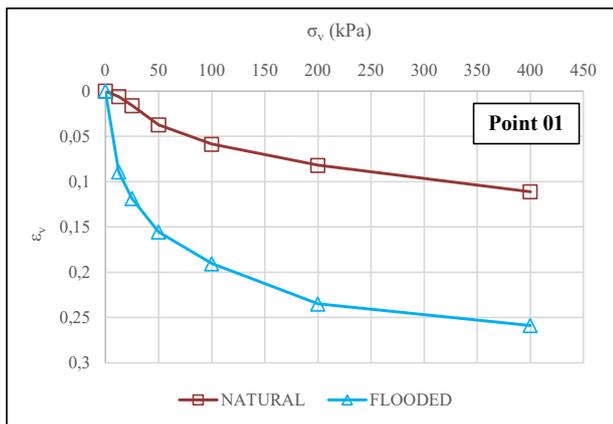


Figure 4 Stress x strain curves for Point 01.

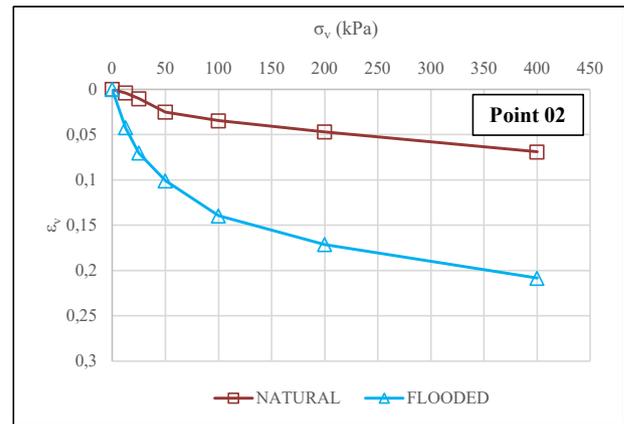


Figure 5 Stress x strain curves for Point 02.



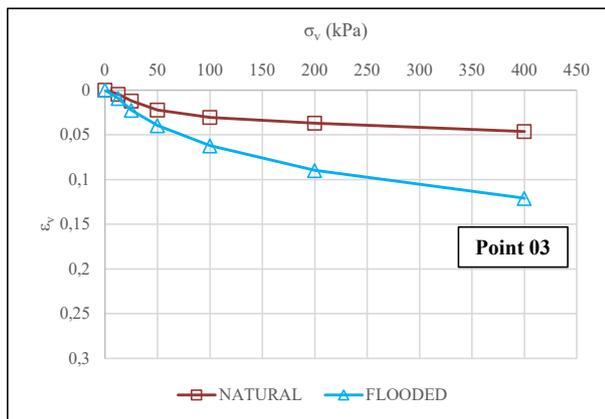


Figure 6 Stress x strain curves for Point 03.

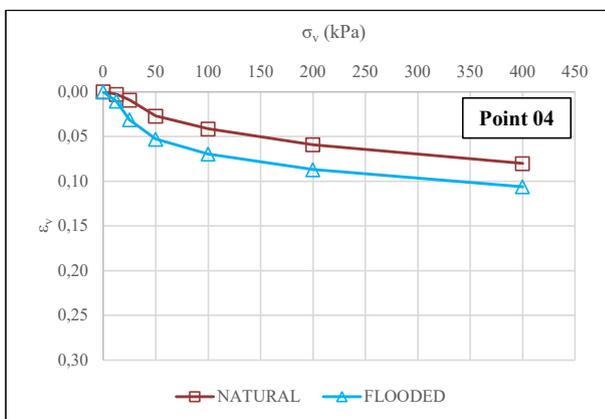


Figure 7 Stress x strain curves for Point 04.

## 4 Numerical Analysis and Results

In the numerical analysis of this study, the hypothetical earth dam was assumed to be located in a symmetric V-shaped river channel, that is, the geometry of the embankment gradually transitions from the central cross-section towards both abutments). The dam foundation was assumed to be impermeable and incompressible and three longitudinal sections were chosen to be studied: Section A, located at the maximum height of the embankment, and Sections B and C, both on the same side of the embankment, as shown in Figure 8.

In the analysis, different configurations were also considered for each section, varying: (a) the embankment materials; (b) the strains in the embankment for each testing scenario (Points 01-04); and (c) the influence of draining elements. In each of these situations, the collapse and its consequences were analyzed.

In the simulations, the maximum height for the embankment was 10.0m. Adopted maximum water level

was 8.0m (2.0m below crest height). Both upstream and downstream slopes were 1V:2H. Dam crest was 200.0m long. Section 01 was 46.0 m wide (bottom edge) and 10.0m high. Section was 7.0m high and Section 03, 4.0 m. The analysis of the strains in each section considered the maximum water level (8.0m).

### 4.1 Influence of the Position on the Compaction Curve

The first carried out analysis assessed the different levels of strains that occurred in the hypothetical earth dam when soil stiffness was varied along the dry branch of the compaction curve. The simulations analyzed the maximum cross-section (Section 01), which was assumed to be homogeneous (only one material, with no drainage elements).

Figure 9 shows the contour of the deformed dam after filling, when the soil parameters were varied from Point 01 to Point 04 (see Table 1), the different points on the dry branch of the compaction curve, as shown in Figure 1. The strains in Figure 9 were enhanced ten times in order to facilitate the perception.

Figure 9 shows that on the dry branch of the compaction curve there is a tendency of loss of soil stiffness as the scenarios approach the point that corresponds to OMC, which is seen through the improvement in the collapse deformations (Point 04). However, the soil compacted at the conditions of Point 03 (moisture content 2.68% below OMC) also had a very good performance, with strains quite similar to the soil compacted at the OMC (Point 04).

The upstream side had greater displacements, for this is the region where more elements are filled with water, after the embankment reached the steady condition (stabilization of percolation). This also resulted in smaller deformation moduli (loss of stiffness).

### 4.2 Transient Analysis Along the Longitudinal Profile

The software Slide made it possible to determine water flow and pore pressures, whereas UNSTRUCT helped evaluating the influence of the filling period on the development of deformations caused by local soil collapse due to changes in the stiffness as a consequence of the changes in the moisture content.

Thus, if the temporal changes in the reservoir level are known, the pore pressures of any desired time can be determined. Hence, the deformations in the embankment at this specific moment can also be established.

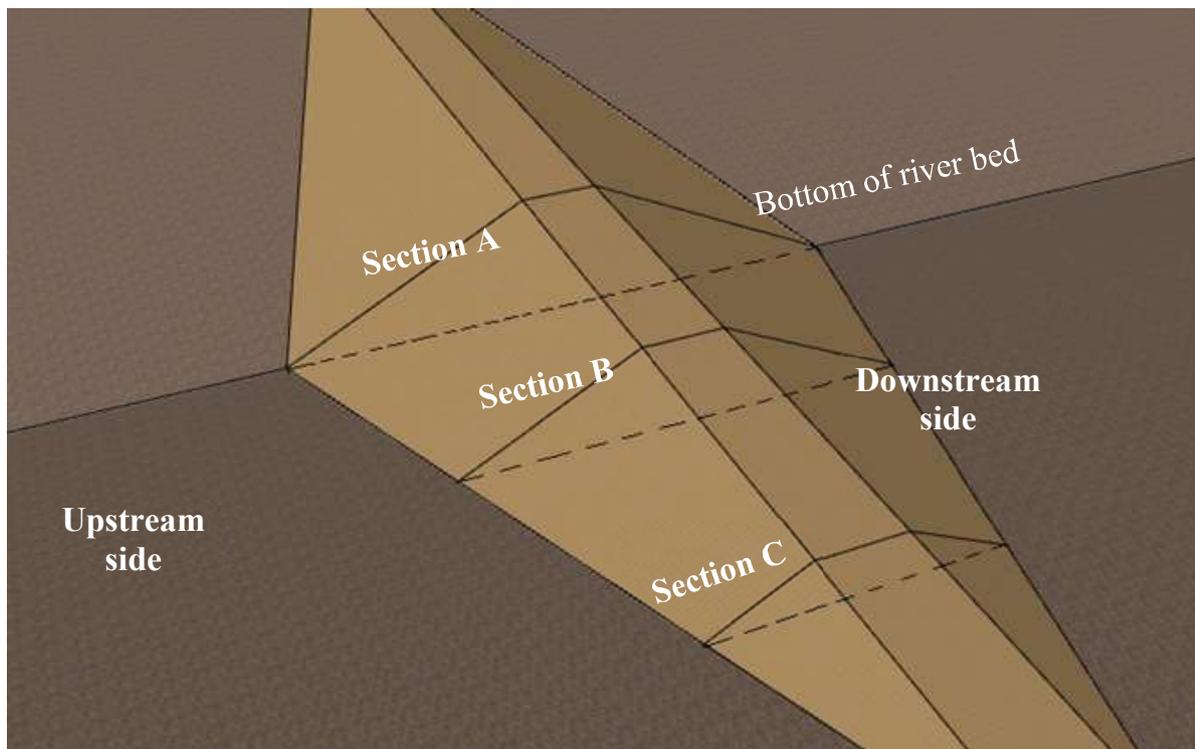


Figure 8 Hypothetical earth dam and the analyzed sections (A, B, and C).

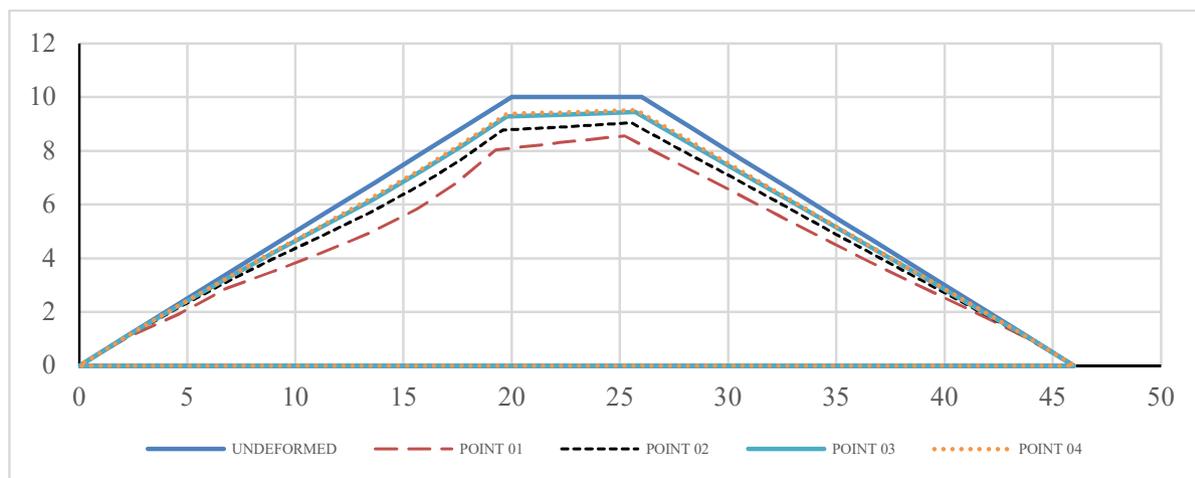


Figure 9 Strain pattern when soil parameters were varied on the dry branch of the compaction curve (Points 01 to 04).

Another factor that can be evaluated are distortions that can occur along the various longitudinal cross-sections of the dam due to differential deformations, since the geometry plays an important role on the displacement of nodes in the simulated mesh.

Thus, soil deformation occurred in three cross-sections of the hypothetical dam and in four different periods

of time after complete filling (3 months, 6 months, 12 months, and 48 months) were assessed, considering the parameters of Point 04: moisture content = 13.66% (OMC), dry unit weight = 1.85g/cm<sup>3</sup>, void ratio ( $e$ ) = 0.44, and porosity ( $n$ ) = 0.31.

The dam was considered to be homogeneous (i.e., only one material used in the embankment, with no drainage element) and that it was completely filled immediately (in

a very short period of time). In the analysis, the parameter “percolation through the embankment” was varied over time, until a steady condition was reached.

The evolution over time of deformations on the dam contour considering the conditions of Point 04 can be seen in Figures 10, 11, and 12, where deformations were increased 20 times to facilitate the visualization.

It was noticed that the strains over time were greater as the analyzed cross-sections increased in size/area. In Section A (maximum section), collapse occurred and it was the most severe. In Section C (the smallest one), strains were almost non-existent, even after 48 months of filling. This can be seen in Figure 13, which shows the evolution of strains in the dam.

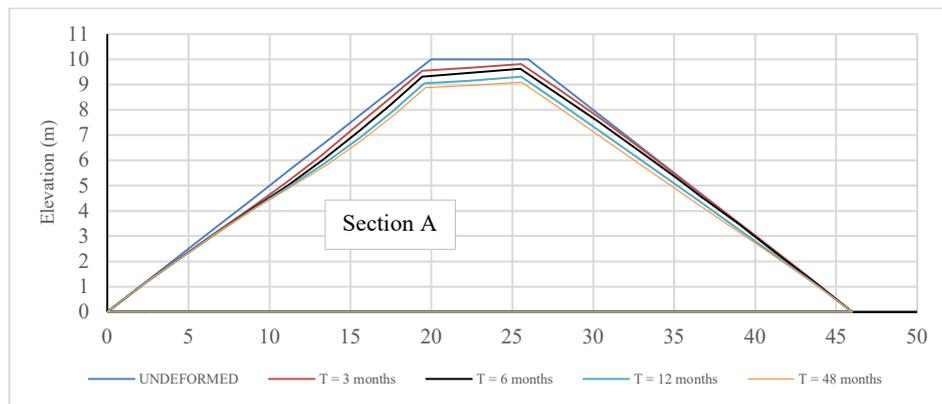


Figure 10 Deformations over time in Section A.

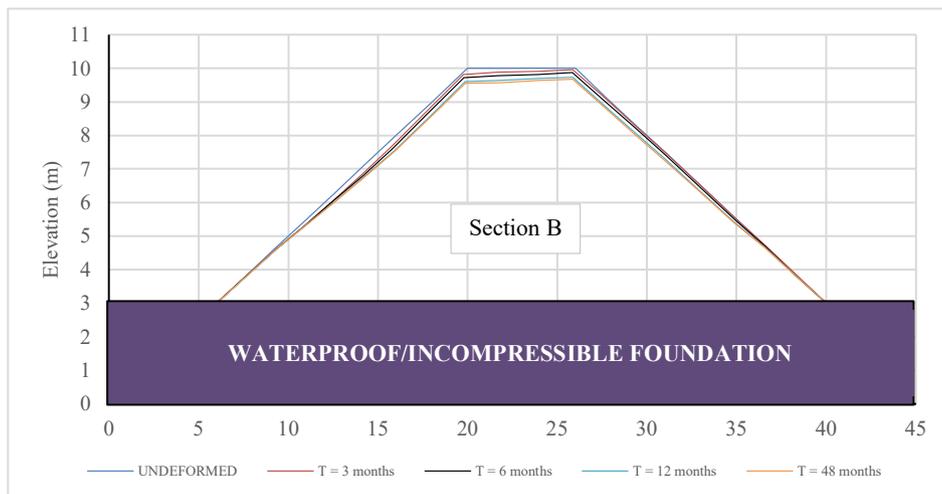


Figure 11 Deformations over time in Section B.

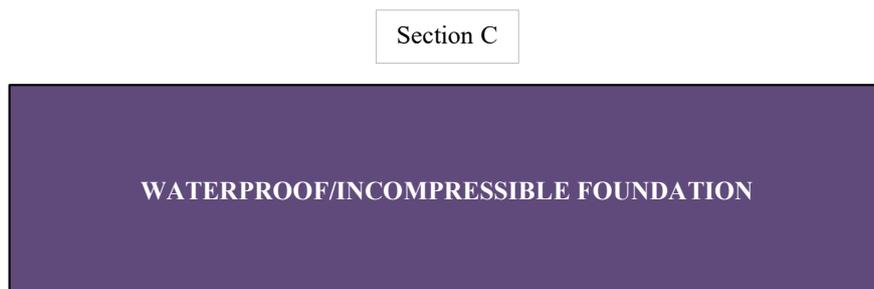


Figure 12 Deformations over time in Section C.

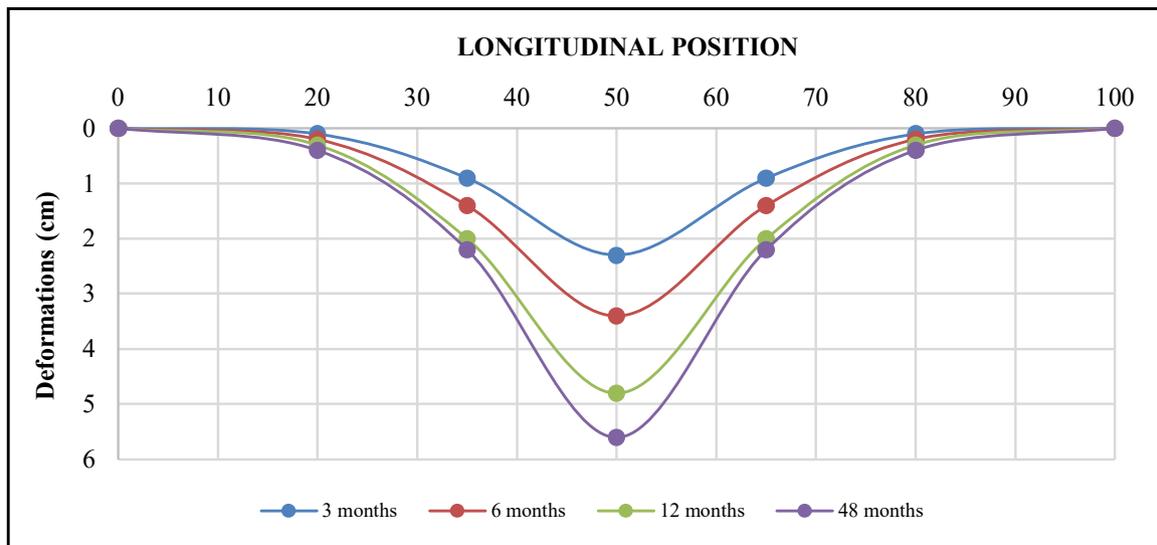


Figure 13 Strains over time along the longitudinal profile of the dam.

### 4.3 Analysis of Dam with a Drainage System

Another scenario that can be analyzed using UNSTRUCT is the use of drainage elements in the embankment. For this analysis, Section A (maximum section) was chosen and four situations were considered:

1. Dam built with homogeneous soil in the conditions of Point 03 and no drain;
2. Dam built with soil in the conditions of Point 03, with a drain;
3. Dam built with upstream soil in the conditions of Point 03 and downstream soil in the conditions of Point 02, with a drain; and

4. Dam built with upstream soil in the conditions of Point 03 and downstream soil in the conditions of Point 01, with a drain.

The goal was to evaluate the behavior of the hypothetical dam with and without drainage elements, as well as varying the compaction parameters in the downstream slope when a drainage element was present. The steady-state flow configuration for the homogeneous dam without a drain and with upstream and downstream slopes in the conditions of Point 03 is shown in Figure 14.

The steady flow configuration for dams with a drain, upstream slope in the conditions of Point 03 and downstream slope varying among the conditions of Points 03, 02, and 01 is shown in Figure 15.

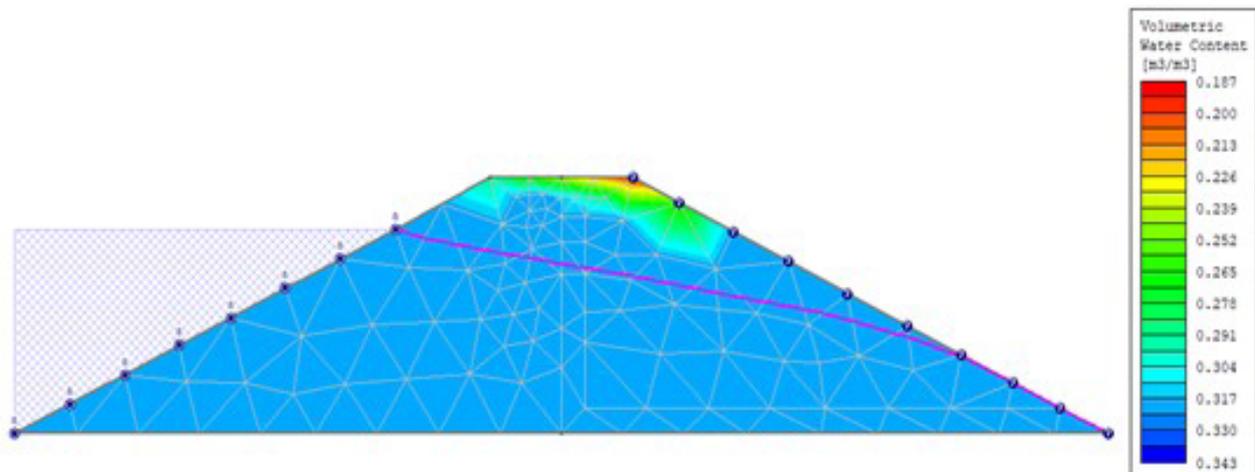


Figure 14 Water flow in the homogeneous dam without a drain.

The strains found for the different tested configurations are shown in Figure 16, where strains were increased by 20 times to facilitate visualization.

It was observed that the largest strains occurred in the scenario with a homogeneous dam without a drain. The presence of a drain clearly cooperated to reduce strains (and therefore diminish the risk of collapse), regardless of the compaction conditions of the downstream slope.

This suggests that a more economical and stable scenario for an earth dam should indeed include a drain and also have the upstream slope compacted at OMC (or pretty close to it) and the downstream slope a little below OMC conditions. This will save a lot of water, a very scarce resource in semi-arid regions, where earth dams are usually the main choice of impoundment structures to retain water.

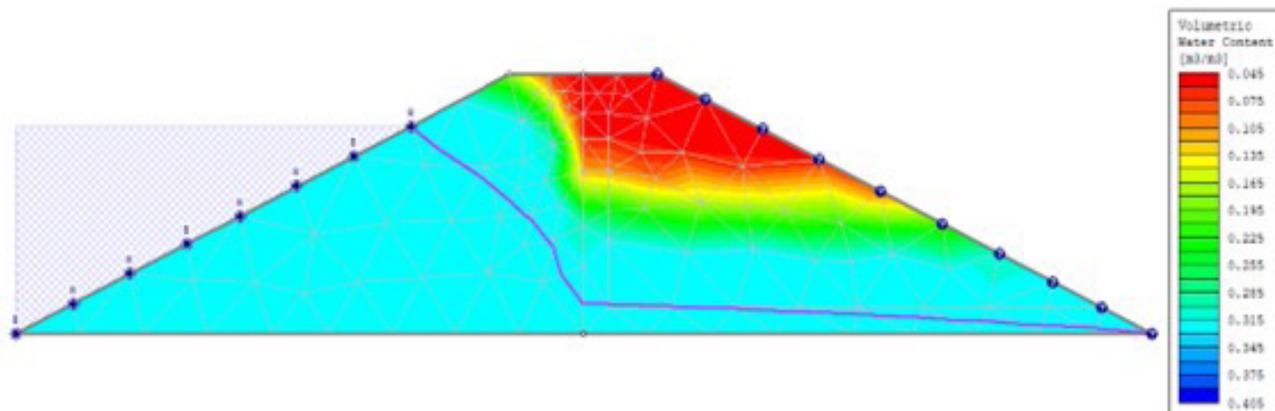


Figure 15 Water flow in the homogeneous dam with a drain.

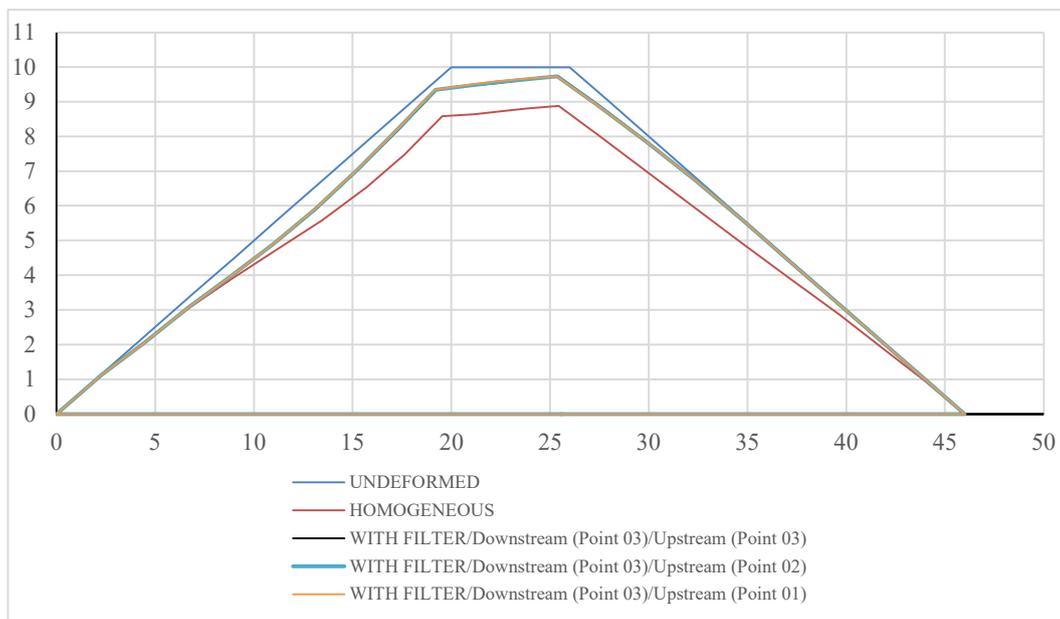


Figure 16 Strains for the different evaluated scenarios.



## 5 Conclusions

In this study, the numerical analysis provided results consistent with standard theory. The software UNSTRUCT proved to be easy to use and excellent (both from practical and theoretical standpoints) when it comes to modeling unsaturated soils and considering stiffness variation, which is not properly taken into account by other commercial geotechnical softwares.

The first analysis considered a homogeneous earth dam with the soil in four different scenarios on the dry branch of the compaction curve for standard Proctor energy, in which moisture content and dry unit weight were varied.

In the simulations, the largest collapses occurred for the soil in the worst considered conditions: lower moisture content and lower dry unit weight (Point 01). As the moisture content and dry density increased along the dry branch of the compaction curve, the simulation results showed increasingly smaller strains. As expected, the best scenario (with the smallest strains) corresponded to the situation when the soil was compacted at optimal moisture content and maximum dry unit weight.

The variation of strains along the longitudinal axis of the earth dam were also assessed by considering irregular configurations of the dam foundations which usually induce cross-sections with peculiar geometric shapes. The simulations showed that greater strains/deformations occurred in the cross-sections with the largest dimensions and that these differential deformations (distortions) between adjacent sections are possible cracking zones.

Changes in the deformations over time after the dam is filled can be fairly well predicted by modeling distinct scenarios with the software UNSTRUCT and comparing the results with parameters obtained *in loco*, if the dam is instrumented.

The use of a vertical drain proved to be efficient in reducing deformations due to soil collapse, since it limits the percolation of water to the upstream area of the dam and reduces the occurrence of deformations due to loss of stiffness when compared to the scenario without any drainage structure installed.

The vertical drain was also efficient in scenarios where soil compaction conditions in the downstream area

were varied, indicating that its use leads to a reduction in the amounts of water needed for compacting downstream areas without incurring in major changes in the deformation by submersion.

In general, UNSTRUCT provided a more realistic analysis of the behavior of the hypothetical earth dam, as it considers aspects inherent to unsaturated soils and allows assessing variations in diverse scenarios.

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**Author contributions**

**Romário Anderson Guerreiro Maia:** conceptualization; formal analysis; methodology; validation; writing-original draft. **Francisco Chagas da Silva Filho:** conceptualization; formal analysis; methodology; validation; writing review and editing; supervision; visualization.

**Conflict of interest**

The authors declare no conflict of interest.

**Data availability statement**

Model data and reference datasets can be downloaded from: <https://repositorio.ufc.br/handle/riufc/67524>.  
Scripts and code are available on request.

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