RUSSELL ON PEANO'S AXIOMATIZATION OF ARITHMETIC

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Dedicated to the memory of Michael Wrigley (1953 - 2003)

The use of set theory is a common practice in the treatment of philosophical problems in the areas of philosophy of logic and philosophy of language, among others. Thus, when there are difficulties in understanding concepts such as *state of affairs*, *propositions*, *truth*, *meaning*, etc., set theoretical notions are commonly employed in order to elucidate these concepts. Moreover, there is a feeling among philosophers that if it is not possible to characterize a concept in this way we must eliminate it from our theory. Without a doubt, Quine was the most important philosopher to assume this attitude, and it is no coincidence that in a well-known passage in chapter VII of *Word and Object*, when he discusses the conception of *elucidation*, the example he gives is the set theoretical definition of *ordered pair*.

The main source of this conception of philosophical elucidation was the *logicist program*[1] . The impact of Russell and Frege's conceptions on Carnap, for instance, was decisive in his adoption of a certain way of conceiving philosophical analysis, and until the end of his life, he defended the idea that Frege's approach to arithmetic had provided a satisfactory solution for a difficult philosophical problem[2] . As a matter of

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(1) Of course, the works of mathematicians like Zermelo, Kuratowski, von Neumann, and others, had a great influence in the emergence of this conception. However, their ideas will not be discussed in this paper.

(2) See, for instance, his reply to Strawson's paper in Schilpp, p. 935.

ANALYTICA
volume 8
número 2
2004

fact, the *logicism* can be seen as the main source from where Carnap derives the idea that the proper task of philosophy is to provide *explanations* of concepts that can be found in every day life or in the early stage of scientific development. It is not a coincidence that on different occasions[3] he gives as examples of this method the Russell-Frege characterization of numbers and Tarski's work on truth, two results obtained within the framework of set theory.

Russell was always more concerned than Frege with the problem of obtaining some philosophical justification for the set theoretical characterization of arithmetic. On several occasions he considered this problem and tried to obtain a satisfactory solution to it. Along this line, the problems that we will examine from now on are: what argumentation does Russell use in order to justify his conception that numbers are sets? Is it sound? What do we attain, from a conceptual point of view, from this approach? This will be done mainly from Russell's argumentation presented in the first chapters of his book *Introduction to Mathematical Philosophy*[4] .

The first thing that we must do in order to obtain satisfactory answers to these questions is to analyze Russell's criticism of the axiomatization of arithmetic obtained by Peano in 1899. In the beginning of his book, in chapter one, we find the following statement:

> It is time now to turn to the considerations which make it necessary to advance beyond the standpoint of Peano, who represents the last perfection of the "arithmetisation" of mathematics (…) but we shall give some of the reasons why Peano's treatment is less final than it appears to be. (Russell, 1985, p. 7)

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(3) See *Meaning and Necessity*, pp. 7-8 and *Logical Foundations of Probability*, pp. 5 and 17.

(4) I will not discuss Benacerraf's argumentation presented in "What Number Could not Be", because I think that is possible to refute Russell's position without appealing to the existence of different set theoretical characterizations of numbers. Moreover, as Benacerraf himself has acknowledged ("What Mathematical Truth Could Not Be-I") his argumentation has several problems.

ANALYTICA
volume 8
número 2
2004

According to Russell, Peano's work could not be considered the last step in the elucidation of the concept of *number* because there were problems in interpreting the primitive terms in Peano's axioms[5] that could not receive a satisfactory solution.

It is possible to conceive Peano's basic notions (*zero*, *number*, and *successor*) as either variables or constants. There are three basic problems that we have to face if we take the first alternative. If they are variables it is possible to find an indefinite number of interpretations satisfying the axioms. For instance, if '0' has its habitual meaning, 'number' means the even numbers, and 'successor' refers to the operation of adding two to a given number, it is easy to see that all Peano's axioms are true in this interpretation. Thus, Peano's system does not accomplish its main task, namely its axioms do not allow us to distinguish between the series of natural numbers and any other progression[6] .

The second problem is that we do not know if the objects defined by Peano's axioms exist because if the primitives of the system are variables we must assume that the set of axioms is a definition of natural numbers, and it is a well-known fact that from a definition it does not follow that what is defined exists. In Peano's system even if we agree that its definition is acceptable, we do not have any assurance that the natural numbers really exist.

> (…) If we adopt this plan, our theorems will not be proved concerning an ascertained set of terms called 'natural numbers', but concerning all sets of terms having certain properties (…) But from two points of view it fails to give an adequate basis for

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(5) Peano's axiomatization has three primitive terms (*0*, *number* and *successor*) and five axioms: 1. 0 is a number; 2. The successor of any number is a number; 3. No two numbers have the same successor; 4. 0 is not the successor of any number; and, 5. Any property that belongs to 0, and also to the successor of every number which has the property, belongs to all numbers.

(6) A *progression* is a series that satisfies four conditions: 1. It has a first term; 2. Each member of the series has a successor; 3. There is no member that occurs twice in the series; and, 4. Every term can be reach from the start in a finite numbers of steps.

ANALYTICA
volume 8
número 2
2004

> arithmetic. In the first place, it does not enable us to know whether there are any sets of terms verifying Peano's axioms (…). (Russell, 1985, p. 10)

The last of Russell's objections is that the task that natural numbers have to accomplish is twofold. They have to verify the totality of the arithmetical formulae, and they must allow us to count common objects in our daily life; in other words, besides their theoretical function, natural numbers have a practical one and the existence of an infinite number of interpretations of Peano's system would make it impossible to fix a precise meaning for numeric expressions.

> (…) A system in which "1" meant 100, "2" meant 101, and so on, might be all right for pure mathematics, but would not suit daily life (…) we want our numbers to be such as can be used for counting common objects, and this requires that our numbers should have a definite meaning, not merely that they should have certain formal properties (…). (Russell, 1985, pp. 9-10)

Nevertheless, Russell attracts attention to the fact that we already have knowledge of the meaning of primitive terms independently of Peano's axioms, and it is from this fact that he begins to consider the second option mentioned above, namely the primitive terms have to be conceived as constants. From this point of view it is quite natural to consider that these concepts are of such a nature that no further analysis is possible. However, despite the naturalness of this solution Russell rejects it:

> (...) all that we can do, if we adopt this method, is to say "we know what we mean by '0' and 'numbers' and 'successor', though we cannot explain what we mean in terms of other simpler concepts". It is quite legitimate to say this when me must, and at some point we all must; but it is the object of mathematical philosophy to put off saying it as long as possible. By the logical theory of arithmetic we are able to put it off for a very long time (…). (Russell, 1985, p. 9)

According to Russell, Peano stopped his analysis too early, and in order to produce an adequate theory of arithmetic we must avoid assuming primitive

ANALYTICA
volume 8
número 2
2004

concepts unless we are sure that these concepts cannot be analyzed into simpler ones. Russell defends this idea with two kinds of arguments; on the one hand he uses technical results that he has obtained during his attempt to reduce mathematics to logic, and on the other hand he makes use of philosophical objections to show how Peano's account is unsatisfactory from a conceptual point of view.

Russell, independent of Frege, discovered how it is possible to obtain set theoretical definitions for the primitive terms of Peano's arithmetic. Thus, he defines *natural number* as the posterity of 0 with respect to the relation *immediate predecessor*; *number 0* as the class that has as its only member the empty set; and defines *successor* in the following way:

> The successor of the number of terms in the class a is the number of terms in the class consisting of a together with *x*, where *x* is any term not belonging to the class. (Russell, 1985, p. 23)

The next move is to prove Peano's axioms with the assistance of these definitions. From the definitions of *number* and *zero* Russell shows that two of Peano's axioms (number one and number five, see footnote 4) become true by definition. Thus, according to him, the principle of induction is what distinguished natural numbers from all other numerical entities:

> We shall use the phrase "inductive numbers" to mean the same set as we have hitherto spoken of as the "natural numbers". The phrase "inductive numbers" is preferable as affording a reminder that the definition of this set of numbers is obtained from mathematical induction. (Russell, 1985, p. 27)

Afterward, with the definition of *successor* Russell proves the following two axioms: that the successor of any number is a number, and that 0 is not the successor of any number. However, there is a problem in proving the third axiom because, if the number of individuals in the world is finite, it is false. The only

ANALYTICA
volume 8
número 2
2004

solution is to assume the *axiom of infinity*[7] that cannot receive any justification within the framework of *logicism*.

In relation to the philosophical aspect of Russell's criticism, it is not an easy task to understand his position since his ideas are presented in a brief way. In the end of chapter two we find the following passage:

> So far we have not suggested anything in the slightest degree paradoxical. But when we come to the actual definition of numbers we cannot avoid what must at first sight seem a paradox, though this impression will soon wear off. We naturally think that the class of couples (for example) is something different from the number 2. But there is no doubt about the class of couples: it is indubitable and not difficult to define, whereas the number 2, in any other sense, is a metaphysical entity about which we can never feel sure that it exist or that we have tracked it down. It is therefore more prudent to content ourselves with the class of couple, which we are sure of, than to hunt for a problematical number 2 which must always remain elusive. (Russell 1985 p. 18)

The idea is that, at first sight, it seems to be odd to identify numbers and sets; however the advantage of this approach is that numbers are elusive entities, whereas classes are not. Russell's conclusion is a little surprising: in spite of the unnatural consequences of the identification of these two types of entities, this is the only way to obtain a satisfactory explanation of the concept of *number*.

Is Russell's point of view acceptable? Are numbers sets? Is Peano's approach unsatisfactory? In order to answer these questions we must analyze Russell's remarks and establish if the reasons he provides are enough to confirm his conclusions.

First we must consider the case in which Peanos's primitive terms are conceived as variables. It is a historical fact that within the *Italian school*[8] it was a

(7) Russell's formulation of this can be found in chapter 13 of *Introduction*: if *n* be any inductive cardinal number, there is at least one class of individuals having *n* terms.

(8) This school was composed by Veronese, Pieri, Burali-Forti, Betazzi, Fano, among others, and all of them worked under Peano's influence.

ANALYTICA
volume 8
número 2
2004

common practice to adopt this approach in order to explain how an axiomatic theory works. However, if we adopt this view on the nature of the axiom it becomes very difficult to give an account of the several components of the axiomatic method. For instance, if the primitive terms are variables until we interpreted them the axioms are neither true nor false, because, according to this view, they are not propositions but propositional functions. But, from this perspective, for instance, how do we understand the concept of *proof*? A proof is a procedure that allows us to establish the truth of a theorem from the truth of the axioms. With this new approach it becomes impossible to maintain that one of the most important tasks of a proof is to produce knowledge and justification, since the notion of *truth* has disappeared from the basis of the theory.

A possible solution is to use the concept of *interpretation*, but then the problem indicated earlier by Russell appears again: which interpretation is the right one? It is important to bear in mind that if we interpret Peano's primitive as variables what we are doing is to move to another level and, in this way, we are no longer dealing with natural numbers, but with sequences in general. By the way, Hilbert's system of geometry had the same problem and Frege saw it clearly; when Hilbert says that his axioms are definitions of the primitive terms he is changing the level and working with a second-order geometry. In this case the interpretations are instantiations of more general concepts and not definitions of the primitive terms. Then, what is important to see is that the objection Russell raises is not concerned with the nature of the natural number concept, but one that involves the problem of how to understand the nature of primitive terms in an axiomatic theory. Russell is right in defending the idea that to take the axioms as definitions is not the right way of interpreting Peano's work, but this does not imply that we must reduce numbers to a more primitive entities; the only conclusion that we get from this fact is that we have to conceive the axioms in another way.

The second objection Russell raises is that if we treat the primitive terms as variables we will never be sure that what they define exists. This is a serious problem only if we think that what is prior is the theory and not the domain that is

ANALYTICA
volume 8
número 2
2004

described by it. If we adopt Aristotle's idea and take axioms as fundamental truths that describe a specific domain, what we have to do is to assume the existence of some entities. From this perspective, Peano's axiomatization has the same problem that any axiomatization has, namely it cannot assure the existence of everything, some entities must be assumed from the beginning. Of course then a new problem arises: how do we know what entities must be assumed? Again, what is important to understand here is that this problem shows that a certain way to look at the axiomatic method is not correct; this has no relation with Peano's approach that assumes that the concept of *number* is a primitive one.

The idea according to which natural numbers, besides satisfying the arithmetical formulae, must allow us to use them in our daily life, is a correct one. The problem, again, is to understand what the role-played by axiomatization is in this context. If we demand that Peano's axiomatization gives the meaning of number words we, again, must face the objection of multiples interpretation; but if we assume that Peano's axioms reflect the main features of a concept that goes before the axiomatization, then there is no problem; it would be odd to assume that before Peano's work nobody knew how to count! People have informal knowledge of arithmetical notions and any systematization must be in accordance with them, otherwise we always can affirm that the theory does not agree with the main insights about natural numbers, and must be rejected.

Now we have to examine Russell's statement according to which Peano had stopped his analysis too early. It is clear that what he has in mind is the fact Peano's axioms can be proven with the assistance of set theory. But, from this fact can we conclude that the domain of arithmetic is reducible to that of set theory?

From the technical considerations uncovered earlier it is possible to establish several philosophical conclusions, and in principle none has any sort of precedence in relation to the others until we provide some kind of justification that explains the choice of a particular one. One possible conclusion is to infer that set theory is committed with an ontology that includes natural numbers. Another possible conclusion is that set theory is a very strong theory and it is this fact that allows us to obtain *representations* of the main results of arithmetic within it. This

ANALYTICA
volume 8
número 2
2004

approach does not imply that numbers are sets, but that with the assistance of sets we can simulate the structural properties of numbers; in this case the two notions do not need to match completely. The conclusion is that technical results alone do not establish philosophical thesis; we always need some philosophical premise that must be sustained by appealing to general principles.

As a matter of fact Russell himself recognizes this when he says that 'we naturally think that the class of couples is different from the number 2'. Frege in *The Foundations of Arithmetic* also draws our attention to the same problem[9]:

> Another type of case is, I admit, conceivable, where the extension of the concept "equal to the concept F" might be wider or less wide that the extension of some other concept, which the could not, on our definition, be a Number; and it is not usual to speak of a Number as wider or less wide that the extension of a concept; but neither is there anything to prevent us speaking in this way, if such a case should ever occur. (Frege, 1980, §69)

It is interesting to see that until today this problem has received little attention and it is frequently considered a minor detail with no importance. Thus, we can read in Enderton's book on set theory:

> (…)This construction of the numbers as sets involves some extraneous properties that we did not originally expect. For example:
>
> $$0 \in 1 \in 2 \in 3 \in \ldots$$
>
> and
>
> $$0 \subseteq 1 \subseteq 2 \subseteq 3 \in \ldots$$
>
> But these properties can be regarded as accidental side effects of the definition. They do not harm, and actually will be convenient at times. (Enderton, 1977, p. 67)

(9) It is important to realize that Frege's conception on the nature of *extension of concept* is different from the actual conception of *sets*.

ANALYTICA
volume 8
número 2
2004

A possible solution to this problem would be to adopt Russell's position as suggested in his book *Our Knowledge of External World*:

> (…) In comparison with this merit, the question whether the objects to which the definition applies are like or unlike the vague ideas of numbers entertained by those who cannot give a definition, is of very little importance. (Russell 1995, p. 210)

Here we have several problems. The first is that Russell simply assumes that the notion of *number* is a definable one; but it is not so obvious that this is the case. Some type of argumentation must be presented, and what we see is that from Russell's criticism of Peano's work, nothing prevents us from treating the concept of number as a primitive one.

Perhaps Russell's additional thesis is that the notion of *number* is too elusive whereas we have a clear understanding of the concept of *set*. But is this true? With the set theoretical definition, do we have a precise definition of number? But is this enough? Not from a philosophical point of view. If the number 2 is an elusive entity, what do we say about the empty set? It is surprising that a philosopher like Russell who knows the philosophical problems related to set theory very well assumes such a position. What is a set? And what to say about singletons? By the way, what are sets according to Russell's conceptions? In chapter XVII of *Introduction to Mathematical Philosophy* after some considerations that pretend to show that sets are neither individuals nor propositional functions he says:

> When we have decided that classes cannot be things of the same sort as their members, that they cannot be just heap or aggregates, and also that they cannot be identified with propositional functions, it becomes very difficult to see what they can be, if they are to be more that symbolic fictions (Russell 1985 p. 184).

Russell's position is like that of a physicist, who after verifying that light's behavior is different from anything that we have ever seen, concludes that light

ANALYTICA
volume 8
número 2
2004

does not exist![10] Problems like these show that in order to obtain an ontological reduction of arithmetic to set theory we need, in addition to technical results, some kind of explanation that shows the intelligibility of this maneuver.

At several occasions Russell made use of *Ockham's razor*[11] in order to justify his approach, and later Quine will assume a similar attitude in relation to numbers and sets: if one type of entities is sufficient to get all we need, why assume another one? To presuppose both entities would be a violation of the *principle of parsimony*. However, we must be careful in using *Ockham's razor*, otherwise we can end up with a very economical system that explains nothing. A good example of this is provided by Quine in his paper "Things and Their Place in Theories" where he argues that we must abandon physical objects in favor of pure space-time, and that in the sequence, we can substitute space-time for quadruples of numbers according with a system of coordinates. In this way all we need in our ontology is the hierarchy of pure sets:

> (…) There are no longer any physical objects to serve as individuals at the base of the hierarchy of classes, but there is no harm in that. It is a common practice in set theory nowadays to start merely with the null class, form its unit class, and so on, thus generating an infinite lot of classes, from which all luxuriance of infinites can be generated. (Quine, 1981, pp. 17-18)

What does this argument prove? Nothing. We need some kind of conceptual explanation that gives some intelligibility to this reduction, and to appeal to

(10) Here we have a very interesting situation. It is a well known fact that light has a dual nature, in other words, at the level of quantum mechanics we only will obtain a satisfactory account of most of central facts about it if we assume its dual character, namely, that it is neither particle nor wave but both. It is very difficult, if not impossible, to find a physicist that from this fact concludes that light is a 'logical fiction' or that quantum theory is inconsistent and must be abandoned. Why not treat set theory in a similar way?

(11) The more common formulation of this principle is: entities should not be multiplied beyond necessity.

ANALYTICA
volume 8
número 2
2004

Ockham's razor is not enough. It is difficult to imagine a philosopher or a physicist who would be satisfied with this approach in relation to the question of what there is. This kind of wild reduction accomplishes only one task, which is to create a huge confusion in relation to the philosophical role of set theory.

The second problem Russell raises is that those opposed to the set theoretical approach only have 'vague ideas' about numbers. These ideas can be vague but they exist and are the main source from where any characterization of arithmetic must start. Of course, it can happen that in developing our basic arithmetical intuitions, we have to revise some of these ideas, what is difficult to understand is the proposal that the intuitive notion of *number* is useless. Later, Skolem will be astonished when he discovers non-standard models of arithmetic[12]. This is a good example of problems that we get in when we forget the basic intuitions with which we start our study of arithmetic[13].

At some points Russell seems to suggest that the difficulty to find a definition of the concept of *number* is an evidence in favor of the set theoretical approach. However, from the fact that we cannot find a satisfactory definition of numbers we can simply conclude that this concept is a primitive one. By the way, the fact that it is hard to explain a concept does not mean that it does not exist[14]:

> The mathematician's patterns, like the painter's or the poet's, must be *beautiful*; the ideas, like colors or the words, must fit together in a harmonious way. Beauty is the first test: there is no permanent place in the world for ugly mathematics (…) It may be very

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(12) Of course, that the existence of these models are an immediate consequence of the incompleteness of arithmetic established by Gödel in 1931. However, it was Skolem that for the first time constructed such a model.

(13) Kripke is absolutely right when he affirms: '(...) Certainly the philosopher of 'possible worlds' must take care that his technical apparatus not push him to ask questions whose meaningfulness is not supported by our original intuitions of possibility that gave the apparatus its point" (Kripke, p. 18)

(14) See, for instance, what happens with colors, they are indefinable but nobody will deny their existence because of that.

ANALYTICA
volume 8
número 2
2004

> hard to *define* mathematical beauty, but that is just as true of beauty of any kind – we may not know quite what we mean by a beautiful poem, but that does not prevent us from recognizing one when we read it. (Hardy, p. 368, 1990)

After all, what are the conclusions that can be established from the analysis of Russell's criticism of Peano's work? The first one is that mathematical results are not enough when the problem is to give a satisfactory answer to an ontological question. Russell, in some way, tries to provide these additional assumptions by appealing, basically, to three theses, and it is from them that we must obtain our last conclusions. The principles he adopts are the *Ockham's razor*, the idea that number are elusive entities, and that the concept of *set* is not difficult to define.

There are a lot of problems associated with *Ockham's razor*. How must we understand this principle? What is its exact formulation[15]? When we say that entities should not be multiplied beyond necessity, what exactly we mean? The answer to this will dependent on the meaning of the word 'necessity'. Necessity for what? If our problem is to get an intelligible ontological reduction, the need of some conceptual explanation is a necessary condition in order to justify this maneuver. In this sense Russell's use of *Ockham's razor* is not acceptable. In any way, the application of this principal involves difficult problems that have been discussed[16], and it must be used very carefully.

The problem with the idea that we will obtain a satisfactory characterization of the natural numbers identifying them with sets is that there are other approaches to the concept of number that seem to do justice in a more appropriate way to our intuitions about numbers. The basic idea is that numbers different from set have instances, and in this way is very natural to conceive them as properties. In a passage from *Mathematical Logic*, Quine expresses this idea very well:

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(15) See the conclusion of Burgess and Rosen's book, for instance.

(16) See, for instance, Chateaubriand's paper "Ockham's Razor".

ANALYTICA
volume 8
número 2
2004

> To say that the Apostles are pious is to attribute a property to each man among the Apostles; and to say that we are unfortunate is to attribute a property to each individual among us. To say that the Apostles are twelve, on the other hand, is to attribute a property rather to the class of Apostles; and to say that we are seven is to attribute a property to us as a class. These properties twelve and seven, symbolically, 12 and 7, are properties of classes; or, in keeping with our custom of treating properties as classes (§ 22), they maybe construed as classes of classes (…); Quine (*Mathematical Logic*), p. 237.

The problem is that Quine reduces properties to sets, but there are certain approaches that do not take this step. Indeed, Russell himself considers this possibility in his book *Introduction to Mathematical Philosophy*:

> (…) It would be possible, though less simple than the procedure we adopt, to regard the number of a collection as a predicate of the collection (…)(Russell 1985 p. 44).

Carnap in *Meaning and Necessity*[17] and in *Introduction to Symbolic Logic* developed this idea in some detail. For instance, in this last book in chapter A §17 Carnap defines numbers as properties. First he defines some auxiliary predicates in the following way: $1_m(F) \equiv \exists xFx$; $2_m(F) \equiv \exists x\exists y(Fx \wedge Fy \wedge x \neq y)$; $3_m(F) \equiv \exists x\exists y\exists z(Fx \wedge Fy\ Fz \wedge x \neq y \wedge x \neq z \wedge y \neq z)$; etc. The next move is to use these predicates in order to define natural number: $0(F) \equiv \neg 1_m(F)$; $1(F) \equiv 1_m(F) \wedge \neg 2_m(F)$; $2(F) \equiv 2_m(F) \wedge \neg 3_m(F)$; and so on[18]. Of course two important problems

(17) Chapter III §27.

(18) Of course, someone can object that the concept of *number* is indefinable and that this type of characterization does not do justice to it. The problem here is to determine what kind the definition is used and how it helps us to clarify some problems related to the ontology of arithmetic. The fact that a concept is primitive does not mean that we cannot give some kind of explanation of it. One of the reasons that we have advanced so little in the understanding of the concept of *set* seems to be that many people think that since set is an indefinable notion we must give up all our hope of obtaining a characterization of it that can help us better understand its nature. This is an error that has done

ANAlyTICA
volume 8
número 2
2004

remain: are they extensional or intensional entities? Numbers as properties apply to sets or to other concepts? In any case, what is important here is to perceive that we can present numbers as entities that are not sets.

In relation to the concept of set, what we have seen is a huge difficulty in obtaining a minimal explanation of its nature. As we said before, some elementary notions like *empty set*, *singleton*, etc. until today have not been properly understood, and some central problems like the *continuum hypothesis* remains without solution. Hence, the thesis that sets are more intelligible than numbers is untenable. Therefore, the most reasonable approach is to assume that sets and numbers are different types of entities that have a close relationship, and see the set theoretical characterization of numbers as a useful tool that can help us to understand the real meaning of the central concepts of arithmetic.

Revisão de: ***Ethel Menezes Rocha***

RESUMO
Meu objetivo nesse artigo é examinar a preocupação de Russell com a justificação filosófica da caracterização da aritmética pela teoria dos conjuntos como é apresentada em sua argumentação nos primeiros capítulos de seu livro Introduction to Mathematical Philosophy. *Através desse exame procurarei responder as seguintes questões: quais os argumentos que Russell usa para justificar sua concepção de que números são conjuntos? São bons argumentos? Do ponto de vista conceitual qual é o ganho que se tem a partir dessa concepção?*
Palavras chave: *Russell - teoria do conjunto - números*

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much harm in areas like arithmetic and set theory which are very general and deal with some of the most basic notions of our conceptual framework.

ANALYTICA
volume 8
número 2
2004

ABSTRACT
My main purpose in this article is to examine Russell's concerns with the philosophical justification of the set theoretical characterization of arithmetic presented in his argumentation in the first chapters of his book Introduction to Mathematical Philosophy. *Through this exam I will seek to answer the following questions: what argumentation does Russell use in order to justify his conception that numbers are sets? Is it sound? What do we attain, from a conceptual point of view, from this approach?*
Keywords: *Russell - set theory - numbers*

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volume 8
número 2
2004

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ANALYTICA
volume 8
número 2
2004

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