

Wittgenstein “Great Analysis” and Frege’s construal of number as a property of properties

Araceli Rosich Soares Velloso

UFG

INTRODUCTION

According to Wittgenstein in *Tractatus Logico-philosophicus* (*TLP*), “Analysis” is a technical term for a reductionist process, which aims to find, by means of a unique descending chain of definitions, a universal and complete basis for each and every proposition of our ordinary language, resulting in a final analysis composed only by elementary propositions (EP). At the end of this process, each one of those propositions should correspond in a perfect isomorphic manner to its own projected *state-of-affairs* and the latter alone will be its truth-maker *and* its sense. Thus, in *TLP*, each elementary proposition should have a completely independent and determined sense. The final goal is to construe every other proposition as a truth function of this universal basis. In accordance with this goal, one could explain the meaning *and the truth conditions* of every other proposition as a function of that elementary level. We are going to refer to this complex and lengthy analytical process as “The Great Analysis”¹.

As a consequence of its very construal, *The Great Analysis* is required in *TLP* as a condition of possibility of attributing a completely explained and determined sense to any proposition which intends to represent reality. In the *tractarian* framework, a proposition has a determined sense *if, and only if*, it can represent reality in a complete and thorough manner. For Wittgenstein in *TLP*, this analysis is an indispensable requisite for any explanation of the relation between

1 Cf. Velloso (2014).

language and reality as a depiction of the latter by the former. Understood in this way however, the radical theory of language presupposed by the author of *TLP* seems to imply even more strange consequences. The one which interest us most here is exactly the presupposition of an isomorphic relation between language and reality. This presupposition implies the establishment of a perfect isomorphic and one-to-one relation between EPs, the singular elementary level of our language, and the *state-of-affairs* projected by it.

In *TLP*, the projective relation should initially consist in a correspondence between each *genuine* name, concatenated inside the EP, and an element (a simple object) of the projected *state-of-affairs*. Actually, the "isomorphism" thesis in *TLP* seems to imply a little more than that. Already in the notebooks Wittgenstein was speculating about the possibility of building a model, a linguistic one, embedded in each EP. These "models" required that the "configuration"² of *genuine* names in EPs should be the same as the "configuration" of *simple* objects in the projected *state-of-affairs*. In other words, both, EPs, and the projected *state-of-affairs*, should have the same *logical form and the same multiplicity* (WITTGENSTEIN 1993, SRLF p. 30). At the end of analysis, we would have a linguistic proxy for each *state-of-affairs*. Furthermore, this strategy would have the further advantage of explaining the sense of all propositions univocally and determinately through a process of linking them directly to their own projected *state-of-affairs*.

The process of analysis, understood as a device for allowing a compositional theory of meaning like the one proposed in the *TLP* looks very attractive, at first. Nonetheless, there, Wittgenstein provides us with a scanty sketch of how this unusual process could take place in all its minute details. He assumes it as a possibility condition of the complete determination of sense and provides no further elucidations, leaving for us only with a bare sketch. As it is well known and can be attested in his only published paper – "Some Remarks on Logical Forms" –, when he finally provides us with a first draft of how the logical form of EP would look like, he found himself wrapped in so many difficulties that he finally ends up aborting the whole enterprise.

Our goal in this paper is not to go over all those difficulties, nor to determinate one principal reason behind the abandonment of the project of a "*single* completely analyzed form for every expression", as he later refers to in his (*PI* §91). Our aim is to compare his confident

² We are using "configuration" here in the same sense as Wittgenstein uses the notion of "structure" *TLP* (cf. 2.034), cf. also Griffin (1964, p. 52). The crucial aspect here is the complete absence of properties within PEs.

proposals in *TLP* with his hesitating remarks in the *PI*, and with some further new twists of his latte middle period. By this comparison we hope to throw some light into very important worries Wittgenstein has about two very central and important ideas proposed by Frege in §§18-24 of *The Foundation of Arithmetic (FA)*. The first one is a sequence of arguments showing the urgency of making a fundamental and radical distinction between aggregates and sets. Frege offered those arguments in order to criticize the idea that numerical attributions would be first order predications. The second was Frege's solution to those problems, his construal of the notion of "numerical attribution" as a second order predication, a predication over properties. Our suggestion is that those concerns were the ones which prompted Wittgenstein to develop his "aspect seeing" approach and led him to employ it as a criterion for choosing between alternative paths our analytical process could follow. If we are successful in our argument, we will also be showing a way to understand why it doesn't make sense for Wittgenstein to talk about one, and only one, complete analysis of each proposition in the *PI*.

Our first goal will be to present Wittgenstein's two different views on the "Great Analysis", the tractarian and his later views. In order to accomplish this goal, we will begin by presenting a more detailed construal of just how such process of analysis could be carried out at the time of the *TLP*.

I. THE "GREAT ANALYSIS" IN THE *TLP*

In *TLP* 3.25, Wittgenstein declares: "A proposition has one and only one complete analysis". We have proposed to view this great analytical process as a step-by-step process³. The process begins with unanalyzed propositions of our ordinary language, which describe complex events extended in space and time. The intermediary steps are the replacement of our ordinary propositions for propositions which describe all the possible alternative spatial-temporal sub-events of the initial complex-event. Finally, the third step would be to make a last replacement of each proposition introduced in the second step by a disjunction of conjunctions of elementary propositions. Those propositions must have the same logical form which would have the atomic *state-of-affairs* projected by them. Our investigation is related to the problems

3 Cf. Velloso (2014).

of the first two steps only, because it is there that the arguments we want to explore, opposed to a unique and complete analysis of each propositions, are effective. In the sequel then, some elements of these two initial steps proposed in our investigation will be further investigated.

I.1 THE FIRST STEP

The ordinary propositions of the initial step are those of our day-to-day language. At first sight, there seems to be nothing wrong with them, they are just ordinary sentences. A proposition like "John is sitting on his favorite armchair, in his office, at some specific date and time" is in order just as it is, as Wittgenstein himself often emphasizes:

In fact, all the propositions of our everyday language, just as they stand, are in perfect logical order. (1961, TLP 5.5563)

So, as far as only mere ordinary communication is at stake, it seems that we wouldn't need any further elucidation of the meaning of our propositions. The whole point of Wittgenstein's logical analysis in *TLP* is not connected to ordinary communication, but to the philosophical role of language as a representational device though. And it is as a representational device that our language must depict reality in a thoroughly and completely determined manner.

Let's explore the idea which seems to be behind Wittgenstein's proposal a little further. On the one hand, he argues that if one evaluates the *state-of-affairs* which our ordinary singular propositions usually project, one will find that they are not very specific about all the alternative sub-events that compose the complex *state-of-affairs* or situation projected by them. On the other hand, Wittgenstein suggests that a language at least implicitly has to represent reality in a thorough and complete manner. Furthermore, we could add to this assumption that reality would itself be composed of completely particular events. Each fact in the world is completely unique and determined to its last minutest sub-event. From all these presuppositions, Wittgenstein seems to conclude in the *TLP* that ordinary propositions do not represent reality in a "direct, exact manner". Or, in other words, what speakers usually do with language is only to describe a quite underdetermined cluster of possible events, which could, any single one of them, be what their proposition is projecting.

Let us return to the example above, the proposition “John is sitting in his favorite armchair in his office at some specific date and time” or else, even the more general: “It rains, at time t at place p ”. If one pays attention to these examples and to the *state-of-affairs* those propositions should project, one will notice that they include dissimilar and complex situations. At time t and place p , John could be sitting in his chair in several different ways. The rain could consist in a multiplicity of different distributions of drops and each drop could be described in several dissimilar ways. Both, the proposition describing John’s position and the one describing the rain, could be made true by whole lot of very diverse complex-events. Thus, propositions like those ones should be considered as “semantically general” statements, statements which could be made true by a variety of different *state-of-affairs*.

From the examples above, it becomes clear that at the end of the second step of our analysis we should have something like a disjunction of propositions representing the various alternative truth maker events. Each proposition will describe a different complex-event which could have made the initial assertion true. Finally, at the third step, each of those disjoints will have to be converted in a conjunction of EPs, thoroughly representing each event to its last detail. They will look like a list of occurrences of each possible aggregate of molecules of john’s body or of each drop of rain.⁴

The *semantical generality* we’ve been talking about is a very particular kind of generality and it implies a very specific “indetermination” of our ordinary propositions as well. In order to better explain it, we will have to say first what it is *not*. We are not talking about the generality expressed in general quantified statements like those we find in TLP 5.52, nor about the generality involved in polysemic terms, such as “rose” for the color, and “rose”, for the flower. Actually, the kind of indetermination we meant is characteristic of a descriptivist account such as Frege’s.

Let us step back a little. According to Frege’s famous distinction between sense and reference, a singular term refers to an object only by means of its modes of presentation. In turn, his account of “mode of presentation” is usually construed as involving some sort of definite

⁴ Our examples are all physicalistically construed. We’ve chosen this option because a phenomenalist construal of the TLP would involve us directly with the problem of time, which was a characteristic concern of Wittgenstein’s so-called “middle period”.

descriptions, i.e., a way to identify the referent of a singular term by means of the concepts or properties that uniquely describe it. So, we could treat all nominal parts of our unanalyzed propositions as singular terms which contains or presuppose some definite descriptions. Under this Fregean descriptivist account, singular terms always use properties as an indirect way to identify the object they purport to refer to.

Wittgenstein's view of "apparent" singular propositions in *TLP* follows, according to our proposal, the same Fregean approach. The examples are again that of the rain or of John sitting in his favorite chair. If one considers those propositions, one should say there is a definite description hidden under each nominal part: the name "John" could be construed as describing a specific body with a variety of alternative details; "the rain" could also be described as a distribution of drops of different size and shapes, falling at diverse place and times. What we are suggesting is that in Wittgenstein, just as in Frege, these hidden descriptions are the ones which are actually fixing the reference of those "singular" terms.

Now, with this descriptivist approach, we can return to our main point: an indetermination in the way our terms fix their referent. The point is that a variety of alternative situations, or spatial-temporal extended portions of the world, could satisfy our hidden definite descriptions. In *TLP*, Wittgenstein intends to solve this problem by replacing every group of identifying properties by another one, which gives a list of names of each atomic component event in each alternative complex situation. At the end of the tractarian analytical process, one should thus get only simple names of atomic aggregated events and no general terms. Following James Griffin (1964, p. 57), we argued elsewhere⁵ that Wittgenstein's main goal with *TLP*'s Great Analysis was to get rid of all those unspecific properties and replace them by simple names:

Where are any general terms now? Evidently all get analyzed away. But note that on the fully analyzed level instead of speaking of 'the broom' what we now speak of are 'these particulars in this configuration and these other particulars in that configuration, &c'. So, it looks as if configurations *take over the role of general terms.* [My emphasis] (GRIFFIN, 1964, p.52)

The main point of the analyzing away all general terms, as well as “apparent” singular ones, is to obtain a completely and *independent* formulation for every atomic event, a formulation which does not contain that “semantical generality” we’ve been talking about. In our construal, this was Wittgenstein’s solution in *TLP* for the linguistic indetermination identified in our ordinary singular statements which are supposed to be about complexes:

Every proposition which seems to be about a complex can be analysed into a proposition about its constituents and about the proposition which describes the complex perfectly; i.e., that proposition which is equivalent to saying the complex exists. (Appendix I - 1913, p. 101)⁶

[...] When a propositional element signifies a complex, this can be seen from an indeterminateness in the propositions in which it occurs. In such cases we know that the proposition leaves something undetermined. [...] (TL 3.24)

Thus, we are forced to conclude that, as long as the proposition still includes general terms, or make reference to “complex ordinary objects” by means of identifying properties, the process of analysis must go on, for it still requires further determinations. According to our proposed version of analysis, one would still have to go further in the attempt to identify the conjunction of unique *state-of-affairs* which was meant to be projected. Wittgenstein apparently had something along these lines in mind when he asserts that, at the end of analysis, we should individuate our projected complex event only by means of simple names, names which have only referents. As we argued in a previous paper, the best way to understand those simple names should be as coordinates names representing the occurrence of each possible atomic event (VELLOSO 2016, p.). So, at the end of analysis, one remains with a list of names of each atomic space-time sub-event of the projected complex-event. According to this construal of Wittgenstein’s position, the occurrence of any empirical body would be considered as a space-time complex aggregated event, a spatial-temporal extended process. In a note to the passage above, Wittgenstein makes the following comment: “*Russell for instance imagines every fact as a spatial complex.*”.

6 Cf. TLP 2.0201.

I.2 THE SECOND STEP

In a well-known passage of his latter book the *PI*, Wittgenstein gives us a brief sketch of how the transition from the first to the second intermediary step should proceed. As he says in the passage, the transition involves analyzing away from our ordinary propositions those expressions which are not really singular.

When I say: "My broom is in the corner", – is this really a statement about the broomstick and the brush? Well, it could at any rate *be replaced by* a statement giving the position of the stick and the position of the brush. And this statement is surely a further analysed form of the first one. – But why do I call it "further analysed"? – Well if the broom is there, that surely means that the stick and the brush must be there, and in a particular relation to one another; and this was as it were hidden in the sense of the first proposition, and is expressed in the analysed proposition. [My emphasis] (§ 60)

One first important point is that the idea of "analyzing away" here doesn't simply mean to take out one expression and replace it by another. "Analysis" for Wittgenstein is a peculiar process in which we replace the whole original proposition by a completely set of new *ones*. It seems therefore that we could *not* undertake the process of analysis in a sort of "mechanical" way, i.e., simply dividing the proposition into parts, or even making some sort of grammatical investigation which consider the words as substantives, adjectives, verbs and so on. What we have at our disposal to replace the unanalyzed proposition are still propositions, each and every one of them, and they should collectively express, in its analyzed form, the space-time sub-events of each of the various original complex-situation being projected. They are thus a kind of physical or phenomenological elucidation, made from a reductive and extensional perspective, of the various initially projected alternative events.

The original problem of how to analyze ordinary propositions became thus, in this construal, a problem of establishing how to replace our ordinary, and apparently singular, propositions with an entirely new set of propositions of a different kind. On this reconstruction, the problem of how we could carry on the process of analysis becomes the problem of finding a procedure of arriving at the second level of whole new propositions, which, nonetheless represent the same series of alternative possible events. In order to reach a solution to this

problem, let's return once again to the terminology of the *TLP*, where we find Wittgenstein introducing the concept of an "internal relation":

A proposition about a complex stands in internal relation to the proposition about a constituent of the complex. (TLP 3.24)

The solution suggested in this passage is to circumscribe a kind of semantic relationship, an internal relation, standing between the unanalyzed proposition and the second level explanatory ones. Our next move should thus be an investigation on just what such an "internal relation" is supposed to involve and how exactly could we obtain new propositions, which maintain an internal relation with the original propositions, to replace the initial one.

There is a general rule by means of which the musician can obtain the symphony from the score, [...] and, using the first rule, to derive the score again. That is what constitutes the *internal similarity* between these things which seem to be constructed in such entirely different ways. And that rule is the law of projection which projects the symphony into the language of musical notation. It is the rule for translating this language into the language of gramophone records. [my emphasis] (TLP 4.0141)

Following Wittgenstein's remark, our suggestion here is that we could obtain new propositions, which explain the meaning of the original ones, if we consider the various *state-of-affairs* projected by the original propositions and describe the sub-events that compose each of those complex-events. In *the PI*, §60, example, the situation described by the original proposition could be something like the one of "the broom being in the corner". This situation could be further redesigned, in turn, as "the being in the corner of the broomstick", "the being in the corner of the brush" and "the broomstick's connection to the brush". As Wittgenstein put it, this process resembles a translation inside one and the same language, a kind of paraphrase process, from a simpler to a more elaborate way of describing the same set of alternative situations. With this translational process in mind we could establish a similarity relation between the two descriptions of the same initial projected complex-event. As he affirms in *TLP* 3.343: "*Definitions are rules for translating from one language into another.*"

Our suggestion here is to take Wittgenstein's remark about translation as an elucidation of the supposed "internal relation" which should hold between propositions of the first and the second steps. Translation will be settled by rules and those rules act in the same way as definitions, relating internally the two linguistic descriptions – the second, a translation of the first.

Let's now see what more *TLP* has to tell us about such internal relations:

A property is internal if it is unthinkable that its object should not possess it. (This shade of blue and that one stand, eo ipso, in the internal relation of lighter to darker. It is unthinkable that these two objects should not stand in this relation.) The contraction of a symbol for a complex in a simple symbol can be expressed in a definition. (*TLP* 4.123)

In this passage, Wittgenstein says that properties or relations are internal, when it is unthinkable that the "objects" related by them are not related in the way described. Under this characterization, they are essential to the very identity of the objects described. In *the PI*, §60, example, we are supposed to understand first what a Broom is, univocally and unambiguously, and also what exactly means for it "to be in the corner". It is only after being able to describe particular component sub-events of our "being-in-the-corner" complex-situation that we will be ready to proceed with our analysis. We thus normally proceed: a "broom" is a "broomstick connected to a brush". Through these definitions, or translations, expressions of the original propositions would be analyzed away and replaced by different ones. This process will go on until we finally reach the primitive signs. This is what Wittgenstein tells us in this passage:

Every sign that has a definition signifies via the signs that serve to define it; and the definitions point the way.

Two signs cannot signify in the same manner if one is primitive and the other is defined by means of primitive signs. Names cannot be anatomized by means of definitions. (Nor can any sign that has a meaning independently and on its own.) (*TLP* 3.261)

The final goal is to reach an end product composed only by sequences of primitive signs. Understood in that way, our process of analysis resembles that of a descending chain of definitions, which take us from the top – our ordinary language – to the lowest level – that of

the elementary propositions. The EPs in their entirety form a kind of “primary language” which would then, and only then, be able to connect language to reality.

Finally, but not less important, according to our initial *desideratum*, Logical Analysis in the *TLP* will be a search for a complete and determined elucidation of the sense of each proposition. If we maintain an “internal similarity relation” between the unanalyzed and the full analyzed propositions, this *desideratum* will be accomplished. It follows from this *desideratum* also that there cannot be *another way* to analyze the original propositions which could still project the same state-of-affairs, no diverse way to describe the initial projected series of complex-events. Otherwise, one would not have one, and only one, correct and complete analysis. So, the option of having more than one way of analyzing a proposition would imply that our process of connecting language to reality and understanding this relation as a representational relation would have been deficient.

The character of necessity and essentiality of those internal properties, as we said, is the one responsible for the completeness and uniqueness of *TLP*'s Great Analysis. This is the point where, according to our proposal, Wittgenstein's philosophical position suffer a change in the latte middle period, between 1933 and 1939, in the following manner. In *TLP*, Wittgenstein strongly believed in one, and only one, internal relation between any two sets of propositions. This internal relation will eventually bring us to a unique and foundational lowest level of language. In *the PI*, this belief change in a radical way. This change consists in a movement from a foundational and compositional view of language, and here we are talking of a universal and all-inclusive language, to an anti-foundational and non-universal view. In *the PI*, instead of one basic level of EPs and a truth function that determines the sense of any proposition from the lower to the upper levels, in the *PI*, Wittgenstein talks of different “language games” or different “grammars”. Moreover, in each one of those different parts of language we will have multiples and diverse ways to elucidate the sense or any proposition. In this broke up scenario, one will not be allowed to talk anymore of a unique and complete descending chain of definitions, which leads to the analyzed form of each ordinary proposition. Our purpose in the following sections will be to gain a better understanding of the reasons behind this significant change in Wittgenstein's philosophical approach to language.

II. THE PHILOSOPHICAL INVESTIGATIONS' ARGUMENTS AGAINST THE "GREAT ANALYSIS"

In the *PI*, Wittgenstein criticizes his old tractarian conception of language. He previously construed language as a complete and unique calculus. According to this account, the sense of each and every proposition has to be completely and uniquely determined by a truth function applied to the lowest level of elementary propositions. The descending chain of definitions obtained through the analytical process would have to be responsible for establishing, via the elementary propositions, a unique truth maker and a unique meaning for every EP. In the *PI*, he makes the following evaluation of his old proposals:

But now it may come to look as if there were something like a final analysis of our forms of language, and so a single completely resolved form of every expression. That is, as if our usual forms of expression were, essentially, unanalyzed; as if there were something hidden in them that had to be brought to light. When this is done the expression is completely clarified and our problem solved.

It can also be put like this: we eliminate misunderstandings by making our expressions more exact; but now it may look as if we were moving towards a particular state, a state of complete exactness and as if this were the real goal of our investigation. (WITTGENSTEIN, *PI*, §91)

This is what Wittgenstein himself thought he had done in *TLP*. He calls it "*investigations [...] trying to understand the essence of language – its function, its structure*". (*PI*, §92) His claim in the *TLP* was that one should be able to find a completely determined and unique sense for each proposition. He states that before he thought that when a sense is attached to a proposition, it must be an exact one; it must not be vague, for what could a "vague sense" be like? A "vague sense" will be sheer nonsense, was his position throughout the *TLP* (*PI*, §99). According to his previous view, a logical and universal structure laid underneath our ordinary language and it was this structure which was responsible for the determination of thought as a unique correlate-picture of the world. In a scenario where thought would be idealized, turned into "pure crystal", our propositions would have their grammar and their truth conditions completely determined in a univocal manner, prior to all experience (*PI*, §94-8).

In the *PI*, after presenting his old position, he expresses his doubts about them. The main one was about the completeness and uniqueness of the semantical elucidation of any proposition. Now, his tractarian view turned out to be a pursuit of chimeras. Hence, in the *PI*, he repeatedly investigates whether two diverse and equally satisfying ways of analyzing the same proposition would not be possible. In the *TLP*, there is but a single propositional description of the correct collection of simple elements, or atomic events, from which a complex is composed and *that* is the terminus of the analysis. In *the PI*, he speculates about more than one way to describe how a complex could be composed of its parts. The following passage states this point about the “chimera of pursuing simples” at the end of the analysis:

But what are the simple constituent parts of which reality is composed? – What are the simple constituent parts of a chair? – The bits of wood of which it is made? Or the molecules, or the atoms? “Simple” means: not composite. And here the point is: in what sense “composite”? *It makes no sense at all* to speak absolutely of the “simple parts of a chair”. [My emphasis] (WITTGENSTEIN, *PI* §47)

The passage ends up with a somewhat enigmatic remark: “*It makes no sense to speak absolutely of the ‘simple parts of a chair’*”. Does the strangeness comes from the suggestion that one could not analyze the material composition of a chair in a more atomized way and proceed with this analysis until reaching some sort of completion? Is he suggesting that there is more than one aggregation of “simples” from which our complexes could have been made of? To elucidate this enigmatic remark, we should look to the beginning of that passage. There he had said that the answer to this question depended on the way one understands the word “simples”, within quotation marks in the passage. His suggestion was that we should not understand “simples” as absolute and unique elements from which everything is made of, but as being “simples only in a relative way”. Here, our suggestion is that Wittgenstein had in mind a relative, as opposed to an absolute way of speaking about the simple parts of an event-situation. Furthermore, this relativistic approach to “simples” involves another very important element. According to it, the elucidation of the composition of any complex-situation would depend on the way one looks at it and no single way to look at the chair would be better, or more primitive, than the others.

Let us explore this option a little further, introducing into our discussion Frege's formulation of the aggregate/set distinction (1953, FA §§18-22). In Wittgenstein's example, we have at least three different ways to construe a chair's structural composition: "bits of wood", "molecules" and "atoms". If we consider the "bits of wood" approach, our next step would be to pursue a conjunction of propositions describing each one of those "bits". On this account, the bits of wood would be the "simple parts" of the chair. Now, if we consider alternatively the "molecules" approach, we will have another very different conjunction of propositions, with a diverse content, for they will have to describe, not bits of woods, but instead each molecule that is within that chair. In this second approach, molecules will be the new "simple parts" of the chair. The same will happen with atoms, and so on. Linking Frege's ideas to a well-known wittgensteinian terminology, we are going to refer to those alternative options as being "different aspects to see the same situation", "different ways of looking at the same complex", or, simply the "seeing as argument", from now on.

Summing up, Wittgenstein's in the last passage of his late book the *PI* insists on a dependence of the way our analysis should go on upon the elected "way we look at a complex-situation". According to the later Wittgenstein, the same complex-situation, or event, could be "looked at" in various different ways. If we have a chair in front of us and take the "molecule" approach, as a physicist would do, we would be able to "see it as" a cluster of molecules and count thousands of occurrences of them. But if we employ the "bits of wood" aspect, we would "see it as" a collection of "undetached bits of wood", as, say, a carpenter would do, when he constructs a chair. Furthermore, according to the last option of "seeing the chair", we would count just, say, six simple parts – and not a billion, as in the "molecule" option.

III. "ASPECT SEEING", THE TRACTARIAN LOGICAL ANALYSIS OF PROPOSITIONS CRITIQUE IN A FREGIAN ACCENT

The claim that a distinction between aggregates and sets must be made was, of course, Frege's in FA. There, he offered the following similar argument against Mill's account of numbers:

Here the definite article in the phrase 'the characteristic manner' is a mistake right away; for there are very various manners in which an agglomeration can be separated into parts, and

we cannot say that one alone would be characteristic. For example, a bundle of straw can be separated into parts by cutting all the straws in half, or by splitting it up into single straws, or by dividing it into two bundles. [...] The number word “one”, again, in the expression “one straw” signally fails to do justice to the way in which the straw is made up of *cells* or *molecules*. [my emphasis] (1953, p.30)

The affinities between this passage and that of §47 of the *PI*, quoted above, are clear:

But what are the simple constituent parts of which reality is composed? – What are the simple constituent parts of a chair? – The bits of wood of which it is made? Or the molecules, or the atoms? “Simple” means: not composite. And here the point is: in what sense “composite”? *It makes no sense at all* to speak absolutely of the “simple parts of a chair”. [My emphasis]

In this passage, Wittgenstein proposes the very same list of alternative ways to see an aggregate as Frege had done before: “*the expression ‘one straw’ signally fails to do justice to the way in which the straw is made up of cells or molecules*” (1053, FA §23). Our suggestion is that both philosophers are proposing the same question, namely: is there one single unique and characteristic manner of how a chair could be chopped off into its constituting parts? It seems that the idea here is quite the same: one can choose between *various* ways to see an *aggregate*, complex-situation, or event, and no one could be considered to be *the characteristic manner* of dismembering it. According to Frege, each one of those various ways, each chosen *aspect* of the aggregate, produces a different number, because it would imply seeing it as different collections of objects.

Another very important point is that we can generalize Wittgenstein’s “aspect seeing” approach and take it, not as a local list of alternative for a particular situation, but as a unified, general point of view. Consequently, the “molecule” view, the “cell” view, or even, evoking some latter examples given by Quine, the “und detached parts” view⁷ and the “complement” view⁸, are all general alternative ways to see how a complex event can be construed as made of objects and to see those objects as its constitutive parts.

7 (QUINE 1960, chapter 2). Cf. also (VELLOSO 2009, 2011).

8 (QUINE 1992, p. 32).

Another feature of Frege's argument which was also explicit in Wittgenstein's remarks in the passage quoted above, is that, to view a situation as composed of objects is just a consequence of applying to it one "aspect", one way of looking at some given situation. Before "choosing one determined aspect of the chair", one cannot, neither talk about, nor count any objects there.

But one should be careful here. There are also differences between the two philosophers. The most important one is a difference on emphasis. For Frege, there would be no unique way to count the objects which are involved in any aggregate, but this is just because numbers are not first-order concepts, but second-order ones, and they are applied to "zero level" reality only through first-order concepts' intermediation. But objects of "zero level" are for Frege an assumption of his foundational approach to language. In Wittgenstein's version, Frege's argument is used to produce a variety of aspects of one and the same situation, leading us to consider as "simples" a diverse collection of objects. For Wittgenstein, the main conclusion was that it would "make no sense" to talk about an "absolute collection", which is more fundamental than all the other alternatives.

Commenting again his ideas in the *TLP*, Wittgenstein directly criticizes himself once again:

My view in the *Tractatus Logico-Philosophicus* was wrong: because I didn't clearly understand the sense of the words "a logical product is hidden in a proposition" (and similar words), because I too thought that logical analysis would have to bring hidden things to light (as do chemical and physical analysis). (BT, 28, 82)

This is all connected as well with the false concept of logical analysis that Russell, Ramsey and I used to have. Such that one awaits an ultimate logical analysis of facts, as one waits for a chemical analysis of compounds. An analysis that actually enables one to find, say, a 7-place relation, like an element that actually has the specific weight 7. (BT, 112, p.385)

The logical analysis in the *TLP* was a search for an elucidation of the underlying sense of our propositions. It was based on the belief that there should exist a unique universal domain of simple objects, which are the starting point of a compositional process for establishing the sense and the truth-conditions of any given proposition. This process will be like "analysis and synthesis", two sides of the same movement of composing and decomposing propositions into words, and facts into simple elements. In the *BT*, *Wittgenstein* sustains a radically different

attitude towards language. In this passage, he doesn't take a proposition as a complex unit which should be decomposed in a unique way anymore, in order to uncover its real logical structure. He realizes that one can encounter different groups of objects which could be individuated and identified in any given situation. And, of course, no single one of them would be more "fundamental" than the others.

Now that we have presented and discussed, albeit quickly, Frege's and Wittgenstein's handling of what appears to be one and the same problem, we have to move on to finer details involved in those discussions. Our main goal will be to further support our proposal of approximating Frege's "aggregates and sets distinction", or the "more than one way to segment the same aggregate" arguments, with the notion of "aspect seeing" adopted by Wittgenstein in the *PI*. To accomplish this goal, we have to discuss two points related to changes in Wittgenstein's thought in his intermediate period: (1) First, we will investigate a few aspects of Wittgenstein's "internal" and "external" distinction as applied to properties, in the early intermediate period; and (2) then finally discuss Wittgenstein's reaction to Frege's construal of numbers as "properties of properties".

IV. INTERNAL AND EXTERNAL PROPERTIES DISTINCTION IN THE EARLY INTERMEDIATE PERIOD

To reach a better understanding of Wittgenstein later alternative proposal, we need, as we've said above, to investigate the dichotomy between "internal and external" properties. As we will argue, this dichotomy is directly related to the "aspect seeing" idea, as well as to Frege's aggregate/set distinction. In the following passage, Wittgenstein discusses Frege's definition of number employing just that terminology, internal vs. external properties:

We could say, as an approximation, that number (quantity) is an external property of a concept and an internal property of its extension (of the list of objects that fall under it). A number is a schema for the extension of a concept. (2005, BT 116 p. 399)

Let's first examine the idea of an "internal property". What does it mean to be the property of an extension or, more precisely, of a list? For Frege, properties (concepts) *have* extensions, but

they could not be said to be "properties of their extension", they are rather "properties" of *objects belonging to that extension*. Consequently, being a property of an extension should be something distinct from being a first-order concept. Now, if we consider the following statements: "*The sign for the extension of a concept is a list*", "[the] *number (quantity) is [...] an internal property of its extension*" and finally "*A number is a schema for the extension of a concept*", the conclusion is that a *number*, being an internal property of the extension, is a schema for the sign for the extension. It is not quite clear from the passage, neither the reason why Wittgenstein introduces the notion of "schema", nor the nature of those new entities, the "schemas".

To clear things up a bit, then, let us remember a crucial point here. According to Wittgenstein's ideas at the beginning of the intermediate period, one cannot make meaningful propositions about internal properties, a point reminiscent of TLP.

It is nonsense to say of an extension that it has such and such a number, since the number is an internal property of the extension. But you can ascribe a number to the concept that collects the extension (just as you can say that this extension satisfies the concept). (1975, [PhBm XI-119] p. 141)

On the one hand, an internal property cannot be meaningfully asserted about an extension, because things could not have been otherwise. Now, if there are no other alternative way things could be, the assertion of that fact would be a necessary statement. But, according to Wittgenstein, at least in the TLP, only bipolar contingent propositions could be meaningfully asserted, necessary ones could not be properly asserted. On the other hand, within the same passage, Wittgenstein gives us an example of what can be meaningfully asserted:

We can say that there are 2 circles in this square, even if in reality there are 3, and this proposition is simply false. But I cannot say that this group of circles is comprised of 2 circles, and just as little that it's comprised of 3 circles, since I should then be ascribing an internal property. (1975, [PhBm XI-119] p.141)

So, one can say – meaningfully – that there are "two circles" which satisfy the property of "being in this square". The reason is that it is contingent how many circles we draw in the

square, or it's an external property of the things identified by the concept "being a circle".⁹ Now, "3" is an internal property of the *extension* of the concept "being a circle and being in this square" and, so, something necessary or essential to that list. Wittgenstein position at this regard at that time seems to be that an external property can be asserted about a concept, although an internal property cannot be asserted about its extension.

In this passage from the BT, Wittgenstein tries to elucidate further the nature of such linguistic entities, number-statements:

A number is a schema for the extension of a concept. That is, as Frege said, a statement of number is a statement about a concept (a predicate). It's not about the extension of a concept, i.e. a list that may be something like the extension of a concept, but a number-statement about a concept has a similarity to a proposition saying that a determinate list is the extension of the concept. I use such a list when I say "a, b, c, d, fall under the concept $F(x)$ ": "a, b, c, d," is the list. Of course this proposition says the same as $Fa.Fb.Fc.Fd$; but the use of the list in writing the proposition *shows* its relationship to " $(\exists x, y, z, u). Fx.Fy.Fz.Fu$ " which we can abbreviate as " $(\exists ||||x).F(x)$." (2005, [BT 116-583] p. 399)¹⁰

At the beginning of this passage, Wittgenstein evokes Frege to explain his point: "a statement of number is a statement about a concept". He then introduces an example: "a, b, c, d, fall under the concept $F(x)$ ". According to him, one cannot assert an internal property of a list like that. What one can do with such lists is to use them to make a statement about a concept. At the end of the passage, he attempts to correlate the singularity of the list with the generality expressed by the existential quantifier. We can observe in a number-statement like that list, the presence of 4 letters naming 4 objects, respectively. Following Wittgenstein, this number-statement "says the same as" " $Fa.Fb.Fc.Fd$ ". The advantage of the last formulation is that it *shows* its internal connection to a general statement, $(x, y, z, u). Fx.Fy.Fz.Fu$, since we can count the 4 letters in the first and the 4 parameters in the second. A further important point about this passage is that, in its abbreviated form, the general statement was rewritten employing but 4

9 For Frege, "two" is a second-order concept asserted about the first-order concept "being a circle in the square".

10 This passage was written between 1029-33.

bars, " $(\exists ||||x).F(x)$ ". The bars are not variables, but parameters, i.e., they mark the syntactical occurrences of a finite number of objects within that list. Lastly, we could point out that such bar notation is a way of referring, and at the same time exemplifying, the objects within the list.¹¹

One additional point about Wittgenstein's notational device is that it reminds us of the distinction between "asserting" and "showing" in *TLP*, for in this notation the four bars also "display" the cardinality of the list. It is as if, although we cannot *assert* the internal property about the list, we can *exemplify* the list inside the statement as an instrument for asserting something about its relation to the concept from which it derives.

The distinction between internal and external properties has the advantage of establishing a clear connection with that other important idea we've been discussing, that of "aspect seeing". In *the BT*, Wittgenstein himself makes the correlation explicit:

We don't *see* that something can be looked at in a certain way until it has been so looked at. We don't see that an *aspect* is possible until it is there. [...]

Indeed, seeing the *internal relation* can in its turn only be seeing something that can be described, something of which one can say: "I see that this is the way things are"; so really it can only be something like the signs for correlations (such as connecting lines, brackets, substitutions, etc.). And everything else has to be contained in the application of the sign for the general rule in a particular case. [My emphasis] (2005, [BT 134-721] p. 478-9)

In this passage, Wittgenstein claims that we cannot "see" the internal relation until signs are used to represent it. Nevertheless, he admits the possibility of describing it, when he says that: "seeing the internal relation can in its turn only be seeing something that can be described".

11 In his Doctoral Thesis, Nakano discuss this passage and gives a slightly different interpretation. There, Nakano was trying to answer the following question: Was Wittgenstein inclined to accept Frege's idea that number is a predication about a concept in the beginning of the intermediate period? For him, Wittgenstein's reason for not recognizing the assertion of an internal property as meaningful is just that "number is a constitutive property – an aspect – of the extension and, therefore, cannot be depict by a proposition" (2015, p.75). Nakano's conclusion is that, with this statement, Wittgenstein is not implying anything more than that the assertion: "a, b, c are 3 objects" doesn't add nothing more than what is said by the other assertion: "a, b and c".

This could sound paradoxical though. After all, can one, or not, make a meaningful statement about an internal property?

Wittgenstein solution seems to involve again the “asserting” and “showing” tractarian distinction. In fact, after seeing it, we “become capable” of describing what we saw, but only by means of “signs for correlations (such as connecting lines, brackets, substitutions, etc.)”. This notational device will in a certain sense introduce the new concept, which was previously overlooked. The procedure consists in extracting the new complex concept by means of the introduction of an extra notational device.

This procedure is reminiscent of Frege’s proposal of a fertile analysis.¹² The idea, common to both viewpoints is that one does not describe something which is already there explicitly, but sees it in the signs and in the internal propositional structure. As Wittgenstein says: “*We don’t see that an aspect is possible until it is there*”. The idea is sort of “bringing to light or uncovering” something which was already there, hidden in the predicative structure of our propositions.

In face of all such arguments, it is plausible to read Wittgenstein as employing rather the notion of “aspect”, instead of “concept”, but with a purpose somewhat similar to Frege’s. The similarity between the two philosopher’s approaches consists in that both recognize the need of an intermediary in order to be able to know just what to count. In Frege’s case, we would need a pivot concept, and in Wittgenstein’s, a new way of “seeing”. The former’s solution of viewing number as second-order properties is also reviewed by Wittgenstein in 1939, as we are going to discuss in the following section. So, in the next section, we will explore convergences and divergences between these two approaches to this whole line of argument.

V- GOOD AND BAD ASPECTS OF FREGE’S DEFINITION OF NUMBERS AS PROPERTY OF PROPERTIES ACCORDING TO WITTGENSTEIN.

In his 1939’s lectures, Wittgenstein mentions Frege’s definition of number as “*a colossally difficult step that had to be taken*”:

12 Cf. (RUFFINO 1991)

In the definition of number, Russell and Frege made one great step – a colossally difficult step that had to be taken. Frege defined a number as a property of a property. It is not a property of a heap of apples. But it is a property of 'the property of being an apple lying on this chair'. This made one thing very clear: the relation between number and property. (1976, XVII p. 166)

In this passage, he is referring specifically to Frege's famous distinction between aggregates and sets, from the initial paragraphs (§§18-27) of the *FA*. As we've stressed before, numbers are not first-order concepts for Frege, and so cannot be attributed directly to "zero level" reality elements. According to that philosopher, a number attribution demands a first-order concept – a *pivot concept*, which tells us what to count: chairs, bits of woods, molecules, etc. As we know, for him numbers appear as "properties of properties", or "second-order concepts".

The second step in order to connect Wittgenstein's "aspect seeing" approach to Frege's "aggregates and sets distinction" is quite apparent in the following passage, where Wittgenstein shows his awareness and concern regarding the very problem Frege's definition of number was supposed to be a solution. In the following passage, he gives as an example the concept – "being five nuts on the table" – and explains Frege's idea in details.

Take Frege's statement that a numerical statement is a statement about a concept. This means that if we say, for example, "There are five nuts on the table" the five is not predicated of a heap but of a concept. We don't say that what we see here has the property five; because what we see here may have any number – one or a million; is it the number of atoms, for instance? But the concept "nuts on the table" has the property five. This is a very great clarification as far as it goes. (1976, XXVII-I p. 262)

In this passage, one could say that Wittgenstein at least does think very highly of Frege's arguments and is clearly in agreement with them. He even accepts some of its consequences, e.g., that one has a lot of different ways to segment (or "see") the same *aggregate or situation*, different aspects one could consider in other to describe a situation and count the number of objects in it. As he puts it at the end of the passage: "*This made one thing very clear: the relation between number and property.*" In this further passage below, we find again Wittgenstein recognizing the value of Frege's arguments, but, unlike the previews philosopher, not entirely

assuming a crucial consequence of the aggregate/set arguments: the thesis that attributions of numbers *always* and *necessarily* involve second-order predications.

What we normally mean by number is not at all always a property of a property. Because we would not know what has that property. Yet Frege's definition has made an enormous amount clear. (1976, XVII p. 168)

According to Wittgenstein, Frege enhanced a lot our comprehension of the nature of numbers by bringing to light a very difficult and delicate philosophical point, but he is not entirely satisfied with Frege's proposal. According to him, Frege didn't formulate a totally clear and unambiguous solution to the problem he has identified. Quite on the contrary. According to Wittgenstein, Frege's alternative seems to misformulate the linguistic nature of numerical attributions. In his view, Frege's (but also Russell's) formalizations of those statements lead to some confusions which were not there before.

In fact, it makes confusions. I do not mean it is valueless. But it does not show the point of anything; it leaves everything as it is. It makes language a trifle more explicit, leaving all the confusions. It makes certain points clearer. It does not go into detail. It avoids certain limited confusions. And I don't think that doing this work was a simple thing; quite the opposite. It translates arithmetic into a language in which we see certain points, which we did not see before, and get into certain confusions which we would not have got into before. (1976, XXVII-I p. 264-65)

He adds to this remark some examples of a kind of predicative analysis of our ordinary propositions made by Frege which could have been at the root of the confusion.

Take another case: Frege, when he had said that a numerical statement was about a concept, went on to say that a numerical statement was predicating something of a predicate. He would say that "a man on this sofa", for instance, is a predicate; because we say "Turing is a man on this sofa", just as we say "This sofa is green". And so when we say "There are two men on this sofa", we are saying that the predicate "man on this sofa" has the predicate "two". -*This is both a clarification and a confusion.* There is a temptation to talk of a predicate of a predicate. One had the word "predicate"; and at first no one would have talked of a

predicate of a predicate. People used to talk of subject and predicate in logic. And then Frege said, "We have a predicate of a predicate." One of the great things was the jingle. It was a grand discovery. And if you like it, you had better stick to it. (Compare "A class of classes".) [my emphasis] (1976, XXVII-I p. 262-63)

On the one hand, then, Frege's *enormous* discovery was a "pointer", and a warning, which set us in the right direction. The warning is that it is only by means of *first-order concepts* that we will be able to "see" objects in any situations and thus treat them as units. In other words: we count *apples, fruits in this chair, nuts on the table, bits of woods* or even *atoms* or *molecules*. But, on the other hand, the proposal also leads us to some confusions of its own.

These confusions begin when we treat the statement "There are two men on this sofa" in the same way as we treat the other two statements: "Turing is a man on this sofa" and "This sofa is green". The point shared by both, Frege's and Russell's accounts of language, is that our ordinary statements should be revised and converted into a new logical language, unambiguous and clear. In this new language, all propositions, including the numerical attributions, should have a nominal and a predicative part. Following this line of thought, one could end up thinking that, even if a statement does not contain any singular term, one could still consider its general term as the nominal part. As we are going to explain below, the quantifiers would have to assume the role of properties, i.e., they will have to be properties of properties.

As Frege emphasizes (1977, p. 48), the construal of quantifiers as second-order concepts could be also advocated if we take into consideration the way one negates those plural statements and derive its contradictory form. For example, the negation of "All mammals are land-dwellers" is not "All mammals are not land-dwellers", but "Not *all* mammals are land-dwellers". According to him, therefore, what we deny is what he calls the "second-order concept 'All'" and not the grammatical subject-concept "Mammals". Furthermore, the logical subject, about which the logical predicate "all" is denied, is the first-order concept "being mammals and so land-dwellers".¹³

In the following passage, Wittgenstein gives us further indications of just how the treatment of quantifiers as second-order concepts will led us into confusions:

Again, take: "I met a man" and "I met John". This look very much alike; it looks as though "a man" were like "John". – Frege expressed this in an entirely different way from Russell. Russell expresses "I met John" by " $f(a)$ " and "I met a man" by " λx ". – This is in a way similar to what Frege said about predicating the number, not of the heap of nuts, but of the concept; because this also cleared the grammar enormously and made certain misunderstandings impossible. (1976, XXVII-I p. 262)

In the second part of the passage, Wittgenstein seems to be alluding here to the fact that both, Frege and Russell, treat the expression "there exists" as a second-order concepts – the quantifier – which should then be applied to first-order concepts. Wittgenstein emphasizes that we have numbers only after being able to "perceive" a situation with the "glasses" of first-order concepts (the "aspect seeing" idea). So, he agrees with Frege that number could not be applied directly to objects, as first-order concepts would be. This part seems to be indisputable. And, as we've seen before Wittgenstein takes this move to have "*cleared enormously the grammar*".

Despite approximating both Russell and Frege in this respect, in the first part of the passage above Wittgenstein also makes a curious remark about a dissimilarity between their use of the existential quantifier in singular existential propositions. He took that example from Russell's book *The philosophy of Logical Atomism*, where the later offers a formalization of the statement "I met a man". For Russell, this case is not a case of a proposition, but of a propositional function. In his own words:

A propositional function is simply any expression containing an undetermined constituent, or several undetermined constituents, and becoming a proposition as soon as the undetermined constituents are determined.

Russell's idea is that great mistakes steams from the confusion between propositions and propositional functions:

Propositions can only be true or false, but propositional functions have these three possibilities. It is important, I think, to realize that the whole doctrine of modality only applies to propositional functions, not to propositions. (1972, p. 65)

In Russell's account, statements like "I met a man" have an undetermined component and should be analyzed as a propositional function. Singular statements should be analyzed as definite descriptions in turn, for they are propositions and not propositional functions, and so are either true or false. In neither case Russell maintains though, the ordinary language formulation with respect to the nominal part of that statement. For Russell, all singular terms must be analyzed away from language, as he argued in his paper "On Denoting" (1905).

In Frege's account, on the contrary, one must use singular referential terms to refer to particular objects or individuals, and existential quantified statements to talk about properties. In this second case, we always have the same interpretation: a proposition containing a second order predication. Contrary to Russell, Frege doesn't have an intermediary general expression which says, about an undetermined element of the domain, that it has some property, though.

Frege has also a different account of singular propositions. On Frege's well known account, singular terms are a necessary part of any formal language. For him, singular terms within singular sentences *cannot* be formalized as descriptive noun phases, nor as undetermined propositional functions such as "there is an x such that ..." (as had Russell suggested in his theory of descriptions). In Frege's view, although singular terms are in a certain sense a kind of definite description, for they make reference to objects through their identifying properties – their "mode of presentation" –, they should not be treated as sentences, but as terms. Furthermore, any singular statement must have a nominal position occupied by a singular term, otherwise we would have no subject matter to be talked about. In Frege's account, even when we have an ordinary proper name, as Aristotle for example, we could replace it by some singular term, which contains identifying concepts.¹⁴ So, for him, we do have a way to use concepts to refer to objects of our empirical experience and we cannot translate those referring expression – singular terms – into sentences.¹⁵ Our suggestion about the passage quoted from Wittgenstein is that

14 Cf. Frege's explanation of the ambiguity of ordinary proper names in (FREGE 1977, p. 57-58). There he suggests that in a perfect formalized language they should be replaced by the appropriate singular term, which refers to the object by means of its mode of presentation (interpreted as some conjunction of identifying properties).

15 In the introduction of the book *Logicism and the Philosophy of Language, Selections from Frege and Russell*, Sullivan suggests that "Russell's semantics of quantification is identical to Frege's, in that quantifiers behave semantically as functions from predicates to truth-values. However, definite descriptions, phrases of

he seems to be more comfortable with Frege's position than that of Russell's. Wittgenstein's difficulties with Russell's formalization of general existential statements is that, according to it, one is allowed to refer to an undetermined thing of the domain, but in Frege's.

According to Wittgenstein, not only Russell's notion of "propositional function" is mistaken, but it is the very source of a "muddle":

This business of "the property of being a man on this sofa" is a terrible muddle. – "This is a horse." But "Here is an x which is a horse" – no. The idea of all these things being predicates "man", "circle", etc. – is a mass of confusions, and is of course embedded in Russell's symbolism:

$(\exists x)$

This comes from the English "There is a man such that. . ." But nothing in English provides for "There is an x which is a man." What would you say it is that is a man? (1976, XVII p. 167)

Here is the muddle then: the existential quantifier used by Russell's construal of propositional functions allows an undetermined way of referring to things in our domain and thus predicate properties about them. As we've already said, with his definite descriptions, Russell offers a means to analyze away all singular terms from ordinary language. The predicative structure of these statements, which result from the transformation of singular terms into sentences, would again imply the unwanted formulation: there is an x of an unspecified nature, picked up from a universal domain, which have the property P . According to Wittgenstein, the tangle here is that one cannot "pick up an unspecified thing".

The problem of picking up an unspecified thing out of a domain is the same which lead Frege to adopt his construal of quantifiers as second order properties: one can "see" the same aggregate or situation in a variety of ways, and each one of them gives way to a different collection

the form 'the F ', are counted as referring expressions by Frege, but as quantifies noun phrases by Russell. This Theory of description is both, one of the most important differences between the views of Frege and Russell, and one of the most important ideas or developments in the philosophy of language." (SULLIVAN 2003, p. 78) Cf, also (CHATEAUBRIAND 2001, p. 98-101).

of objects. Nonetheless, in the clear and unambiguous form of Russell's and Frege's formalized language, the use of existential quantifiers leads us to think, quite erroneously, that one could pick up "bare objects" to be the values of a propositional function or definite description variable. What is being left out here is *Frege's own argument*, according to which one does *not* have just one unique "characteristic manner" to pick up some object. To deal with the arguments which led Frege to construe quantifiers as second order properties and to count objects, Wittgenstein's proposal would be to apply the "aspect seeing" view.

Now, part of Frege's solution involves "embedding", so to speak, a conceptual description into the singular terms. This first move brushes-off the difficulty we've been talking about just from the singular part of language. Nevertheless, Frege's second move is the construal of quantified statements as second order propositions and this move produces an underdetermination regarding the subject of our general statements. Under Frege's formalization, they will be translated into a collection of "incomplete" propositions.

But consider Russell's idea of generality for example. And also Frege's idea that a numerical statement is about a predicate. This is all right only in a grammatical sense: in so far as in English or German, when we say "There are five circles in this square" or "There have been five thunderstorms in the week", etc., these are predicates only in the sense that we can form a proposition "So-and-so is a circle in the square." In most cases this is not so.

In "There are three circles in the square" we don't say anything about things which *are circles and are in the square*. But: "All the geometrical figures in this square are circles" - then it is *the geometrical figures* which are circles. [My emphasis] (1976, XXVIII-I p. 268-69)

Wittgenstein's point is that a concept can be absent from the nominal part of any determined proposition, only if one has a statement of the form: "the 'so-an-so' is a circle" and it should be accompanied by an ostensive act. The reason is what we have here is a case of a demonstrative use of an expression, the expression "so-an-so" in quotes¹⁶. So, if it is a *proposition* which is intended, and propositions must always be able to be true or false according to Wittgenstein view, one must have some way to identify the subject. Only after "picking up"

16 In this case, we could use "this" or "that" also.

a selected area of the geometrical figure by means of a first-order concept (or perhaps an expression supplemented by a demonstrative act), something could be truly predicated about it. So, if our statement is about a figure containing a square, and some circles drawn within it, the statement “All the geometrical figures in this square are circles” would make perfect sense. As we are going to show with some examples in the next section, this same drawing could also be “seen” from a lot of other different perspectives, and this means to apply the “aspects seeing” approach in order to segregate, or “see” it, as alternative collections of objects. For example, we could take the same figure as being about “circles being inside a square”. Then we would say that “all the circles drawn in this paper are inside the square”.

An important point about those examples, is that all those alternative segregations will be compatible with the same empirical situation. Therefore, Wittgenstein main point against Russell’s, but also against Frege’s conception of generality, is that one cannot apply a predicate to an unqualified thing. One has to segregate the situation into objects first, applying some “aspect view” to it. Only after that one can proceed and predicate something regarding that subject. At the end of this process we will finally obtain a meaningful proposition, one which can be genuinely true or false. As Wittgenstein says in the following passage, in ordinary language, it *never* makes sense to talk about *bare* objects, propertyless objects.

The truth is that the way of writing a generality: is taken from ordinary language. Only in ordinary language we never say, “There is a thing which is a man and has grey trousers.” We never talk about bare individuals. We say instead, “There is a man who has grey trousers”. (1976, XXVIII-I p. 268-69)

Wittgenstein reminds us here that, although Russell’s treatment of formalized existential propositions is taken “*from the [ordinary] English ‘There is a man such that.’*”, nothing in ordinary English accounts for “there is a *thing* which is a man and ...”. His suggestion is that we should choose the ordinary language treatment of existential propositions and not Russell’s. The mistake of Russell’s account of generality is that it erroneously assumes that one could talk about an “unspecific bare individual” without mentioning any of its properties.

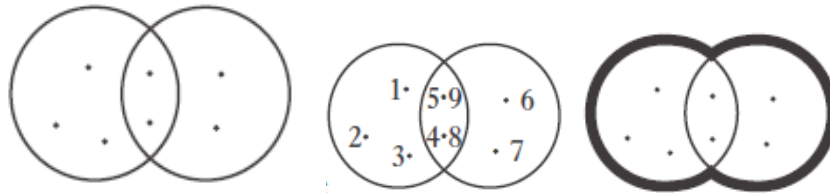
Summarizing the conclusion obtained in this section, Wittgenstein agrees with Frege’s arguments in FA, but doesn’t accept Russell’s and Frege’s construction of generality, which in

fact goes against their initial insight and implies an erroneous way to construe general statements about existence.

VI- SOME EXAMPLES OF THE "ASPECT SEEING" ARGUMENT IN THE *PI*

Now that we can employ the notion of *aspect*, the idea of there being "more than *one way of seeing* the same *aggregate/event situation*", in a similar way Frege would have employed his idea of the need of first order pivot concepts, let us look again at some other passages where Wittgenstein is employing it.

Are there 9 dots or 7 in these two circles? The way this is normally understood, there are 7. But do I have to understand it this way? Why shouldn't I count the dots that jointly belong to both circles twice: It's different if we ask: "How many dots are there in the borders outlined in bold?" (2005, [BT 116-589] p. 403)



In another characteristic example extract from the BT, Wittgenstein addresses the problem of not having only one characteristic method of dividing the rectangular figure below:

We can describe the way a rectangle is divided by saying: It is divided into 5 parts, or: Four parts have been cut out of it, or: Its division-schema is ABCDE, or: You can traverse all the parts by crossing 4 boundaries or: The rectangle is divided (i.e. into 2 parts), one part is subdivided, and both parts of this division are subdivided – etc. *I want to show that there isn't only one method of describing the way it's divided.* [my emphasis] (2005, [BT 115-577] p. 397)



Following this same line of thought, Wittgenstein gives yet another very striking example in §48 of his *PI*.

'RRBGGGRWW'

1	2	3
4	5	6
7	8	9
r	r	b
g	g	g
r	w	w

There, he makes proposes some queries about the figure containing squares:

Here the proposition is a complex of names, to which corresponds a complex of elements. The primary elements are the colored squares. "But are these simple?" – I do not know what else you would have me call "the simples", what would be more natural in this language-game. But under other circumstances I should call a monochrome square "composite", consisting perhaps of two rectangles, or of the elements color and shape. But the concept of complexity might also be so extended that a smaller area was said to be 'composed' of

a greater area and another one subtracted from it. But I do not know whether to say that the figured described by our proposition consists of four or of nine elements! Well, does the proposition consist of four letters or of nine? - And which are its elements, the types of letter, or the letters? Does it matter which we say, so long as we avoid misunderstandings in any particular case?

This example is quite emblematic, because Wittgenstein is presenting straight out here all these diverse ways of counting the simple elements in question and claiming that they all should have an equal status, no one is more fundamental than the other. If we use the "types of letters" view, we will have to count 4 types in the figure, but if we use the "letters" view, we must count 9 *in the same figure*. Moreover, if we "see it as" areas, we could have 1 big square or 9 little squares. We also have an unexpected alternative: "the complement view".¹⁷ In this latter alternative, the ostensive gesture indicates the subtracted area, but what we want to point to is really everything else besides that area. As Wittgenstein's put it: the whole paper but one square, or even the nine square holes altogether.

Our suggestion is that in all those passage Wittgenstein is employing Frege's argument. The alternative ways involve each one of them diverse "aspects" of one and the *same* event. Rephrasing in Frege's vocabulary, they result from the application of different first-order concepts to one and the same aggregate. And when we do this we apply different second-order concepts, and different numbers are suitable to those first-order concepts.

VI- CONCLUSION

In the TLP, there was the goal of analyzing away all general terms from the EPs through a unique chain of definitions, which should terminate in a complete and extensional disjunction of conjunctions of EPs for each unanalyzed proposition. In our construal above, the process of analysis in *TLP* aimed to eliminate all the qualitative parts of any proposition and replace them, in a reductionist vein, by mere sequences of EPs, composed only by simple objects' names. The abandonment of this foundational and reductionist view of language, however the account one provides for it, is a quite well-known trait of Wittgenstein's Philosophy.

17 Once again, reminiscent of Quine's idea of "cosmic complements" (1992, p. 33).

In this paper, some aspects of this anti-reductionist approach were revisited and discussed. Within this discussion, other important notions emanated from Wittgenstein early works, like “internal and external properties”, the “show and saying distinction”, as well as Wittgenstein’s reactions regarding Frege’s famous definition of numbers as “properties of properties” were also tackled and discussed. Our aim was to understand better Wittgenstein’s own self-critique of TLP’s “Great Analysis”. We’ve tried to show that Wittgenstein’s self-critique, as well as the ensuing alternative ways to analyze ordinary propositions, were both a result of Wittgenstein’s concerns *vis-à-vis* Frege’s arguments in FA and above all, a result of his construal of Frege’s definition of numerical attributions as involving second order predications.

As an outcome of the confrontation of Frege’s position with Wittgenstein’s “ideas”, we’ve proposed that the latter has developed the “aspect seeing” view as an alternative to the elder philosopher’s construal, which took first-order concepts as the subject of second-order numerical attributions. According to Wittgenstein’s account, Frege’s solution was a pointer to a new way of seeing the whole issue, one which helped clarifying a lot of confusion. Nonetheless, those arguments led to Frege’s and Russell’s new treatment of generality. In Wittgenstein’s view, both treatments were wrong and led us into a muddle: the interpretation of the existential quantifier that comes with it.

We also claim that Wittgenstein’s critique of his old mentors’ construal of generality was not the same. He differentiates Frege’s from Russell’s conception of how to formalize propositions involving existence. Both Wittgenstein’s preceptors are descriptivists, of course, for according to their proposals one can only refer to an object by means of its properties. But, on Russell’s descriptivism, all singular statements should be translated into a formalized version in which singular terms must be analyzed away. For Russell then, every statement about individuals which is not a definite description can be formalized as a statement about unspecific “things” within a domain. This lack of specificity comes directly from his theory of descriptions. According to it, the so-called propositional functions are not about particular individuals, but about “unspecific ones”. On Wittgenstein’s opinion, Russell’s existential quantified propositional functions are missing something, namely, their nominal part. The younger philosopher claims that, when one wishes to say something about a subject, one must employ the “aspect seeing” view and segregate objects within the represented situation. For him, as well as for Frege, only after determining the subject matter of one’s statement can something ever be asserted about it.

In this topic, Frege seems to offer a better approach than Russell's. For Frege, one should not dispense with singular terms unreservedly. They are an essential part of all singular propositions, in Frege's opinion. Nonetheless, Frege's account of the existential quantifier still imply an assertion about an unspecified thing of the domain. For Wittgenstein then, Russell's and Frege's account of generality leads us "into a muddle", the muddle of not knowing what, if anything, has the property predicated of the "undefined x ". Wittgenstein's uneasiness towards the existential quantifier is a warning regarding the misleading postulation of a universal domain of predication. For him, unless we identify whatever we are talking about, it makes no sense to assert any predication about that at all.

A corollary of our discussion of Wittgenstein's criticism regarding Frege's and Russell's treatment of generality is the requirement that the nominal part of any proposition had to be fixed as a precondition to propositionality, something capable of being true or being false. We've found an intimate connection between such precondition to propositionality and the "aspect seeing" view, for precisely these aspects are supposed to do the task of fixing the nominal part of such statements.

We then moved back to our main concern, i.e., to explain why a proposition would have more than one analysis. Our hypothesis was that those descending chains of definitions, or internal relations, which would guide us in TLP's Great Analysis through the whole analytical process, all the way to its terminus, should not be a unique. On our construal of Wittgenstein's view in PI, the analysis of propositions must depend on the "aspect" or the "way of seeing" the situation described. If one is interested in a physical elucidation of the nature of a table, one could analyze the proposition "This is a table" as being about a collection of organic molecules and follow this path to go on with such analysis; but, if one is interested in constructing a table for her/his sitting room, it is better to "see" the subject matter of the unanalyzed propositions as a bunch of undetached table parts instead.

In order to exemplify our thesis, we quoted some passages from the *PI* and from the *BT* where, according to our proposal, the "aspect seeing" approach is being employed. We've given examples of propositions in different linguistic context: empirical context, geometrical figures, drawings and even mathematical contexts. In all those examples, we noticed that there were some relations connecting the alternative "aspect seeing" to one and the same "situation".

However, they are also different views, for they would be employed alternatively for analyzing the same initial proposition. In *the PI* and forward, Wittgenstein holds that it doesn't make sense to take a proposition out of its context of use and analyze it any "unique manner". It is not that one cannot analyze propositions, but it won't be a context-isolated and unique procedure.

Our investigation is therefore a grammatical one. Such an investigation sheds light on our problem by clearing misunderstandings away. Misunderstandings concerning the use of words, caused, among other things, by certain analogies between the forms of expression in different regions of language. – Some of them can be removed by substituting one form of expression for another; this may be called an "analysis" of our forms of expression, for the process is sometimes like one of taking a thing apart. (§ 90)

RESUMO

o propósito desse artigo é circunscrever e discutir a autocrítica, feita por Wittgenstein no período de 1933-39, a uma das teses mais fundamentais do Tractatus (TLP): "Há uma e apenas uma análise completa da proposição" (3.25). Chamaremos esse procedimento peculiar de a "Grande análise". Os argumentos de Wittgenstein contra a sustentabilidade da sua antiga tese podem ser encontrados em algumas passagens do livro Investigações Filosóficas (IF), bem como em passagens do Grande Dátiloscrito (BT). Conforme será argumentado nesse artigo, essa autocrítica é uma consequência direta da preocupação de Wittgenstein, no período intermediário, com duas importantes propostas feitas por Frege nos §§18-24 do Os Fundamentos da Aritmética (FA) em relação o seu entendimento da noção de "atribuição numérica". Nosso objetivo principal será mostrar de que maneira o movimento de autocrítica feito por Wittgenstein em relação a sua antiga concepção de "Grande Análise" teria sido influenciado por suas preocupações, no período intermediário de sua obra, com essas duas propostas feita por Frege no FA a respeito da noção de "atribuição numérica". A fim de completar tal tarefa, será apresentada inicialmente de modo breve uma versão de como seria a "Grande análise" no TLP. Em seguida, selecionaremos e discutiremos algumas passagens do IF nas quais Wittgenstein recusa a completude e unicidade envolvida em sua prévia concepção tratariana de "análise". Procuraremos destacar com clareza uma noção que estaria envolvida de um modo essencial na autocrítica de Wittgenstein: a noção de "ver como/ver aspectos". Como será argumentado, essa noção, assim como a autocrítica feita por Wittgenstein, foram ambas diretamente influenciadas pela noção fregiana de "atribuição numérica". Para completar essa segunda tarefa, teremos de revisitar e discutir algumas noções pertencentes ao período inicial da filosofia de Wittgenstein – como a de "propriedades internas e externas" e a distinção entre "dizer e mostrar" – ambas noções discutidas no TLP. No decorrer de nossa discussão, apresentaremos uma crítica feita

WITTGENSTEIN "GREAT ANALYSIS" AND FREGE'S CONSTRUAL OF NUMBER AS
A PROPERTY OF PROPERTIES

ANALYTICA
volume 21
número 1
2017

por Wittgenstein a Russell, mas também a Frege, sobre a concepção de generalidade, ou mais especificamente, sobre o uso do quantificador existencial, proposta por ambos.

Palavras-chave Wittgenstein; Análise de Proposições; Predicação de Segunda ordem; Visão de Aspectos.

ABSTRACT

The purpose of this article is to discuss Wittgenstein 1933-39 self-criticism of one of the most fundamental tractarian theses: There is one and only one complete analysis of the proposition (3.25). Wittgenstein's arguments against his previous thesis can be found in some passages of the Philosophical Investigation (PI) as well as in the Big Typescript (BT). As we shall argue, Wittgenstein's self-critique is directly related to two important ideas proposed by Frege in §§18-24 of The Foundations Arithmetic (FA) with respect to his construal of "number attributions". Our main goal in this paper will therefore be to show how Wittgenstein's self-criticism of his old conception of the tractarian "analysis" would have been influenced by his concerns in the middle period of his work with these two proposals made by Frege in FA regarding the notion of "numerical attribution". Our first task will then be to connect Wittgenstein's self-criticisms to his former conception of "analysis" in the Tractatus Logico-Philosophicus (TLP). To accomplish this first goal, we will have to briefly present a version of how such analysis would look like in TLP. Our next task will be to pinpoint some passages of the PI in which Wittgenstein rejects the completeness and uniqueness involved in his previous construal of the analytical process. We will try to circumscribe clearly a crucial notion involved in Wittgenstein's self-critique: the idea of "seeing as/seeing aspects". Our claim is that this notion, as well as Wittgenstein's self-criticism, were both directly influenced by Frege's construal of the "number attribution" presented in FA. To accomplish this second task, we will have to revisit and discuss some further concepts from an early period of Wittgenstein's philosophy, like "internal and internal properties" and the "saying/showing distinction" in TLP. On our way towards accomplishing this goal, we will present a curious complaint by Wittgenstein regarding Russell's, as well as Frege's conception of generality, or more specifically, on the usage of the existential quantifier suggested by both these authors.

Keywords Wittgenstein; Propositional Analysis; Second-order Predication; Aspect Seeing.

Obras Citadas

CHATEAUBRIAND, Osvaldo. 2001. *Logical Forms Part I - Truth and Descriptions*. Vol. I. 2 vols. Campinas, São Paulo: Itala M. Loffredo D'Ottaviano.

FREGE, Gottlob. 1977. "On Concept and Object." In *Translations from the writings of Gottlob Frege*, by Gottlob FREGE, 42-55. Oxford: GEACH, Black; GEACH, Peter.

FREGE, Gottlob. 1977. "On sense and reference." In *Translations from the philosophical writings of Gottlob Frege*, edited by P. GEACH and M. BLACK, 56-78. Oxford: Basil Blackwell.

—. 1964. *The Basic Laws of Arithmetic exposition of the system (BLA)*. Berkeley and Los Angeles: University of California Press.

—. 1953. *The Foundations of Arithmetic*. Translated by J. L. AUSTIN. Oxford: Basil Blackwell.

GRIFFIN, James. 1964. *Wittgenstein's Logical Atomism*. Oxford: Oxford University Press.

NAKANO, Anderson Luis. 2015. "A MATEMÁTICA DAS PHILOSOPHISCHE BEMERKUNGEN: WITTGENSTEIN NO CONTEXTO DA GRUNDLAGENKRISE." *Tese de Doutorado defendida na UFSCar*. São Carlos, São Paulo, - -.

NARBOUX, Jean-Philippe. 2003. "ASPECTS DE L'ARITHMÉTIQUE." *Archives de Philosophie* (Centre Sèvres) 64: 569-591.

QUINE, W.V. O. 1980. "Identity, Ostension and Hypostasis." In *From a Logical Point of View*, by W.V. O. QUINE, 65-79. London, London: Journal of Philosophy.

QUINE, W.V. O. 1969. "Ontological Relativity." In *Ontological Relativity & other essays*, by W.V. O. QUINE, 26-68. New York: Columbia University Press.

QUINE, W.V. O. 1996. "Two Dogmas of Empiricism." In *From a Logical Point of View*, by W.V. O. QUINE, 20-46. Cambridge: Harvard University Press.

—. 1960. *Word and Object*. Cambridge: The MIT Press.

QUINE, Willard V. O. 1992. *Pursuit of Truth*. Cambridge: Harvard University Press.

RUFFINO, Marco Antonio. 1991. "Context Principle, Fruitfulness of Logic and the Cognitive Value of Arithmetic in Frege." *History and Philosophy of Logic* 12: 185-194.

RUSSELL, Bertrand. 1905. "On Denoting." *Mind*, - -: 479-493.

—. 1972. *The Philosophy of Logical Atomism*. London.

SULLIVAN, Arthur. 2003. "Introduction." In *Logicism and the Philosophy of Language: Selections from Frege and Russell*, by Arthur SULLIVAN, 15-88. Sydney: SULLIVAN, Arthur.

VELLOSO, Araceli. 2014. "Wittgenstein's unique "Great Analysis": a consequence of the construal of propositional sense as truth-conditions." *ANALYTICA*, Julho 31: 229-269.

—. 2011. "Revisitando a tese da indeterminação da referência de Quine." *Cad. Hist. Fil. Ci.*, jul-dez -: 309-326.

—. 2009. "Agregados, conjuntos e a tese da indeterminação da referência." *doispontos*, Outubro 1: 109-127.

WITTGENSTEIN, Ludwig. 1979. "Appendix I - 1913." In *Notes on Logic 1914-1916.*, by Ludwig WITTGENSTEIN, 93-107. Oxford: Basil Blackwell.

—. 1976. *Lectures on the Foundations of Mathematics 1939*. Edited by Cora Diamond. HARVESTER PRESS, LTD.

—. 2001. *Philosophical Investigations*. Translated by G. E. M. ANSCOMBE. Oxford: Blackwell Publishing.

—. 1975. *Philosophical Remarks*. Translated by Raymond Hargreaves and Roger White. Oxford: Basil Blackwell.

WITTGENSTEIN, Ludwig. 1993. "Some Remarks on Logical Form." In *Philosophical Occasion 1912-1951*, by Ludwig WITTGENSTEIN, edited by James C. Klagge and Alfred Nordmann, 29-35. indianapolis & Cambridge: Hackett Publishing Company.

—. 2005. *The Big Typescript TS 213*. Edited by C. Grant Luckhardt and Maximilian A. E. Aue. Malden: Blackwell Publishing.

—. 1961. *Tractatus Lógico-Philosophicus*. Translated by D. F. PEARS and B. F. McGUINNESS. London and Henley: Routledge & Kegan Paul.

Recebido em dezembro de 2017

Aprovado em fevereiro de 2018