

# Negation<sup>1</sup>

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What is the meaning of the word “not”? When one denies a proposition, what is one doing? From the beginning to the end of his philosophical career, Wittgenstein tried to establish a link between these two questions, making the answer to the second prompt an answer to the first. But the motivation behind the second question completely changed along the way. From the mid-30’s on, the sense of any linguistic expression will be given by the intersubjectively accorded use of that expression in our lives. This holds good for the noun “apple”, for the adjective “red”, for the number “five”; and it also holds good for the word “not” (cf. PU, §547). When we go back to the *Tractatus*, we find that the many things we do in everyday life with this word have a relation which is at most subordinate to the Tractarian concept of negation. Everyday uses of the word “not” are just a reflection of a logically fundamental activity. Although negation cannot be identified with any ordinary activity, denying is a logical operation — it is the *inversion* of its sense (5.2341). It is my contention that this inversion cannot be simply conceived as a relation, a logical relation, or even as an asymmetric logical relation between propositions which are on the same level. Negation should be conceived in the *Tractatus* as an atemporal *logical action*, and the denied proposition should be taken as the result of an atemporal logical process of *construction*. A brief historical retrospect will provide a more definite horizon to this discussion.

For Frege, the sign of negation designates a concept “under which all objects fall, but the True” (GgA, §6). So, the value of the function<sup>2</sup>

$$\sim \xi$$

is the True, whenever it takes as argument an ordinary object like Socrates, a mathematical object like the number five, or the *Bedeutung* of a false sentence like “5+7=13”. The negation concept only yields the value False when its argument is the *Bedeutung* of a true sentence like “5+7=12”. As far as its logical mechanism is concerned, negation is a concept just like any other: it takes objects as arguments, and its value is always a truth-value. The same could be said of the conditional:

$$\zeta \supset \xi$$

is a binary function whose values are always truth-values — if the  $\zeta$ -argument is the True and the argument is anything but the True, its value is the False; in any other case, its value is the True (GgA, §12). Therefore, we should take  $\zeta \supset \xi$  as a binary concept (i.e., a binary relation) exactly on a par with *being greater than*, or *being the father of*.

1 I thank my student Rodrigo César Castro Lima and an anonymous reviewer for the corrections and observations that forced me to be clearer at crucial points in the exposition.

2 Many subtleties are missed by not using Frege’s original notation, especially those attached to the use of the horizontal function (cf. GgA, §5). Important as they are, those subtleties can be bypassed in this case for the sake of uniformity in exposition.

Up to this point, we are placed at the level of the *Bedeutung*. The value of the function  $\xi$  *is a philosopher*<sup>3</sup> when we take Socrates as argument is the True. Neither the function nor the philosopher is a component part of the True. The same happens with the negation function. It takes an argument, and returns a truth-value – the True or the False, as the case may be. In the same way that in our example the concept  $\xi$  *is a philosopher* is not part of the True, *negation*, the concept itself, cannot be a component part of any truth-value. For let us suppose it is. If Socrates is part of the True just because he is mortal, we will have to admit that he is also part of the False, since he is not a planet, and “Socrates is a planet” is a name of the False. The same reasoning will hold good for any object whatsoever, no exception made to the True and the False themselves. At the level of *Bedeutung*, we have functional (not mereological) compositionality. But at the level of *Sinn*, things are quite different. Parts determine the whole, but they must also be identifiable within it. They cannot vanish, as references do. Even though all references coincide, a difference at the level of a partial sense will immediately imprint a *characteristic* in the sense of the whole – the standard examples involving definite descriptions immediately come to our minds. A partial sense not only *determines* the sense of the whole expression but also *characterizes* it as a nose characterizes a face – by being a component part of it.<sup>4</sup>

This has a very important consequence for the semantics of the logical constants: the negation of a sentence can never have the same sense as a sentence in which the negation sign does not occur. Double negation does not result in a difference at the level of truth-values, but it introduces an inexorable difference at the level of senses. Fregean semantics cannot equate the senses of  $p$  and  $\sim\sim p$  for the simple reason that the sense of  $\sim\sim p$  contains the concept of negation as a component part. Wittgenstein has a very acute perception of this fact.<sup>5</sup> Frege was quite aware that connectives can be defined in different ways, and that different connectives can be taken as primitive. In the *Begriffsschrift*, all connectives are defined in terms of the conditional and negation concepts. But we could construe logic using disjunction or conjunction instead of the conditional, and it is hard to see why the sense of logical propositions should change so completely in virtue of this choice. It is even more difficult to see why  $p \supset q$  should have a different sense according to the definition we choose to give to the conditional connective, and why this sense should be different from the one it has if taken as a primitive sign. All of these choices seem to take place at a purely conventional level, but if Frege’s semantics is adopted they are bound to reflect differences internal to the atemporal realm of Thought.

We find a very similar situation in Russell’s *Principles of Mathematics*. According to Russell, if we are granted the freedom to quantify over a completely unrestricted domain of entities (collectively called “terms”), then conjunction, disjunction, negation, and even the notion of proposition can be defined in terms of a single indefinable notion: implication. This is conceived as a relation like any other – some terms hold it, other terms don’t. Intuitively speaking, Russell’s use of it is such that only propositions hold it, provided the first term is false or the second term is true. It works exactly as our conditional, with the only difference that it makes sense to apply it to any term whatever. So

3 I will use italics whenever I make reference to the functions themselves (as opposed to functional signs).

4 This is true for ordinary contexts, and also for indirect contexts like the expressions and attributions of belief, for instance. Of course, “the Morning Star” does not have sense in itself, taken apart from any sentential context, but within a certain context its sense must be given and must be a component part of the whole.

5 Cf. 5.43, remembering that a “fact”, in the semantics of Frege, cannot be anything but a true thought. Cf. also 5.4 (where Frege is explicitly mentioned), 5.41, 5.44, and 5.254. All these criticisms apply *mutatis mutandis* both to Russell’s “objective propositions” (*PoM*, §51) and to any version of Russell’s “multiple-relation theory of judgment” in which “molecular thoughts” are analyzed with the help of logical constants. See below.

$Russel \supset Russel$

is a meaningful, but false, proposition, exactly as

$$5+7=12 \supset 5+7=13$$

while

$$5+7=12 \supset 5+7=12$$

is a true one. Russell cannot enunciate these conditions without warnings because he defines the notion of “proposition” *in terms* of implication:

[A]lthough implication is indefinable, *proposition* can be defined. Every proposition implies itself, and whatever is not a proposition implies nothing. Hence to say “*p* is a proposition” *p* is equivalent to saying “*p* implies *p*”; and this equivalence may be used to define propositions. (PoM, §16).

The so-defined notion of proposition is subsequently used as an antecedent to define other connectives as derived notions. Negation for instance is defined in the following way:

not-*p* is equivalent to the assertion that *p* implies all propositions, i.e. that “*r* implies *r*” implies “*p* implies *r*” whatever *r* may be. (PoM, §19)

A double negation would become a very complex proposition, and it would be difficult (if not impossible) to avoid the conclusion that “*p*” says something completely different from “ $\sim\sim p$ ” (cf. 5.44). No conceivable notion of synonymy seems to have a place in this kind of semantics.

Russell abandons his former notion of proposition to favor the so-called “multiple relation theory of judgment”. This is not the place to examine the different versions of this theory, and the dead-ends to which it leads. In general, it says that a sentence does not have *any* semantic content in isolation, although it can take part of semantically charged *psychological* contexts such as understanding, meaning, supposing, judging, believing, etc. A relational sentence “*aRb*” is taken by Russell as having no content of its own. It can be understood, meant, judged, etc. by a subject *S*, and in *these* contexts what we have is not a dual relation between this subject and an objective proposition, but a *multiple* relation between *S* and the entities involved in the complex that would make true the belief *that aRb*. One of the problems of this theory is to accommodate the logical vocabulary. If we take the belief that *aRb* as a 4-place relation between *S* (the subject), the individuals *a* and *b*, and the relation *R*, which kind of relation would correspond to the opposite belief? Would it be a 5-place relation between the same entities and an additional one called “ $\sim$ ”? And what would account for the relation between judging that *p* and judging that  $\sim p$ ? We seem to have no means of explaining in which sense these two judgments should be taken as “contradictory”. They are as different as a 4-place relation and a 5-place relation could be.<sup>6</sup>

This is where the “fundamental thought” of the *Tractatus* finds its place: logical constants do not stand for anything (4.0312). They are not names, and do not “characterize” the sense of the propositions in which they eventually occur. In a sentence like “ $\sim\sim p$ ”, negation must

<sup>6</sup> The introduction of logical form as a component of judgments can be thought as an attempt to solve this kind of problem. We would have a 5-place judgment in both cases, with the form of “*aRb*” and the form of “ $\sim aRb$ ” as the fifth element of each. But this would lead us either to an infinite regress (if the form is taken as an ordinary component of the complex), or to a fragment of the old theory of objective propositions (if the form is taken as a completely general existential *fact*).

disappear completely, leaving no traces behind it. *We must be able to analyse it without taking negation as one of its component parts.* If  $p$  is an elementary proposition,  $\sim\sim p$  should also be elementary for the simple reason that “they” are not “two” propositions, but *the same one*. The propositions  $p$  and  $\sim p$  must have *opposite* (and not just different) senses. There is no additional element in  $\sim p$  but only the *inversion* of the original sense. Even though logical constants do not stand for anything, the semantics of the logical vocabulary should account for the fact that

$$p \supset (q \supset p)$$

must be true, no matter which propositions we imagine taking the place of the letters “ $p$ ” and “ $q$ ”, while

$$p \equiv (q \equiv p)$$

admits both true and false fillings. We must finally have an explanation for quantifiers in line with what is said in 4.0312 – they should not be taken as higher order concepts (as Frege did, for instance).<sup>7</sup>

But if connectives and quantifiers do not contribute to the sense of a proposition standing for logical objects, how should we conceive their contribution? It is easier to understand Wittgenstein’s solution if we take the general form of truth-function (i.e., the general form of proposition) as a guide. The recursive scheme

$$[\bar{p}, \bar{\xi}, N'(\bar{\xi})]$$

divides the propositions of language in two kinds: the elementary propositions, which *do not require* the use of the operation  $N$  to be given, and all other propositions, which *can only be obtained* by at least one application of the operation  $N$  to a formally selected group of previously given propositions. Negation is the simplest case. Given the totality of elementary propositions, one can select a single proposition of this totality (let us say,  $p$ ), and apply the operation  $N$  on it. The result will be  $Np$  (or, in the usual way of writing it,  $\sim p$ ) and there will be an internal (purely structural) relation between the basis and the result of the operation. In this case, this structural internal relation will correspond to a truth-functional relation between basis and result: given the value of the basis, the value of the result will also be given. Moreover, the operation can be applied to its own result again and again, generating a series governed by a uniform method of construction:  $p, \sim p, \sim\sim p$  etc. This is called by Wittgenstein a “series of forms”.

It is important to distinguish the syntactic and the semantic level of operations in general, and negation in particular. We get  $\sim p$  from  $p$  by applying a syntactic method of construction that is linked to a specific semantic modification. For Frege and Russell, this modification would be described as the introduction of a new concept by the logical constant “ $\sim$ ”. We have just seen the host of problems arising from this view. Wittgenstein finds a way out of these problems identifying a semantic modification which is not an *addition*. Negation is an *inversion* of the sense (5.2341) – it denies what was being asserted, and asserts what was being denied. But this

7 The Tractarian account of identity falls outside the scope of this paper. It is eliminated as a logical relation between objects in favor of a convention for the use of variables and constants (5.53-5.5352). More exactly, Wittgenstein adopts a convention that is the necessary consequence of the distinction between symbols and signs made in other sections of the book (3.32-3.328). Identity, when it is meaningfully used, is the mere expression of a *substitution rule for signs* (4.241). There is nothing wrong with the expressions “ $p=\sim\sim p$ ”, “ $5+7=12$ ”, “ $\sim=\sim\sim$ ” as long as we do not take them as material propositions. The last one, for instance, means that the signs “ $\sim$ ” and “ $\sim\sim$ ” are interchangeable in a certain notational game. This is not a nonsense. It can even be true regarding certain games. The logical basis for this interchangeability, in turn, cannot be expressed by any means. But the function of the identity sign is not to express it. In some notations for chess, the identity sign is used to indicate the promotion of a pawn (“ $e8=Q$ ”, for instance). The difference is that in “ $5+7=12$ ” the substitution has a logical ballast. But the ballast itself is not part of the convention.

conception only becomes possible once we are ready to admit that the negated proposition has a judgment *incorporated* to its sense from the start, and that it cannot be severed from it. An elementary proposition is primitively and essentially assertive – the picture theory is the theory of how this primitive assertion works. Negation, by its turn, cannot be asserted – it supersedes assertion *as* an opposite assertion. That is why  $\sim\sim p$  and  $p$  are not just logically equivalent propositions (in the plural), but the very same proposition under different guises:  $\sim\sim p$  literally *comes back* to  $p$ , and double negation can be simply erased.

Being an inversion of sense, negation must be seen, not only as a syntactic, but also as a *semantic construction*. It is not on a par with assertion, as two different truth-tables could seem to be. Assertion must be the initial semantic step; negation is always the second at least. This is the basis of the picture theory of propositions, and the very notion of elementary proposition depends on this asymmetry. Elementary propositions are given as a totality of possible assertions, not as a totality of Fregean thoughts. But if so, how is it possible to explain the semantics of the negation sign in complex propositions, like  $p \supset \sim q$ ? If we take  $q$  as elementary, it is certainly not being asserted *within* this proposition, and the negation sign is not indicating the inversion of *this* assertion. Wittgenstein's solution comes with the notions of *joint negation* (marked by the operator  $N$ ) and *formal selection* (indicated in the general form of proposition by the horizontal bar over the propositional variable "ξ"). We make a formal selection of propositions by means of a description of the values of the variable "ξ". We can do that in three different ways (5.501): (i) by a description of individual propositions (when we are dealing with a finite number of them); (ii) by means of a propositional function whose values are the propositions to be selected; (iii) by a recursive law for the construction of a series of propositions. As it is well known, this is (more than) enough for the definition of the usual connectives and quantifiers. Let us take a very simple example to help us in the exposition. Suppose that  $p$  and  $q$  are elementary propositions. The truth-function corresponding to the conjunction of these propositions would be given, for instance, by the following serial procedure:

1. Selection (by enumeration) of  $p$ .
2. Application of the  $N$  operation to the selected proposition:  $Np$ .
3. Selection (by enumeration) of  $q$ .
4. Application of the  $N$  operation to the selected proposition:  $Nq$ .
5. Selection (by enumeration) of  $Np$  and  $Nq$ .
6. Application of the  $N$  operation to the selected propositions:  $N(Np, Nq)$ .

Notice that the formal selection of propositions does not amount to a joint assertion of them (otherwise the final result would be a disjunction). Steps 1, 3 and 5 do not involve any operation (although they are essential to the construction). Steps 2 and 4 are equivalent to the denial of elementary (and therefore, intrinsically asserted) propositions. In both cases, the result is a *denial*, i.e. an *intrinsically asserted negation*. The novelty comes in step 6, where we have an extension of the concept of negation to a (formally selected) group of propositions. In this case, we obtain a *result* (and not a mere selection of propositions, as we have in steps 1, 3 and 5): the joint denial of  $Np$  and  $Nq$ . It comes intrinsically asserted, as any proposition, and is equivalent to the conjunction. Notice that in the process of construction we never use negation "inside" the proposition. It is always applied to a *group* of propositions, each of them intrinsically asserted. It amounts to the *joint inversion* of a determined totality of senses, and the result is always a proposition making a quite definite assertion.

Usual quantification (or what amounts to it in Wittgenstein's notation) uses a new kind of selection — the values of a propositional variable are the values of a propositional function. These values may be infinite in number. It does not matter. From the Tractarian point of view, they form a totality of propositions as precisely circumscribed as any totality can be.<sup>8</sup> Let

$$aRx.xRb$$

be a propositional function whose values are conjunctions of elementary propositions, and let

$$\bar{\xi}$$

be the totality of such values. We can jointly invert the values of these intrinsically asserted conjunctions, and obtain what in our notation would amount to the proposition

$$\sim(\exists x).aRx.xRb$$

If "every proposition is the result of successive applications of the operation  $N'(\bar{\xi})$  to the elementary propositions" (6.001), then the logical relations between  $p$  and  $\sim p$  cannot give the whole Tractarian explanation of the operation  $N$ . Negation goes *from  $p$  to  $\sim p$* , and this is not simply a truth-functional path. It has a constructive side. It has a direction given by the opposition between *elementary* propositions, to which no logical construction is essential, and any other proposition, which *must* be a truth-functional *construction* having elementary propositions as a fixed basis. An operation is something that "must *happen* [*geschehen*] to a proposition in order to *make* [*machen*] another out of it" (5.23, my emphases). This happening is not a logical relation, not even an asymmetrical logical relation between two propositions. It involves, no doubt, a symmetrical relation of contradiction, but it has also a constructive side which logical notions of this kind are not able to exhaust. I am not making reference only to the syntactic side of this construction. The notion of elementary proposition is not purely, and not even fundamentally syntactic. To repeat myself, if  $p$  is elementary,  $\sim\sim p$  must be elementary because they are just two signs for *the same symbol*. There is no component  $\sim\sim$  "within" the sense of  $\sim\sim p$ —this component would be Fregean, not Tractarian. If one wants to see  $\sim\sim$  as Wittgenstein saw it, one must first make it disappear somehow. It disappears as soon as you take the negation sign as the mark of a logical action which can be done and then undone, if done twice.

Of course, the drawing of a formula is a temporal process. But not even the syntactic process involved in the passage from the sign " $p$ " to the sign " $\sim p$ " is a temporal process: in logical syntax, we talk about tokens, not types (3.203). At the semantic level, we will have to talk about the construction of senses, a process which does not have the slightest similarity with temporal or physical constructions. We will have to deal with *actual* infinities of propositions, all of them projected into reality and *meaning* something. Syntactic processes can at least be described. The passage from the sense of  $p$  to the sense of  $\sim p$  stays clearly upon the bounds of sense. There is a precedence here; there is construal; I don't know a better expression to it than "atemporal action". Call it as you may, it will be over there anyway, and I don't think the *Tractatus* can be understood without incorporating this atemporal action<sup>9</sup> to its conception of logic.

8 Notice that if the number of values of a propositional function is finite, the function is just a variant method of selection. Simple enumeration would do the same job. It seems to me that the motivation behind the admission of propositional functions as a principle of selection is (and can only be) to make room for actual infinities that are not linked to recursive procedures.

9 An anonymous reviewer observed that "the text seems compatible with or even dependent on that, in the final analysis, this need for the consideration of something like atemporal acts must already be present in the explanation of the significance of the elementary proposition". And that's true. Atemporal actions are required to account for the figurative relations [*abbildende Beziehungen*] between names and objects. A sign *becomes* a name by its connection with an object, and this connection is not a fact of nature. It is "atemporal", to use a time-honored expression. I explore this issue in several texts, especially

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## Resumo

A negação, no Tractatus, não pode ser tratada como um conceito, tal como acontecia em Frege e em Russell. Se isto acontecesse,  $p$  e  $\sim\sim p$  deveriam ter sentidos composicionalmente diferentes. A negação não pode se resumir a uma relação lógica entre dois enunciados, pois  $\sim p$  deve ser construída a partir de  $p$ , e não o contrário. A negação é algo que devemos fazer para obter uma proposição a partir de outra (5.23). Ela é produto de uma ação, mas esta ação não pode ser natural (ou meramente humana). Ela deve ser atemporal.

**Palavras-chave:** Wittgenstein, Tractatus Logico-Philosophicus, negação, lógica, sujeito transcendental.

## Abstract

Negation, in the Tractatus, cannot be treated as a concept, as in Frege and Russell. If this were to happen,  $p$  and  $\sim\sim p$  should have compositionally different meanings. Negation cannot be taken as a logical relationship between two propositions, since  $\sim p$  must be constructed from  $p$ , not the other way round. Negation is something we must do to get one proposition from another (5.23). It is the product of an action, but this action cannot be natural (or merely human). It must be atemporal.

**Keywords:** Wittgenstein, Tractatus Logico-Philosophicus, negation, logic, transcendental subject.

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in CUTER (2003) and CUTER (2006).