

# Frege and Wittgenstein's debate regarding the notion of "fact" in the *Tractatus*. Is it a Set-theorist or a Mereological notion?

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## INTRODUCTION

A key ingredient, derived from Frege, and which has influenced Wittgenstein throughout his development in the *Tractatus*, was the former's construal of "propositional sense" as "truth-condition". For the older philosopher, a *universal* formal language capable of grounding mathematical concepts in logic would have to somehow involve the relation between "sense" and "propositional truth-conditions". As we will argue in this paper, Frege's confidence on the crucial dependence of truth on the idea of "predication" and, therefore, of the "propositions' truth-conditions" led him, as we will see, to constrain his whole approach by a frequently disputed principle, the *tertium non datur* (Principle of Excluded Middle, PEM, for now on). This principle specified more directly and forcefully in his mature book *Grundgesetze der Arithmetik* of 1884 (GG, from now on) was formulated by him as the requirement that every single concept of his formal system ought to be sharply determined. Carried over from the GG to the *Tractatus*, Frege's demand has been transformed into Wittgenstein's famous principle of "a complete determination of sense" (PCDS for now on) for all propositions, as we read in the *Tractatus* §3.23: "*The postulate of the possibility of the simple signs is the postulate of the determinateness of the sense*".

Frege's philosophical inheritance provides us with at least an important part of the scenario in which Wittgenstein conceived the postulate stated in §3.23. Nevertheless, in the eyes of the younger philosopher, there was a major difficulty blocking Frege's proposals, the elder's notion of "extension" as "a logical object". As it is also widely known, Frege's notion of "extension", as it was encapsulated in his famous Law V, was hopelessly wrecked by Russell's paradox. This paradox directly challenged Frege's construal of "extensions" as "logical objects" in the GG, pointing to an irreversible difficulty at the very core of Frege's logicism.

As one faces the philosophical scenario concerning the foundation of mathematics after Russell's paradox, only two options were available, either to reformulate Frege's notion of "extension", or to maintain it and alter substantively Frege's logical system in order to avoid Russell challenges.<sup>1</sup> As we read the *Tractatus*, Wittgenstein has chosen the first

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1 This adjustment of the logic that underlies the mathematical theory could imply the addition of new axioms to Frege's naïve theory of extensions, as happened in Zermelo-Fraenkel's set-theory, or the facing of a bolder challenge as that of developing a ramified type theory like the one Russell carried out.

alterative.<sup>2</sup> However, not all of Frege's ideas were quite put aside by Wittgenstein. This is the case of Frege's construal of "propositional sense" as "truth-conditions" and its consequence, the demand of a "complete determination for every concept". Even though, as we know, Wittgenstein did not give support to those two claims in its most radical form in his more mature work, we think that it was indeed his main objective in the *Tractatus*.

To implement Frege's ideas on "sense" and "truth" in the *Tractatus*' conceptual frame, Wittgenstein had to make radical changes in the elder's overall approach. Two of those changes are going to particularly call our attention. The more comprehensive one is Frege's construal of "proposition's sense" as "truth-conditions", surprisingly restricted to the empirical domain. Wittgenstein's philosophical choice of taking sense as identical with the determination of proposition's "truth-conditions" is a widely commented issue in the literature.<sup>3</sup> Nevertheless, it always comes as a big surprise when we ponder upon it. The strangeness comes from the proposal that only empirical propositions were supposed to have sense, and so truth-conditions, in the *Tractatus*.

The second change implemented by Wittgenstein upon Frege's ideas about sense was related to the sub-propositional structure of the Elementary Propositions (EPs, from now on).<sup>4</sup> We will argue that the impact of Russell's paradox upon the primary sub-propositional level was devastating. It was so significant for Wittgenstein that he even considered rejecting the very predicative nature of Eps.<sup>5</sup> According to our interpretation, this is the primary reason why he proposes another kind of logical form for those linguistic elements, as well as an alternative logical treatment for all the related resources to a predicative theoretical approach.

In summary, our claim in this paper will be that, in order to explain the *Tractatus*' ontological zero level, without getting involved with Frege's paradoxical notion of "extension", Wittgenstein was obliged to change some important parts of his approach towards the logical grammar of language. It is true that he has maintained Frege's extensional approach for propositional logic, proposing for that part of language a truth-functional analysis. Nevertheless, he rejected Frege's construal "extensions" as "logical objects", or even classes, at the sub-propositional level. Instead, he adopted a new method of analysis for this level. This new method did not analyze EPs into something akin to the subject-predicate form anymore.<sup>6</sup> The idea was to conceive the ontological ground in the *Tractatus* as composed of inseparable complexes. Wittgenstein called those complexes "atomic state-of-affairs" (SoAs, for now on), and their inseparable parts, "simple objects". We will also claim that he proposes the substitution of the predicative structure of EPs and of the correlated set theoretical relation of "set-membership", by the mereological notions of a "complex", and its correlated structural relation of "part" and "whole". Thus, it is central to our

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2 We don't mean to imply that Wittgenstein hasn't discussed Frege's notion of "extension", neither that he hasn't tried, in the beginning of his intermediate period around 1929/30, and in the manuscript 105-106, to understand Frege's idea of extension in his own way, as we said. We are merely saying that Frege's idea of considering extensions as *logical objects*, at least in the way it was originally conceived, was not adopted by Wittgenstein in the *Tractatus* (cf. §5.4 of the *Tractatus*).

3 We are not going to dispute if Frege's construal of "sense as truth-condition" is the same as Wittgenstein's construal of the notion in the *Tractatus*. In section I, the reader will find a characterization of just the relevant elements of Frege's construal of "propositional sense" as "truth-conditions", which he imported into the *Tractatus*.

4 Or singular propositions, in Frege's case.

5 Cf. (VELLOSO, Wittgenstein's unique "Great Analysis", 2014, p. 233)

6 Function and argument, in Frege's terminology.

argumentation the acknowledgment that atomic SoA's parts are not related to one another by the membership relation.

To support our claim, we believe we have two tasks to fulfil: (1) to explain Wittgenstein's restriction of Frege's construal of "propositional sense" as "truth-conditions" solely to empirical propositions as an attempt to cope with Frege's demand of a complete determination for every concept; and (2) to show how the struggle to keep Frege's construal of "sense" as "truth-condition" against Russell's paradox has brought Wittgenstein to restrict this notion even more, refraining to use Frege's construal of "extensions" as "logical objects" at the sub-propositional singular level of EPs and adopting instead a sort of "mereological construal" of the states-of-affairs.

To accomplish our argumentative task though, we believe it is first necessary to introduce and discuss Frege's ideas of "sense as truth-conditions" and of "conceptual sharpness" within his own framework. Against this background, we will be able to finally compare Frege's handling of those notions with that of Wittgenstein's.

Let's now summarize, in very brief lines, our argumentative line. The first section will be dedicated to Frege's ideas. It will deal also with Frege's notion of "extension", but only to specify its role of Frege's system. We will restrict our discussion about this topic to the relevant aspects concerning the subject of section VI, the debate between Frege and Wittgenstein about the later notion of "fact". Section II will present a first overview of Wittgenstein's reaction to Frege's ideas. Next, in section (III), we will introduce a dichotomy proposed by Oswaldo Chateaubriand in the introduction of his book *Logical Forms*. There, Chateaubriand makes a distinction between two kinds of logical analysis: a predicative analysis and a propositional one. In sequence, this dichotomy will be used to justify our claim that the *Tractatus* could and did maintained a diverse approach in relation to these two logical levels.<sup>7</sup>

In section (IV) we will further advance our discussion of Wittgenstein's reaction to Frege's construal of "propositional sense" as "truth-conditions". But this time we will be addressing Wittgenstein's reaction to the elder's famous distinction between sense and reference. In section (V), we will comment upon Wittgenstein's ideas about Eps' logical form. The goal of this whole section will be to present in more detail the claim that Wittgenstein did not adopt a predicative structure at the sub-propositional level of EPs, but instead, a kind of mereological theory. In section (VI), we will finally finish our discussion by evaluating our various claims vis-à-vis the epistolary quarrel that happened between Frege and Wittgenstein. Their debate revolved precisely around the nature of atomic SoAs and about the kind of relation their component parts had to have with each other.

## FREGE'S LOGICAL IDEAS

### L.1 FREGE'S "PROPOSITIONAL SENSE" AS "TRUTH-CONDITIONS"

Wittgenstein's notion of "sense" in the *Tractatus* is unquestionably a notion of sense as "truth-conditions". This means that the sense of any proposition "p" is determined by the

7 On the one hand, one's failure to recognize the distinction between the predicative and the propositional levels of analysis results in a single unified grammar. On the other hand, if one goes in the opposite direction and provides a separate treatment for those two ideas, "sense" and "truth" at the two different logical levels, the result will be quite different. The rules of truth will end up being the subject matter of the propositional logic, and it will be up to sub-propositional part of logic dealing with grammar and meaning.

circumstances where "p" is true or false. (p. §4.063 TLP) Another way to express the same idea is presented in aphorism (§4.431): "The proposition is the expression of its truth-conditions". Despite that, Frege's construal of "propositional sense" as "truth-conditions" was eventually disputed by some interpreters. Dewi Trebaul, for example, who has devoted an entire paper about this topic, believes that Frege's notion of "sense" was only "apparently" truth-conditional.<sup>8</sup> According to Trebaul, there is only one passage in all of Frege's work which could be used to support the thesis defended in the *Tractatus*. (TREBAUL, Sep 2012, p. 6) In fact, in this repeatedly quoted passage from the GG, Frege especially define propositional sense in a way quite similar to the one in the *Tractatus*:

Every name of a truth-value expresses a sense, a thought. Through our stipulations it is determined under which conditions it refers to the true. The sense of this name, the thought, is that these conditions are fulfilled. (GG, §32)

According to this quote, propositional sense is identified with the fulfillment of the proposition's truth-conditions.

There are also those who defend the opposite position. Michael Dummett, for example, in his book *The Seas of Language*, asserts that Frege has explicitly contended for the notion of "sense" as "truth-conditions".

Does the meaning of a sentence consist in its truth-condition? Does the meaning of a word consist in the contribution it makes to determining the truth-condition of any sentence in which it occurs? [...] it has been explicitly contended for by Frege, by the Wittgenstein of the *Tractatus*, and by Davidson. [...] because Frege, in particular, was so great a genius, we have more grasp on what a theory of meaning given in terms of truth-conditions looks like than on the general form of a theory of meaning of any rival kind. (1996, pp. 33-34),

We quite agree with him. We are going to defend in the next section that Frege's construal of "propositional sense" as "truth-conditions" is an outcome of the centrality received by the notion of "truth" in Frege's system.<sup>9</sup> As we are going to argue in the following section, it was not only the separated ideas of "truth" and "sense", but also the connection between them, i.e., the idea of a logical grammar, which was essential to Frege's work.

## I.2 FREGE'S DEMAND OF SHARP DEFINITIONS FOR EVERY CONCEPT

Frege believed that a crucial prerequisite to his formalized language was the requirement that its fundamental elements, the concepts, ought to have sharp boundaries. In volume II, part III, of *Grundgesetze der Arithmetik* (GG), which deals with the Real numbers, this constraint was his primary and almost unique concern. More than that, all through his

8 Trebaul contends that Wittgenstein had criticized Frege's notion of "sense" as being only apparently that of "truth-conditions". To see more details regarding this discussion, cf. (TREBAUL, Sep 2012).

9 Dummett also put things in a very similar manner. In (DUMMETT, 1996, p. 33), he thinks that the choice of using the concept of "truth" as the central one in order to build a theory of meaning is not an easy task. For him "A proof that it is *necessary* to use the concept of truth for this purpose would not, in itself, show how it is *possible* for it to play this role; it would not of itself overcome the initial objections to a theory of meaning in terms of truth-conditions, but it would guarantee that there was a way of overcoming them to be found."

debate on GG, part III, he emphasizes this sharpness as a precondition for any acceptable explanation of numbers.

Thus, there must be no object for which, after the definition, it remains doubtful whether it falls under the concept, even though it may not always be possible, for us humans, with our insufficient knowledge, to decide the question. Figuratively, we can also express it like this: a concept must have sharp boundaries. [...] a concept without sharp definition is wrongly called a concept. Logic cannot recognize such concept-like constructions as concepts; it is impossible to formulate exact laws concerning them. (FREGE, *Basic Laws of Arithmetic*, 2013, p. 69)

As we know, it was one of Frege's crucial targets to construe numbers as objects. And he began his attempt to do so by proposing that logic should best be viewed as the provider of a unique foundational grammar for this universal formalized language. The logical grammar proposed by Frege should only contain rules concerning the notion of "truth". In his formulation, logic's task was "to discern the laws of truth" (1977, p. 1). The derivation of mathematics from logic was possible for him only after the completion of this first task. Consequently, in order to build up this language and its logical grammar, two ideas were essential for Frege: that of "truth" and that of "sense". As we will see hereafter, both ideas are embedded essentially in the background of Frege's formulation of the PEM presented above.

Frege's formulation of PEM involves two crucial elements, as we have said. The first is the idea of "truth". It concerns entire propositions, and consists in the an initial requirement of bivalence, i.e., the warranty that we will ending up assigning truth, or falsity, to any proposition of the language. This initial logical demand depends however on a second element, a grammatical requirement, which is introduced by the idea of "sense". For Frege, certain conditions must be fulfilled by the internal structure of any truth value's bearer. Those conditions were: (1) the truth value's bearer should allow being bisected into two further components, the logical subject and its logical predicate (a fregian function)<sup>10</sup>; and (2) they would be deemed "true" only in case its saturated nominal part were to satisfy its unsaturated predicative one. Once its truth-conditions were fixed, the sense of the entire proposition is fully determined. Frege himself explains this second requirement as follows:

The law of excluded middle is, in fact, *just* the requirement, *in another form*, that concepts [should] have sharp boundaries. Any object either falls under the concept or it does not fall under it: *tertium non datur*. (FREGE, *Basic Laws of Arithmetic*, 2013, p. p. 69)

So, as we can verify in the quote above, the demand of sharpness for all concepts was, in Frege's mind, *just an alternative formulation* of the PEM's universal validity.

The second aspect involved in Frege's formulation of the PEM specifically concerns the internal structure of each single proposition. Consequently, with this formulation, Frege's version of PEM become comparable to Aristotle's presentation of another famous principle of logic, the principle of contradiction<sup>11</sup> (PC)<sup>12</sup>:

10 Following Chateaubriand, we will call "what it is said about" as the logical-subject, and "what is said" as the logical-predicate. As he also points out, this distinction became clearer after Frege's paper "On sense and reference". (CHATEAUBRIAND, 2001, pp. 240-244, cap 2)

11 In classical logic, the PEM and the PC are interdefinable, for, using *De Morgen*, and by the law of double negation, . But in intuitionistic logic, as we know, is not valid.

12 Our quotation of Aristotle here is the one that most clearly states the famous principle and is cho-



Evidently, then such a principle is the most certain of all; which principle this is, let us proceed to say. It is that the same attribute cannot at the same time belong and not belong to the same subject and in the same respect; (Aristotle, *Met.* G 3, 1005b18-21)

Frege's formulation of *PEM* and Aristotle's version of *PC* were alike in many ways worth point out here. The first, more obvious one, is their commitment to "bivalence". This is the most usual way to understand them both. However, there is a second and somewhat unexpected commitment involving their formulations which concerns, not entire propositions and their combination, but their predicative-like internal structure.<sup>13</sup> For both, the commitment to this universal kind of structure is clear.

There are differences also, that worth be pointed out between the two formulations presented. Unlike Aristotle's formulation, on Frege's formulation of the *PEM* one additional condition was imposed: a demand for absolute sharpness of every conceptual expression, as we said. Frege's need of this ulterior condition is explained by his aim to ground the meaning of Mathematical concepts in the semantic of this new formular language of concepts. For Frege, this second commitment cannot be ignored or cast aside, it is a universal constrain on any possible thought and any possible language.

The result of connecting the notion of "propositional sense" with a universal subject/predicate like grammatical form seems obvious, at a first sight, but is not. It links, permanently, propositions' sense to the truth value determined by its internal structure, its logical form. This union of those two ideas, "logical form" and "truth", culminated in Frege's proposal of "a universal logical grammar" and led to the demand of strict sharp boundaries criteria for concepts, as we already said.

The first outcome of Frege's demand for a universal logical grammar was a tradeoff. Either one adopts the analysis of all sub-propositional structure into two parts, the logical subject and its logical predicate (I-subject and I-predicate, from now on) and has to commit to the *PEM* also, or one throw away this kind of analysis and go for another kind of sub-propositional analysis.<sup>14</sup> To put it in another way also, the acceptance of the former, i.e., a predicative-type construction, would oblige one to commit to the later also, the *PEM*.

The other outcome of Frege's universal logical grammar was that he has to introduce the "extensions" as logical objects, for it is a necessary step towards the sharply

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sen to follow Chateaubriand's appeal to this philosopher in other to explain his proposal, as we are going to see next.

13 For Aristotle, the structure will be tripartite: the subject concept, the verb "to be" in the sense of predication, and the concept predicated. For Frege, propositional structure is bipartite: only argument and functions. But we can say that functions are their logical predicates and arguments, their logical subjects, as is going to be explained further. Nevertheless, both has adopted what we could call a predicative sub-propositional structure.

14 Following Chateaubriand's interpretation of Frege's ideas about the internal structure of propositions, we will assume that Frege has not eliminated the subject-predicate distinction in the *Begriffsschrift*, but only reformulated it. "Although it is true that Frege explicitly rejects the traditional subject-predicate distinction in *Begriffsschrift* and does not separate notationally non-logical predicates from non-logical subjects, his logical analysis actually involves a sharpening of the traditional subject-predicate analysis. [...] Frege's logical notation was very badly misunderstood, and people have since felt that linearizing it was an improvement. Yet this linearization had the effect of eliminating the careful separation between logical property and non-logical subjects. As a result, the distinction between what is said and what it is said about became completely muddled. Nevertheless, the problems with aboutness do indeed originate with Frege's treatment of the traditional subject-predicate distinction in *Begriffsschrift*, and he can be blamed for some of the confusion – although it may be misleading to say that he eliminated the distinction." (CHATEAUBRIAND, 2001, p. 244)

determination of all concept-word boundaries. With this accomplishment, Frege has hoped to finally guarantee that the identity symbol used in his system would become well defined for all terms and so all identity propositions would have to be either true or false, *tertium non datur*.

Despite all that, as we've already pointed out in the beginning of this paper, Frege's notion of "extension" as "logical objects" did not accomplish the task its author had expected it to perform. As we know, the value-ranges, so very precious for the later Frege, turned out to be a contradictory concept and engendered a paradox within the philosopher's system. The failure of Frege's notion of "extension" in sharply determining all concepts boundaries has taught us that the construal of "propositional sense" as "truth-conditions" could only be extended to an extensional language, universally, on the price of having to deal with Russell's paradox. In the next section, we discuss how Wittgenstein reacted to this adverse scenario.

## II. WITTGENSTEIN'S REACTION TO FREGE'S NOTION OF "SENSE AS TRUTH-CONDITIONS"

Wittgenstein reaction to Frege's notion of "sense" in the *Tractatus* was quite astonishing. He extravagantly restricted the scope of Frege's construal solely to the empirical and contingent propositions. Wittgenstein's maneuver was undoubtedly surprising and caused a strange twist in our story. The restriction imposed by him banishes all the non-empirical propositions from the sphere of meaningful ones. As one could have guessed, the banishment ruled out an important, we could say even essential part, of Frege's formal system. To have an idea of its magnitude, Wittgenstein excluded as pseudo-propositions, nothing less than all identities, including mathematical equalities (*Tractatus Lógico-Philosophicus*, p. §6.2). Among other serious consequences, this extravagant option seems to disregard completely Frege's initial purpose and, as it were, "throw out the baby with the bath water".<sup>15</sup>

However, we do think Wittgenstein was already, in the *Tractatus*, trying to figure out a way to avoid those disastrous consequences. In our opinion, one should try to understand Wittgenstein's motives in a deeper and more charitably way. Wittgenstein's reaction could be considered as a way to save Frege's project, by restricting it to the contingent part of language, instead of taking it as a critique of Frege's notion of "propositional sense".<sup>16</sup>

Now that we have prepared the way to present Wittgenstein's position as a restatement of Frege's ideas about "sense" and "truth", let's see what line of thought Wittgenstein could have followed to reorganize his master's original plans. One first hypothesis is that Wittgenstein could have blamed Frege's lack of understanding about the multiple tasks of language as a cause for the logicism's failure. For the younger philosopher, since the very beginning of his philosophical work, it was clear that not all linguistic expressions have a representational role. In fact, for the later Wittgenstein, it was even more clear that there were

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15 According to his proposal, mathematical propositions were equations and must, therefore, be treated as pseudo-propositions. Additionally, all logical propositions would be (*sinnlos*) senseless, for they couldn't divide the logical space in two groups of situations: those that would make them true and those that would make them false. One can even say that, in *Tractatus*, there were only two logical propositions: The Tautology, and the Absurd.

16 As was Trebaul way of reading the *Tractatus* and can be confirmed just by reading the title of his paper. (The early Wittgenstein's truth-conditional conception of sense in the light of his criticism of Frege, Sep 2012)

some parts of the language which were not representational devices at all, like: commands, rules, metaphorical talk, performative uses, and the use of formal concepts. Moreover, we think that he was already under just such suspicion when he wrote the *Tractatus*. Be that as it may, the upshot of all these ideas is that he considered acceptable to apply the notion of "sense as truth-conditions" only to those propositions which could have both truth values: the true, but also the false. Those were the bipolar propositions, the ones which should describe some possible event in the world.

Despite surprising maneuver, Wittgenstein maintained a very conventional attitude towards the relationship between general and singular propositions in the area of contingent truths, or truths of facts. In the *Tractatus*, he was still groping, and he continued for a while to entertain Frege's and Russell's account of generality concerning those propositions related to the truths of science in general. The common belief between Wittgenstein's contemporaries was that there had to exist a close connection between the part of language which contains general statements, like the laws of science, and the part which contains singular propositions.

In the next section, we will try to disclose a tension between two logical approaches which are not usually quite distinguished, one from the other. This logical frontier was presented in a very clear way by Oswaldo Chateaubriand. The cohabitation of the two logical approaches distinguished by Chateaubriand will then be the second difference between Frege's conception of logic and Wittgenstein's reception of it in the *Tractatus*. While for Frege, there was no tension between them, as we will see, for Wittgenstein, those two layers could not be easily integrated. Furthermore, this distinction will have a central role in the arguments put forth in the next sections.

### III. THE DIVISION BETWEEN PREDICATE AND PROPOSITIONAL LOGIC

#### III.1 CHATEAUBRIAND PRESENTATION OF THE DIVISION

We now want to discuss in further detail some of the consequences of Frege's formulation of the PEM which we have anticipated in section I.2. As we have commented before, his way of explaining the principle has an element in common with the way Aristotle explained the principle of contradiction (PC). Our claim now is that their formulations revealed two different conceptions regarding the main objective of logic: (1) to discover the laws of truth; and (2) to explain the sense of sub-propositional expressions in terms of *truth-conditions*. To defend this, our strategy will be to follow Chateaubriand's ideas, which uncover this duality and shows how those two different objectives imply two distinct tasks of logic.

In the introduction of his book, *Logical Forms*, Chateaubriand presents in a very direct manner the distinction between the aprioristic and the metaphysical tasks of logic. His way of presenting it involves requesting the assistance of Aristotle's formulation of the "Principle of contradiction" (PC). Chateaubriand quotes two excerpts of Aristotle's *Metaphysics* and claims that the Greek philosopher did recognize the division he intends to make, there:

The principle of contradiction, for example, is formulated both as the principle that the same property cannot apply and not apply (at the same time, in the same respect) to the same subject, and as the principle that contradictory propositions cannot be true – *Metaphysics* 1005b 18 and 1011b 13. (CHATEAUBRIAND, 2001, p. 17)



Still according to Chateaubriand's understanding, the part of logic which consists in a "theory of truth relations between propositions *quite independently of any analysis of the logical structure of propositions* and their linguistic expressions" is what is usually called "propositional logic". According to him, the logic of predicates<sup>17</sup> (or predicate logic from now on), usually considered to be the more inclusive of the two, should be considered instead as the more specialized of them both. He characterizes it as a "general theory of properties and objects based on some specific logical properties and operations". (2001, p. 17)

On Chateaubriand's account, only propositional logic deals directly with laws of truth, i.e., with logic in its *a priori* and most pure sense. Predicate logic does not have this goal, quite the contrary. Concerning predicate logic, the sense and truth-conditions of contingent and also of necessary propositions would depend on the meaning of its predicates. Furthermore, it is predicate logic which deals more directly with how senses are assimilated by language and transmitted to the rest of the system.

In the next quote Chateaubriand advocates an analytical procedure which should uncover "the propositional structure in terms of the categories of objects and properties". He calls those categories and properties "general features of reality". It is this internal structure of propositions, disclosed by the analytical procedure, which finally puts together "sense" and "truth" in a way that would have been acceptable for Frege too.

The connection between predicate logic and propositional logic derives from the analysis of the logical structure of propositions. The modern theory of *logical form* is a theory of propositional structure in terms of the categories of objects and properties and precisely logical properties and operations. Through this structuring, one can *bring together* the logical analysis of the general features of reality with the logical analysis of the truth relations between propositions. (pp. 17-18) [my emphasis]

His proposal of "*bringing together*" the two kinds of logical analysis involves considering both logics as two levels of the same general account of reality. Choosing this path necessarily requires that one provides some sort of "bridge principles" linking the lower metaphysical level to the higher-levels of truth-functional operations.<sup>18</sup> This is the role of the "theory of *logical form*" proposed by Chateaubriand. In his case, as well as in Frege's case, the Logic used to analyze the sub-propositional structure was what the former calls "the predicate logic". Thus, with that union between sense and truth, predicate logic would have the primary role of being a metaphysical theory about reality, as Chateaubriand concludes.

In this sense predicate logic is not a theory of logical truth, or of logical implication, but a theory of reality. Although one can say that the laws of predicate logic are truths of logic, it doesn't follow that predicate logic is a theory of logical truth in the sense of a classification of sentences or propositions.

The distinction between formulating laws of logic as laws of being and characterizing logical truth and logical implication brings out the other aspect of predicate logic. This involves a concern with propositional structure and its relation to reality. (p.17)

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17 Either first-order, or higher order logic.

18 The other attitude would consist in treating the two logics differently, separating their tasks, their compositionality principles, and considering only the propositional logic as being aprioristic, i.e., independent from the world. In this case, truth would depend on predicate logic's account of reality and will remain relative to it. Quine was the one who proposed this division in the first place, and this alternative attitude was adopted by him.

To follow Chateaubriand and Frege path, thou, it is necessary to construe those bridge principles as related to the sub-propositional structure of propositions. The procedure will have to involve, among other elements<sup>19</sup>, logical subjects and logical predicates, plus a satisfaction relation between them, as we already said. It will then be up to the propositional logic to act upon those already established truth-values according to the usual truth-functional operations. Both Chateaubriand and Frege assume those requirements as *desiderata* to the final construction of a universal logical language. According to the former, any philosopher who places the predicate logic at the most fundamental level of his formal system could be considered as being in agreement with Frege about the bridging task of grammar and the predicative logical form.

Chateaubriand and Frege shared a lot, in fact, for both have assumed the necessity of a unified metaphysical theory of concepts and their extensions. In this scenario, it is easy to see how Aristotle's formulation of the PC and Frege's Formulation of the PEM will imply similar consequences: both formulations were grounded in the same general idea, to make truth or falsity depends upon the universality of the predicate/subject sub-propositional relation of satisfaction of the one by the other.

Another point worth mentioning is that Chateaubriand's account of Frege's logical priorities characterize propositional logic as a further development of predicate logic (plus, the usual set-theoretical semantics). So, in Frege's system, over and above the foundational predicative level, one will find the truth-functional layer of propositional logic. On his construction, univocity at the basis is mandatory to guaranty the connection of the second propositional layer over the first level, the predicate logic.

On Wittgenstein's account in the *Tractatus* we can note the same distinction between the two different tasks that should be performed by logic. Besides, in his explanation of propositional logic, the interconnection between "sense" and "truth" was also fundamental. In fact, everything on the truth functional propositional level followed Frege's explanation of the classic logical operations, expressible solely by conjunction and negation.

The sub-propositional level of EP's, thou, was not construed according to Frege's approach. This means that the theory of logical form adopted by him would not be in consonance with the logical grammar proposed by Frege. In its place, the *Tractatus* provided us with an alternative way to account for the logical forms of its EPs. It is true that Wittgenstein also recognized the process of uncovering the propositional logical form as a way to link the meaning of predicates with their potential "truth-conditions", but the final achievement of his analytical process – the "Great analysis" – was quite different. It was definitely not a universal domain of extensions/sets. So, while Frege aimed to settle a "extensional" level, i.e., to have "extensions" considered as the logical objects and as the only objects available at level zero, Wittgenstein did not.

Despite their differences, Frege and Wittgenstein shared two philosophical priorities: the search for a fundamental level of language, and the construal of "propositional sense" as "truth-conditions". Furthermore, regarding propositional logic, we could call both accounts "extensional". As Chateaubriand describes the situation: to equalize "sense" and "truth-conditions" is to place "extensionality", i.e., substitution *salva veritate*, as the fundamental criterion of meaning. (2001, p. 15).

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19 The stipulation of analytical relations between first order concepts, for example. Or the use of quantifiers and symbols of punctuation.

Regarding the *Tractatus*'s sub-propositional level, though, it is not even clear that it should be considered as "extensional", in the sense offered by Chateaubriand. One reason is that the *Tractatus* was not planned to deal with identities and substitution *salva veritate* at this level, as we are going to see next. In contrast, in Frege's system, identity propositions are indispensable elements and constitute the first priority of his formal language. As we are going to see next, there are lots of questions unanswered in that book about EP's logical forms, and we could adopt the hypothesis that Wittgenstein remained in doubt and continued to struggle for a long period over this issue, as this quote seems to attest.

The great problem around which everything that I write turn is: Is there an order in the world a priori, and if so, what does it consist in? (WITTGENSTEIN, Notebooks 1914-16, 1961, p. 53)

### III.2 THE TENSIONED FRONTIER BETWEEN THE TWO LOGICAL ANALYSIS IN THE *TRACTATUS*: THE PROPOSITIONAL AND THE SUB-PROPOSITIONAL LEVELS

Now that we've presented Frege's position and the distinction proposed by Chateaubriand, it is time for us to go back to the debate between Wittgenstein and Frege. In this section we will claim that their disagreement goes deeper than the discussion about the boundaries of sense or the frontier between contingent and necessary propositions. It rather revolves around the scope and impact of the division proposed by Chateaubriand. That frontier line was obliterated in Frege's system by the idea of a unified "logical grammar". However, it was recreated within the *Tractatus* framework.

In Frege's and Russell's formal systems, propositional logic was supervenient on predicate logic. Therefore, there was no room for criticizing the universalization of the subject/predicate. Accordingly, their account of quantified generality, rooted on the same subject/predicate analysis, assumed that singular propositions were just instances of the general ones.

As much as Wittgenstein maintains his mentors' conceptions about generality, as he himself proclaimed, some crucial differences were introduced at the foundational level in the *Tractatus*. Contrary to what we find in Frege's and Russell's philosophical work, in the *Tractatus* we cannot find, neither extensions/sets, nor the "membership" relation. Quite the opposite, the relation established among the referents of sub-propositional terms is characterized by Wittgenstein as an internal relation, holding somewhat mysteriously between simple objects.

Contrarily to the idea of a unified grammar, whereby we find a continuity between the two kinds of logical analysis, in the *Tractatus*, the connection between the way things go at the inner structure of EPs and the way things go between whole propositions at the propositional level was far from clear. The truth-tables consist of an analytical process which should provide the meaning of any molecular proposition in terms of some disjunctive conjunctions of EPs. But, for the EPs' meaning to be successfully determined, we also need an analysis of their logical form. So, in addition to the truth-functional propositional analysis, Wittgenstein desperately needed an explanation of how truth-conditions were determined inside the EPs. Besides, he also needed an explanation of how EP's component's expressions could ever refer to objects. Let's divide our discussion, then, between those two different logical layers<sup>20</sup> and begin by investigating the first one.

20 The propositional and the sub-propositional ones.

In effect, the process of analysis by means of truth tables really ended up with an explanation of whole EPs' truth-value possibilities. According to this step of the process, each truth-table line should represent a possible way the world could have been. In the following aphorism, propositional sense is described as agreement or disagreement with the existence and non-existence of states of affairs.

4.2 The sense of a proposition is its agreement and disagreement with possibilities of existence and non-existence of states of affairs.

4.431 The expression of agreement and disagreement with the truth-possibilities of elementary propositions expresses the truth-conditions of a proposition. A proposition is the expression of its truth-conditions. (Thus, Frege was quite right to use them as a starting point when he explained the signs of his conceptual notation...)

For one example, consider the propositions P and Q combined disjunctively. According to Wittgenstein, the sign which express the truth-conditions of this disjunction would be: "(TT-T) (p,q)", (*Tractatus*, 4.442)). In addition to the more restricted combination of two EPs, he also proposed the composition of a colossal truth-table, an enormous conjunction which should include all EPs. The goal was to obtain a complete description of all possible ways' the world could have been. The lines of this gigantic logical entity would display a modal explanation of reality, each line corresponding to a complete attribution of truth values to the totality of EPs. Wittgenstein called this complete range of possible atomic states-of-affairs' the "logical space".

There has been a large debate in the specialized literature around the exhaustive character and the questionable infinite proportions of this enormous construct, the "logical space". However, this is not our focus here. We just want to emphasize that the completeness and infinite proportions of Wittgenstein's modal construction was not determined in any way by the propositions' predicative structure. The lines of that special truth-table only displayed T and F possibilities, i.e., the actuality or contra-factuality of each possible and independent atomic state-of-affairs. Furthermore, this survey was proposed to be done in an aprioristic and purely logical way by the mathematical idea of "exhaustive distribution". Actually, its construction would demand a simple combinatory rule, which would be entirely neutral concerning the internal structure of SoAs. With this combinatorial rule, one could calculate all the truth and falsity possibilities of  $n$ -propositions just by raising 2 to the  $n^{\text{th}}$  and, thus, obtaining . This result would not require, neither a kind of set theoretical semantic, nor an interpretation of the predicates' constants. The EPs sub-propositional structural components were just inconsequential to this purely truth-functional combination.

4.28 There correspond to these combinations the same number of possibilities of truth—and falsity—for  $n$  elementary propositions.

We could say that, in aphorisms 4.31-4.45, but mainly in all aphorisms of group 5<sup>21</sup>, Wittgenstein has introduced what is nowadays known as a modal explanation of reality.<sup>22</sup> This modal component was present every time he used the two logical modalities: "possibility and necessity". As Raymond Bradley carefully explained in his book (*The Nature of All Being A Study of Wittgenstein's Modal Atomism*, 1992):

21 What we use to call in logical courses as the "Great Truth table of logical possibilities".

22 Cf. also (BRADLEY, 1992, pp. xiii-xv; 24; 122).



In a passage that foreshadows his later metaphor of logical space, Wittgenstein tells us that “Each thing is, as it were, in a space of possible states of affairs.” (2.013). The things and their *de re* possibilities of combination generate this space, this logical space. In short, for Wittgenstein the *de re* modal properties of things are the fundamental ones, both metaphysically and logically. (p. 24)

A very similar strategy for construing the semantical foundation of a formal language was proposed by Rudolf Carnap. The German philosopher has suggested a structure which was quite equivalent to Wittgenstein’s “logical space” called “state-descriptions”. In this exert from BT, Wittgenstein commented Carnap’s and his own attempts.

The idea of constructing elementary propositions (as Carnap, for example, tried to do) is based on a false conception of logical analysis. The problem of that analysis is not: a *theory of elementary propositions* must be discovered. As if principles of mechanics had to be discovered. (alternative text: conception of logical analysis. It views the problem of this analysis as discovering a theory of elementary propositions.) (WITTGENSTEIN, The Big Typescript TS 213, 2005, p. 82e)

As a common ground among the two philosophers, we can pinpoint several shared concepts, but the most relevant for us here are Wittgenstein’s “logical space” and Carnap’s “state-descriptions”. Both structures were conceived as involving the similar ideas of an exhaustive distribution of true values to the atomic propositions of the basis, in a *priori* manner. As it is stated in the quote above, this basis is the result of a careful and methodological process of analysis. So, after the atomic propositions’ evaluation, our philosophers parallelly proposed that the molecular propositions’ meaning should be replaced by an independent logical product of atomic/elementary ones.

According to Bradley, Carnap’s state-description explanation should have been settled in a modal logic scenario.

Chief among the semantic concepts that Carnap employs in Meaning and Necessity are those of state-descriptions [...] Of state-descriptions, he says that they “represent Leibniz’s possible worlds or Wittgenstein’s possible states of affairs.” (p. 9). [...]

Bradley adds also the following remark.

the fundamental ideas that Carnap needed to formulate, in order to develop a full-fledged possible worlds semantics, had already been formulated by Wittgenstein. [...] All that Carnap had to do was to take a good hard look at his state descriptions and to ask: what are they supposed to be descriptions of in some realistic, down-to-earth sense? One natural answer is that they are descriptions of the different possible states of affairs or courses of events (in short, “possible worlds”) in which the speaker of a language could possibly find himself and which he could in principle distinguish from each other. [...] This is all we need to arrive at the basic ideas of possible worlds semantics, (p. 375)

We can conclude safely then that at least some general modal-theoretical approach to language was another point of convergence between Carnap and Wittgenstein.

A final similarity between their works is that they had to face very analogous difficulties. Comparing their proposals on this matter, one could say that their construal of “molecular propositions” as “a result of a unique universal propositional truth evaluated basis” involved the same inconvenient requirement: the logical independence of atomic/elementary propositions.

The following passage from the *Big Typescript* summarizes Wittgenstein's later self-criticism.

Can a logical product be hidden in a proposition? And if so, how does one find this out, and what methods do we have of pulling what is hidden in the proposition into the open? So long as we don't have any methods (for finding it), then we can't speak of something being hidden or possibly hidden.

The question whether a logical product is hidden in a proposition is a mathematical problem. For finding the hidden logical product is a mathematical task. (2005, pp. 82e, BT)

If we were correctly construing the consequences of this hidden logical product's idea, every ordinary proposition could be translated into a logical product of EPs. Furthermore, the conjunction of all those products form one of the world-line of this enormous truth-table.<sup>23</sup> The final result of their method would be the successfully "pulling what is hidden in [all ordinary] propositions out into the open". This successful enterprise in turn should lead to the right explanation of meaning in terms of truth-values. Each conjunctive line of the big truth table could undoubtedly be called "extensional" for the whole construction shows precisely which elements are inter-substitutable *salva veritate* in any molecular proposition.

In this scenario, propositional logic would have gained a central position with respect to its truth-functional nature and of the fruitful logical notion of "truth-tables". Furthermore, it was expected that the truth-functional analysis associated to the "truth-tables" could be the reverse process of an ulterior synthesis in term of the same truth-functions. However, this down and up procedure would depend on the atomicity and logical independence of its elements, plus some other sort of semantical analysis at the sub-propositional level. Unfortunately, this point was quite obscure in the *Tractatus*. Questions like: "How does each conjunctive sequence of EPs depict reality?"; or "How each of them could represent its proper state-of-affairs in a semantically independent manner?" could certainly be posed.

As we have indicated at the beginning of this paper, the answer to these questions would have to be found at the sub-propositional level. It seems to be the task of EPs to provide a "metaphysical theory about reality" through the reference of their component terms. But, if this is its right task for them, some further questions would appear. Does Frege's predicative logic fits in the *Tractatus'* scenario? Or is there another kind of logical grammar which fits better Wittgenstein's ideas?

As we have already discussed in the last section, in Frege's hands, an "extensional" theory of meaning also includes a set-theoretical kind of semantics. This means having something like Frege's logical objects, the "extensions". Frege's extensions should provide a compositionality criterion concerning the sub-propositional context, in two ways. On the subject part, two singular terms connected by an identity relation could be replaced one by the other in any predicative proposition, maintaining its truth value invariant. Whereas, on the predicative propositional part, two co-extensive general terms (predicates) could also replace one another, *salva veritate, in any proposition*

Wittgenstein's "logical space" certainly satisfies the extensionality requirement at the propositional level, it respects the substitution *salva veritate* principle for entire propositions,

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23 Each line of this enormous truth-table represents a possible world, or a possible way the world could be.

as we have already said.<sup>24</sup> But, if we compare Wittgenstein's proposal concerning the sub-propositional level with Frege's principles of substitution *salva veritate*, many questions arise. For example, why Wittgenstein proposed to continue the analytical process and construe another layer underneath the layer of singular propositions? Why should EPs have another kind of logical form, different from the usual predicative one? How is Wittgenstein conceiving such new logical form? Would it include only two kinds of elements, like Frege's singular propositions or Carnap's atomic propositions? Or would it have some other kind of multiplicity? Would it include a reference to time and place, like addresses or locations on a map?<sup>25</sup> Are there any other possible logical form at this fundamental level besides the subject/predicate one? Does this other logical form accept both possibilities: truth and falsity as well? These and other further questions will be investigated in the following sections.

#### IV. WHAT HAPPENED WITH FREGE'S DISTINCTION BETWEEN SENSE AND REFERENCE IN *TRACTATUS*?

Until now, we have tried to equate propositional logic with the laws of truth and predicate logic with a theory of logical form which is responsible for the way sense is introduced in the language, as suggested by Chateaubriand. Keeping in mind that the later kind of logic has to deal with the way sense was assimilated by EPs, we have also tried to understand what was going on at the sub-propositional level in the *Tractatus*. The idea is that Wittgenstein should have provided some explanation of how the logical form of EPs allows its inner components to connect to "sense makers", or whatever they have to connect to in reality, in order to have sense.<sup>26</sup> Furthermore, we have also brought into the discussion the idea that any theory of logical form, like the one described by Chateaubriand, has to deal with compositionality requirements. However, we have encountered some difficulties in applying the same compositionality principles Frege had proposed to the EPs structural components.

Of course, the application of compositionality principles to the internal structure of EPs would depend on how Frege's distinction between sense and reference is assimilated and put to work at the sub-propositional level by Wittgenstein in the *Tractatus*. So, let us now try to understand better how Frege's distinction was received by his fellow junior philosopher and what have been changed in Wittgenstein's approach to them.

##### IV.1 FREGE'S TWO COMPOSITIONALITY PRINCIPLES

Compositionality is a method for reaching the sense of the whole through the senses of its parts. In a first formulation, it came from Frege as well, and we could say that he has thought of it as a sort of "building blocks principle".

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- 24 For Wittgenstein, nevertheless, whole EPs did not name truth-values, as they did in Frege's system. They are said to be true, if the SoA they figurate happens to occur in the world, and false, if this figurative relation does not take place.
- 25 This last alternative frequently appears in Frege's writings in the *Foundation of Arithmetic* when he exposed his process of analysis and on the idea of concepts as structured lists.
- 26 Thinking here on the totality of the logical space.

It is remarkable what language can achieve. With a few sounds and combinations of sounds it is capable of expressing a huge number of thoughts, and, in particular, thoughts which have not hitherto been grasped or expressed by any man. How can it achieve so much? Because thoughts have parts out of which they are built up. And these parts correspond to groups of sounds, so that the construction of the sentence out of parts of a sentence corresponds to the construction of a thought out of parts of a thought. (FREGE, *Logics in Mathematics*, 1979, p. 225)

In the middle period of his work (around 1992 onwards), Frege divided his original *Begriffsschrift's* notion of sense as "conceptual content" in two elements, sense and reference, as it is generally acknowledged. A less familiar idea regarding Frege's system, though is that this separation of sense and reference also originated two different principles of compositionality: the principle of substitution *salva veritate*, regarding referents, and the principle of substitution *salva analyticity*, regarding senses. In other words, in Frege's system, compositionality of senses should be sharply distinguished from compositionality of referents.<sup>27</sup>

The first obstacle to our understanding of how compositionality works in the *Tractatus* concerns identities. In Frege's system, the identities had a crucial status and were in need of evaluation criteria, his famous "identity criteria". In the *Tractatus*, nevertheless, we did not find identities among the genuine propositions.<sup>28</sup> Since identity statements are the ones who assert that two names have the same referent, the absence of identities among EPs in the *Tractatus* lead us to conclude that there could be no replaceability at this level. The only exception would be the replacement of *signs for* abbreviation or simplification purposes.<sup>29</sup> Nevertheless, we can speak of substitutions of expressions (*Ausdrucks*) with the same sense inside *unanalyzed* propositions. What is more surprising though is that, in Wittgenstein's idiosyncratic way of thinking, he still seems to maintain that Frege and Russell compositionality requirement for sense somehow still holds.

3.318 Like Frege and Russell, I construe a proposition as a function of the expressions contained in it.

Accordingly, at the end of the *Tractatus's* analytical process, we will have to find some sort of "atomic semantic blocks", just as we've encountered in Frege's "building blocks theory" with his logical objects. In the next section, we will discuss some possible ways of conceiving this compositionality process.

#### IV.2 WITTGENSTEIN'S OWN DISTINCTION BETWEEN SENSE AND REFERENCE

While Frege's distinction between sense and reference was distributed over all the elements of his *concept-script*, in the *Tractatus* the distinction was obtained in a very different manner. It is true that we also had the two components there, sense and reference (*Bedeutung*). Yet, those two ingredients have received quite distinct treatments. In fact,

27 In the case of propositional logic, compositionality means maintaining the same truth value.

28 5.533 The identity sign, therefore, is not an essential constituent of conceptual notation.

29 In fact, Wittgenstein's position in *Tractatus* is similar to Frege's initial position in *Begriffsschrift*. As we said in Velloso (2009), Frege's first solution to the identity paradox involves the idea of treating names just as signs, when they appear flanking the equality sign in identity propositions. His idea is similar to the use of quotation marks which oblige us to consider, not the referent, but the sign itself.



Wittgenstein has used them in such a way that no linguistic element ended up having both, sense and reference.

3.3 Only propositions have sense; only in the nexus of a proposition does a name have meaning [*Bedeutung*].

Concerning the EPs, Wittgenstein's departure from Frege is evident: the whole EP has a sense and "describe" a situation, but don't have referent.

3.144 Situations can be described but not given names. (Names are like points; propositions like arrows— they have sense.)

On the other hand, names had just reference (*Bedeutung*), but no sense:

3.142 Only facts can express a sense, a set of names cannot.

Despite all that, the distinction established in 3.144 and 3.142 didn't operate everywhere in the *Tractatus*. In the following presentation of the tractarian compositionality principle for sense, Wittgenstein used the term "*Bedeutung*" to characterize an expression (*Ausdruck*) that was not strictly speaking a name.

3.314 An expression has *meaning* [*Bedeutung*] only in a proposition.

3.31 I call any part of a proposition that characterizes its sense an expression (or a symbol). (A proposition is itself an expression.) Everything essential to their sense that propositions can have in common with one another is an expression. [my emphasis]

What could be the appropriate analysis of Wittgenstein's divergent employment of the expression "*Bedeutung*" in §3.314? The answer to this question is quite crucial to our discussion, since it directly involves the difference between events and *complexes*. So, let's try to answer it.

First of all, in §3.314 Wittgenstein was talking about those ordinary propositions' parts which contribute to the sense of the whole proposition. He called them "expressions" (*Ausdrucks*). However, those expressions had also another task within those propositions, similar to that of ordinary names. They stood for physical objects (or complexes).<sup>30</sup> In several passages from the *Notebooks* (NB, from now on) Wittgenstein seems to be worried about what expressions were considered as "names" in ordinary grammar. He was concerned about their behavior in language, for they were still used as names, even when they involved some descriptive content.

To make Wittgenstein's point clearer, let us analyze an example introduced by him some years later, in the *Philosophical Investigations* (§60), the sentence "The broom is behind the door". According to the later Wittgenstein, in the *Tractatus* he thought that the expression "broom" could be further analyzed into: "a statement giving the position of the stick and the position of the brush". Accordingly, the expression "the broom" still involved a descriptive content at that time, an indication that it should be further analyzed. Answering our initial question, the word "*Bedeutung*" occurred also in passages where the respective expressions (as "the broom" of our example) happened to have a double role: they

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30 Wittgenstein's account of complexes is very important to our argumentation and will received a separated treatment in the next section.

described, and, *in a certain sense*, referred to, also. Their referent, what Wittgenstein called "complexes", was identified as what one would normally call a "physical object", a body.

Wittgenstein found those expressions' double role problematic at the NB:

But if there are simple objects, is it correct to call both the signs for them and those other signs "names"? (NB 27.5.15, page 52)

It always looks as if there were complex objects functioning as simples, and then also really simple ones, like the material points of physics, etc... (NB 21.6.15, page 69)

He finally decided not to treat them as "names" at all. The problematic aspect for him, we might guess, was that their supposed referent was a complex also and, so, could be analyzed further into its component parts. Based on those passages, we can assume that for Wittgenstein in the *Tractatus*, any other expression used outside the context of EPs should, strictly speaking, not be called a "name" at all. Unlike names, those sub-components of propositions do have sense, i.e., "stand for" complexes. Using now our interpretative suggestion above, we could say that in §3.314 Wittgenstein was not implying that expressions (*Ausdrucks*) refer to anything. Quite the opposite, they should not behave like genuine tractarian's names. Those simple names must be different from all other language signs:

3.2 In a proposition a thought can be expressed in such a way that elements of the propositional sign correspond to the objects of the thought.

3.201 I call such elements 'simple signs', and such a proposition 'completely analyzed'.

3.202 The simple signs employed in propositions are called names.

3.203 A name means an object. The object is its meaning [*Bedeutung*].

3.221 Objects I can only *name*. Signs represent them. I can only speak *of* them. I cannot *assert* them. A proposition can only say *how* a thing is, not *what* it is.

Names, in Wittgenstein's sense, must only refer. They could never express how things are. Arriving at the Great Analysis' final stage, we shall find only those essential elements, the simple names. Besides, they would be but labels pointing to their referents.

We now seem to be able to enunciate Wittgenstein's predicament in a more direct way. For him, at the level of a *whole* EP, we could speak of its sense as the possibility of its being true, or false. Furthermore, we could combine this "sense" with that of other EPs. The result of combining two or more EPs shall be a new molecular proposition on the same level. From the perspective of the EP's internal structure, though, one would find only simple signs, "the simple names" as their sub-component elements. Besides, each one of those names should have its own referent. Thus, one shall conclude that it would be quite impossible to talk about sub-propositional replaceability of tractarian names, for there were no two names that could refer to the same object. In this last restricted range of simple names and simple objects, the sense of the respective EP would have to be derived solely from the combinatorial possibility of those *names* and their respective *simple objects*.

Looking back to Frege's position on this matter, a singular proposition should contain both: a singular term, the nominal part, and a general term, the predicative part.<sup>31</sup> On Wittgenstein's proposal, though, EPs would only contain names. It is important to note that the difference was not that Wittgenstein's proposal did not involve unsaturated elements in the inner structure of EPs. In fact, it was a little more complicated than this. Despite being names of single entities, simple names seemed to possess an "unsaturated aspect", as the following quote asserts:

2.0121 Just as we are quite unable to imagine spatial objects outside space or temporal objects outside time, so too there is no object that we can imagine excluded from the possibility of combining with others. [...] If I can imagine objects combined in states of affairs, I cannot imagine them excluded from the possibility of such combinations.

In this quote, Wittgenstein was committing precisely to the thesis that simple objects were essentially combinatorial entities. His remark about the impossibility of imagining them separately from their inter-connections inside a SoA suggest that they should not be conceived as individuals.

There are two alternative interpretations available among *Tractatus'* commentators, in the side of a predicative interpretation: to treat simple objects as *qualia* or to treat them as universals.<sup>32</sup> Despite that, as we've said three paragraphs before, the linguistic counterparts of *qualia* or universals are general terms. A first objection to this interpretation is, thus, that normally general terms should not be understood as names of anything singular. Simple tractarian names, on the other hand, point directly to their unique referent. Therefore, the primary reason for not interpreting them as universals is Wittgenstein's own explanation of simple names as labels of absolutely singular and unrepeatable entities. Our conclusion is that tractarian names should not be treated as general terms, even if we accept that they are unsaturated.

Wittgenstein's ideas about the sub-propositional level until now seems quite different from Frege's. For, on the one hand, Wittgenstein denied that EPs could be taken as names, arguing that they should be essentially structured linguistic items. But, on the other hand, he construes each part of an EP as the name of one part of a SoA. Now, this kind of nominalist relation is unusual in a set-theoretical context and fits better the mereological approach, as we are going to see next.<sup>33</sup>

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31 Even for Frege, the important trace which maintains the distinction between singular and general terms is that the former presupposes the extra linguistic clauses of existence and uniqueness, while the later doesn't imply neither of them. Quite on the contrary, for Frege a general term is necessarily uncorrelated by the reference relation to any object and furthermore, *by definition*, cannot involve the unicity condition. According to Frege, predicates (general terms) should be capable of denoting more than one object at a time, an ability which is maintained even when its extension is unitary, or even empty. For example, instead of writing "The Moon", we could write "a moon", and then we are not naming an object but referring to one or more things that satisfy the criteria "being a moon". (The Foundations of Arithmetic, pp. 64, § 51)

32 Cf (FRASCOLLA, 2000, p. 81) where he claims to "unravel the entangled muddle of interpretations which commentators have made over the years" by not adopting his "conception of objects as *qualia*".

33 For a discussion regarding this matter, see (LANDO, 2007). Lando doesn't seem to ever consider a third option, a logical theory which puts the mereological account at the basis and keeps the extensionalist account at the level of general propositions. So, his option was in favor of a homogenous set-theoretical approach in the *Tractatus*.

## V. WHAT CHANGES AT THE *TRACTATUS*'S SUB-PROPOSITIONAL LEVEL.

We will now try to compare some elements of Wittgenstein's explanations of the sub-propositional level, in the *Tractatus*, with another kind of logical theory which also avoids classes and the membership relation. This was the case of logical mereological theories, which have gained prominence in the middle of the 20th century. One example was Henry S. Leonard's Harvard Ph.D. dissertation in philosophy, developed in 1930, which set out a formal theory of the part-whole relation.<sup>34</sup>

Set theory doesn't provide the only conceptual framework for thinking about totalities. The mereological theory of Leonard and Goodman [1940] provides another. (BRADLEY, 1992, p. 211)

In fact, the combinatorial nature of simple objects is reminiscent of that of "combinatorial individuals" in a mereological construal, as Bradley himself recognizes in his book:

I've portrayed the early Wittgenstein as a *de re* possibilistic atomist operating within the framework of a possible worlds' ontology. He is an atomist insofar as all the items in his ontology, linguistic as well as nonlinguistic, are taken to be combinatorial constructions out of simple objects (individuals, things), simple properties, and simple relations.

Bradley also mentions the Australian Philosopher, David Armstrong (*The Nature of Possibility*, 1986) as someone who had also tried to match the idea of combinatorial atomic elements with the modal notion of "possibility". According to Bradley (p. 213), Armstrong thought about his proposal as a "latter-day version of Logical Atomism". He notes that Armstrong has also admitted, as a necessary truth that:

6.3751 [...] particles that are in different places at the same time cannot be identical. (WITTGENSTEIN, *Tractatus Lógico-Philosophicus*, 1961)

Plus, the mereological principle that:

"if x is a proper part of y then x cannot be identical to y". (BRADLEY, 1992, p. 213)

One advantage of bringing those various logical alternatives into our discussion is that mereological theories would provide an example of a single compositionality principle, which explain both, the intentional and the referential aspect of language's expressions. The reason is that all referents were supposed to be concrete individuals of some kind and should encapsulate what can be said about objects. Those simple objects would provide the *truth-conditions* of any proposition through an analytical down-up logical process in exactly the way all logicians' theories about sense have proposed to do. This would solve the duplicity of roles found in Wittgenstein's divergent account of "sense" and of "reference", for the reference of names would engender the sense of the entire proposition.

Another advantage is that the authors mentioned by Bradley could be measured up to Wittgenstein in the *Tractatus*, for trying to explain the notion of possibility by the same idea

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34 Leonard's formal theory evolved into the "calculus of individuals" in (LEONARD & GOODMAN, 1940). The calculus of individuals is the starting point for the post-1970 revival of mereology among logicians, ontologists, and computer scientists, a revival well-surveyed in (CASATI & VARZI, 1999) and in (SIMONS, 1987).



of “atomic elements’ compositionality”. But the main reason supporting this alternative approach is that the internal relation thought by Wittgenstein to hold between simple objects was derived from their disposition and articulation as different parts of a single whole. This idea is also shared by those mentioned author within the mereological framework.

V.1 REVISITING ELEMENTARY PROPOSITIONS’ LOGICAL FORM, WITTGENSTEIN’S VISION IN RETROSPECT.

One of Wittgenstein’s greater difficulties was to give examples of EPs. Nevertheless, we know this much: their isomorphic representants, the SoAs, must be independent one of the other.

2.061 States of affairs are independent of one another.

2.062 From the existence or non-existence of one state of affairs it is impossible to infer the existence or nonexistence of another.

The principle of independence of SoAs and its corresponding principle of the independence for EPs were both fundamental for preserving the notion of “sense as truth-conditions”, as we already said. Assuming then that those logical-independence principles, plus the idea of favoring a mereological approach, we can now proceed in our task of investigating how EP’s logical form should have to look like, according to Wittgenstein.

One suggestion is to once again call upon the assistance of one of the *Tractatus*’s aphorisms:

2.15 The fact that the elements of a picture are related to one another in a determinate way represents that things are related to one another in the same way. Let us call this connection of its elements the structure of the picture and let us call the possibility of this structure the pictorial form of the picture.

According to this aphorism, with the terminus of the GA, one should get a kind of isomorphism between language and reality<sup>35</sup> (the whole logical space, including the actual world). Consequently, at the level at which language would “touch reality”, one would be left only with two elements in an intimate relationship: one linguistic, the Eps; and the other ontological, the SoAs. Also, according to Wittgenstein, they would have to have the same “mathematical multiplicity”.

In a recent paper<sup>36</sup>, we have discussed Wittgenstein’s notion of “logical form” in connection to the “color exclusion problem”. We have provided there a more thorough discussion regarding Wittgenstein’s later, retrospective, opinion concerning EP’s internal structure. His afterthoughts are contained in the following Big Typescript’s (BT) passage:

35 Usually, an isomorphism is understood as a relation between two languages, where the truth of one can be transferred, *salvo veritate*, to the other. But one could also understand “Isomorphism” as it is frequently used in the *Tractatus*’ secondary literature, as a kind of correlation between language and reality.

36 (VELLOSO, Wittgenstein’s Construal of “Numbers” as “Schemes” and the Color Incompatibility Problem, 2021)

Then, when I wanted to carry out such an analysis of a color statement, it became apparent what I had imagined analysis to be. I believed I could understand a color statement as a logical product  $r \& s \& t \dots$ , the discrete factors of which indicated the *ingredients* (if there were several) that the color ("color", not "pigment") consisted of. Then, of course, it also had to be said that these are all the ingredients, and this requirement has the effect that  $r \& s \& t \& S$  contradicts  $r \& s \& t \& u \& S$ . Then the color statement would run as follows: "Now these colors (or: this color) and no others are (is) in this place". BT, 340e-341e

We have suggested in that paper that we should extract two lessons from Wittgenstein later ruminations. The first one is that SoA, including the atomic ones, should be complexes and complexes, by definition, have parts.<sup>37</sup> The second lesson is that atomic SoA's parts should be identified with what Wittgenstein called "ingredients". In the quote above, he suggests also that they are the basic simple elements out of which the atomic SoA should have to be constituted. Besides, he says also that a sequence of simple names could be correlated to an ontological sequence of "ingredients", univocally. According to him, those ingredients will be instants of time, points in space, and, finally, color's ingredients (brightness, intensity, and chroma).

In that paper we've proposed to identify "the discrete factors [...] indicat[ing] the *ingredients* that the color consisted of" together with the other ingredients for time and location with coordinate names. In this explanation, propositions will consist of an articulated list of coordinate names. Now, if SoAs are independent of each other, and completely singular and unrepeatable, then simple names are the labels of their singular and unrepeatable parts. Besides, with the analytical procedure carried out in the *Tractatus* we should be able to finally reach the terminal stage where those articulated structures, and all its parts, should be finally encountered. So, at this terminal stage, when all predicates, including the color predicates, were analyzed away, as Wittgenstein has expected them to be, we would be left only with those final "ingredients", plus a residual predicative like expression: "Occurs!". Not being an ordinary predicate, this last residual assertion would not compromise EPs' logical independence. Aside from that, those final ingredients would act as the true building blocks in the *Tractatus*' system, upon which the principle of compositionality of sense would operate.

Comparing Frege and Wittgenstein's accounts regarding atomic propositions' internal structure, we realize that they went on in quite different directions. For Frege, the predicates of two singular propositions would be equal<sup>38</sup>, if, and only if, their extensions are the same. Additionally, Frege thought he had solved the problem of identity between concepts by appealing to his notion of "extension" as logical objects<sup>39</sup>. Wittgenstein's goal was a quite different one. He aimed to extend the analysis of singular propositions and reach the lowest possible level of decomposition. At this bottom, he expected to find only names of "ingredients", or "elements". In a nutshell, we could say that what Frege calls "objects", Wittgenstein considers to be still "complexes", in need of a further analysis.

In the previous section, we have pondered over Frege and Wittgenstein's differences

37 For Frege, this thesis does not hold. Let's remember that the elephant trunk is not part of the set of elephants, neither belongs to it in way. For Frege, the elephant trunk is another object, distinct from the elephant and with an equal grade of complexity. We are going to see that Frege's arguments go in this same line.

38 In GG, §9 -11, Frege uses the sing of identity in place of the equality sign to compare two predicates in one of the first formulations of axiom V.

39 Cf. (VELLOSO, Frege's two notions of "extension", 2023).

of opinion concerning what should be considered as the lowest element in the ontological hierarchy underneath one's formal language. Our next argumentative step will be to test that interpretation by seeing how our suggestions fit into the epistolary debate between Frege and Wittgenstein about the notion of "fact".

### FREGE AND WITTGENSTEIN'S DEBATE REGARDING STATE-OF-AFFAIRS

We have been arguing that Wittgenstein in the *Tractatus* had tried to put into operation two notions which he had inherited from Frege, those of "sense" as "truth-conditions" and of "logical grammar". We have also argued that a peaceful cohabitation of those two notions within Frege's system was no easy task, whatsoever. It required the introduction of Frege's notion of "extension" as "logical objects" in order to be able to represent concepts by their respective extensions. The strategy had a high cost for him, since it was precisely the notion of "extension", introduced by Frege in *GG*, which revealed to be self-contradictory at the end. We can certainly say that Frege was not the only mathematician who had adopted the idea of extension/sets as logical habitants at level zero. But he was surely the only one who had remained utterly uncompromising and had rejected axiomatic solutions, which, for him, would only pragmatically avoid paradoxical consequences and would end up just concealing real conceptual inadequacies.

As we have also commented, Frege's extensional explanation in *GG* involves more than just adopting extensions as logical objects. His extensions are indeed the only inhabitants of his universal domain, aside from the remaining two truth objects.<sup>40</sup> This movement puts the sub-propositional subject/predicate structure at the center of his system, as the ground-level of his metaphysical conception of reality. Frege's motivation was clear. As we've been emphasizing from the beginning of this paper, his extensions are meant to provide him with an identity criterion and so, to satisfy his request of a sharp definition for all concepts.

As we had also argued, Wittgenstein had made a parallel proposal of searching for a "complete determination of sense" at the *Tractatus*. So, one would expect that he would also need an alternative way of accomplishing his proposal, whereas Frege's logical objects, the extensions, turned out to be a bad alternative. In fact, Frege and Wittgenstein went on in different directions regarding the philosophical explanation of this sub-propositional level. For instance, instead of adopting Frege's strategy of accepting extensions as fundamental logical objects, Wittgenstein chose another kind of building block: atomic complexes, composed by his mysterious "simple objects".

In section V, we suggested that Frege and Wittgenstein had offered two completely different explanations of the way language relates to reality at the bottom level of the hierarchy. For the former, since the universal domain was inhabited by objects of a logical nature, on the language side singular terms would refer to those objects and general terms would be represented by them. For the latter, the universal domain was inhabited by atomic complexes, composed of simple objects. Besides, there was only one referential relation between language and reality at this level, tractarian simple names directly pointing to tractarian simple objects. This happened despite the fact that both of them had adopted, each

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40 According to Frege, "I consider as an object everything that is not a function, e.g., numbers, truth values and the courses of values introduced above" (FREGE, *The Basic Laws of Arithmetic* exposition of the system (BLA), 1964, p. §2). A more detailed discussion on the subject can be found in (Ruffino, 2000).

in its own version, the notion of "sense as truth-condition". We are ready now to finally test the explanatory power of our proposal in clarifying Frege's questions about the construal of the sub-propositional level in the *Tractatus*.

For this purpose, we are going to use their famous exchange of letters regarding the notion of "fact" in the *Tractatus*. Two published letters from Frege to Wittgenstein in the year of 1919 preserve that very specific debate. Unfortunately, we have only Frege's part of the conversation, for Wittgenstein's responses to them were lost. However, we believe it is somehow possible to "fill in the remaining gaps". For this purpose, we will rely on some remarks by Frege as well as some quotes by Wittgenstein's later texts. We will use for this purpose the *Tractatus*, Wittgenstein's manuscript MS 105 (dated from the first half of the year 1929) and TS 214 (from around 1931, in the *Philosophical Grammar*).

In the letter of 28 June 1919, Frege commented on one of Wittgenstein's leading ideas in the *Tractatus*:

You write: "It is essential for the thing that it can be a component of an object-state." Can now a thing be also a component of a fact? *The part of the part is part of the whole. If a thing is a component of a fact and every fact is part of the world, then the thing is also part of the world.* For a better understanding, I wish examples in order to see, as I trust I would, what corresponds in language to the fact. [my emphasis] (FREGE, Frege's Letters to Wittgenstein about the *Tractatus*, 2003, pp. 23-24)

We might perhaps safely guess that here Frege was referring to the aphorisms:

The world is all that is the case.

1.1 The world is the totality of facts, not of things.

1.2 The world divides into facts.

2.01 An atomic fact is a combination of objects (entities, things).

If we consider them all together in that sequence, it seems that Frege was criticizing Wittgenstein for introducing a transitive relationship of part and whole with respect to the tractarian notion of "fact". This is exactly how Giorgio Lando has interpreted this same excerpt in his paper "Tractarian Ontology: Mereology or Set Theory?". In Lando's opinion, Frege is saying that, if the *Tractatus* was indeed a mereological theory, then those notions would have to be transitive regarding one to the others. But since this is not the case, then either we *reject* one of the initial aphorisms 1, 1.1 and 1.2 or rather conclude that the *Tractatus* is downright *inconsistent* (LANDO, 2007, p. 248).

As we will see next, of course there seems to be a lack of compositional transitivity within this group of aphorisms. However, as much as we agree with Lando on this particular point, we will argue that there is another, better way to accommodate those aphorisms all together. We would like to avoid Lando's two alternatives: either accept that the conjunction of these three aphorisms shows the *Tractatus*' inconsistency, or immediately dismiss as foolish one of the first entries of this famous and important book.<sup>41</sup> Our suggestion, as it has perhaps become already clear throughout this paper, is that Wittgenstein adopted a double logical approach – mereological and nominalist – at the level of simple objects<sup>42</sup> and predicational for the rest of language.

41 "Therefore, if one is disinclined to concede that *Tractatus* is inconsistent [...]" (LANDO, 2007, p. 248).

42 The sub-propositional level of EPs and their respective SoAs.



For the sake of trying to clarify their debate, we will begin by agreeing with Lando in that Frege has found Wittgenstein's writings quite unclear about that point. According to the first quoted aphorism: "the world is the totality of facts, not of things". However, Wittgenstein didn't quite clear up the question as to how it could be that, if *the world consists in the totality of facts*, and if, according to the aphorism 2.01, *a fact is made up of things*, we could end up concluding that *the world could not be made up of things*? Our first impression, therefore, would be that Frege was entitled to react to just this absence of a transitivity in the relations involving *objects, facts* and the *world* within the *Tractatus*. We still think, nonetheless, that we have good reasons to claim that Wittgenstein did not make the mistake which Lando attributes to him. To support this position, we will first suggest that Frege's notes did not imply a straightforward accusation of just such incoherence.

Frege's remarks were in fact quite accurate and went directly at the heart of their divergence: the very notion of "fact". His objections could be reformulated more or less on the following lines: Wittgenstein was wavering between adopting an extensionalist account at the lowest ontological level of the *Tractatus* and adopting another, quite divergent kind of explanation for that very same level. In page 24 of this same letter, Frege introduced the example of Vesuvius' lava, in order to clarify the topic.

I wish to have an example because Vesuvius is component of an object-state. Then it would seem also that the components of Vesuvius must be components of this fact; the fact would thus also consist of solidified lava. This would not seem correct to me. (FREGE, Frege's Letters to Wittgenstein about the *Tractatus*, 2003), p.23-24

Frege's objection was reasonable. If one assumes an extensionalist notion of "fact", without any additional physical explanation, the fact is "the eruption of Vesuvius," and the Vesuvius' lava, or the Vesuvius concrete spatial-temporal parts, its physical body's, etc., are not part of this fact. In extensional theories, the membership relation between Vesuvius and the predicate's extension of "that which erupted" is not one of a part and a whole, as Lando noted (2007, p. 248). So, it would be wrong to claim that Vesuvius' lava has a relation, either of "membership", or of "being a part of", to the extension "the things which erupt". It would also be wrong to conclude, from the circumstance that the Vesuvius is a member of the volcanoes set, that Vesuvius's lava is also a member of that set.<sup>43</sup> What is correct to affirm is that Vesuvius's lava has no relation whatsoever with this set, neither as a part with respect to a whole, nor as an element with respect to a set, the set of volcanoes.

Our next step will then be to try "filling in the gaps" by presenting a possible response from Wittgenstein's side. The first preliminary comment about this conjectured answer to Frege's objection is that, since the NB, Wittgenstein had already been extensively discussing the notion of "complex". Even his whole construal of the GA leaned on the argument that the very idea of a "complex" logically presupposes that of "simple components". Now, concerning the notion of fact, we have, of course, aphorism 2.06.

43 In the introduction of the Book "Set Theory and Its Logic", Quine present the relation which holds between the elements and a set or class in an elucidative way. His example are seven pairs of shoes. "Sets are classes. [...] We can say that a class is any aggregate, any collection, any combination of objects of any sort; if this helps, well and good. But even this will be less help than hindrance unless we keep clearly in mind that the aggregating or collecting or combining here is to connote no actual displacement of the objects, and further that the aggregation or collection or combination of say seven given pairs of shoes is not to be identified with the aggregation or collection or combination of those fourteen shoes, nor with that of the twenty-eight soles and uppers. In short, a class may be thought of as an aggregate or collection or combination of objects just so long as 'aggregate' or 'collection' or 'combination' is understood strictly in the sense of 'class'." (QUINE W. V., 1963, p. 1)



(We call the existence of states of affairs a positive fact, and their non-existence a negative fact.)

Nevertheless, Wittgenstein didn't deal in the *Tractatus* with the distinction between "complexes" and "facts" in the same straightforward way he did in the *Philosophical Grammar* (TS 214, around 1931). In the later, he was quite clear about their distinction. Next, our discussion will thus include those later writings as an attempt to fill in the gaps as adequately as possible. The first basic distinction made by Wittgenstein was the following:

Complex is not like fact. For I can, e.g., say of a complex that it moves from one place to another, but not of a fact. But that this complex is now situated here is a fact. (WITTGENSTEIN, *Philosophical Grammar*, 1074, pp. 199-201)

In the sequence, he addressed also Frege's concerns about the mixing of those notions:

[..] To say that a red circle is composed of redness and circularity, or is a complex with these component parts, is a misuse of these words and is misleading. (Frege was aware of this and told me.) It is just as misleading to say [that] the fact that this circle is red (that I am tired) is a complex whose component parts are a circle and redness (myself and tiredness). (WITTGENSTEIN, *Philosophical Grammar*, 1074, pp. 199-201)

In this quote, Wittgenstein seems to be in perfect agreement with Frege. Facts are not the same as complexes and should not be identified with the referent of a true proposition. In as much as one adopts a predicative approach to the propositions' structure, it is misleading to treat a fact as a complex composed of those two "parts": the referent of the subject, and the referent of the predicate. Wittgenstein continued his reasoning by trying to spot the root of this misunderstanding. Thus, he adds the following remark:

The root of this muddle is the confusing use of the word 'object'. (WITTGENSTEIN, *Philosophical Grammar*, 1074, p. 201)

Or, to put in a more precise way, in his opinion, mixing facts and complexes is the result of two conflicting attitudes: 1) correctly assuming that the "actualization of some propositional content" is what one should call "a fact"; 2) but wrongly considering that the fact should be identified with the complex which is asserted as existent.

Returning to the specific context of the *Tractatus*, we realize that there Wittgenstein's proposal was in tune with this first way of approaching the disentanglement of the two notions, "complexes" and "facts", which he expresses in those later excerpts. Actually, he had proposed in the *Tractatus* that each EP should depict one possible SoA, one possible and atomic complex. Next, he added that the ones which pictured how things actually were in the world would be the true ones, and all the others, the false ones. It was also essential to this process that none of those EPs would contain an internal negation. All of them would just depict some possible SoA, some combination of objects. This is important, since it shows that complexes and facts are originally thought as two distinct things. A fact would be an existent SoA, an existent complex.

Our discussion has now reached a very delicate point, for the compositionality between complexes and their parts was at the very core of the *Tractatus*' ontology. Problematic statements like those which relate complex to their parts would have to be part of any

reasonable language regarding the actual world. So, in one way or another, those statements would also be included in both accounts, Frege's and Wittgenstein's as well.

Let's consider first then Frege's case. An extensional theory like his is based on a unique relation of "membership" among the objects of the universal domain. So, Frege's ontological building blocks, as we emphasized in section IV, were not originally related to one another by the part and whole relation. In order to establish that kind of mereological relationship between Vesuvius' parts and the whole of all volcanoes in the world, Frege would have to include in his system a physical theory (such as atomic physics, for example), which would reduce any physical body to a set of atoms by incorporating within language some law-like analytical propositions as, for example, all bodies are made out of atoms. Only by this addition would it be possible to arrive to the whole concrete time-extended process of the Vesuvius's eruption.

There would be a high price to be paid, nevertheless. We would have to presuppose an enormous group of new first level concepts capable of explaining the combinatorial basis of reality as composed by aggregations of singular atoms. Furthermore, for each aggregate, it would be necessary to add a singular term identifying that extension at the level of first order properties/concepts. In GG, Frege had a second-level function which names each extension, his "smooth-breathing" operator. Together with the backslash operator, a "unary first-level, mapping objects to objects", Frege could have named any singular object or physical body.<sup>44</sup>

Now, it is Wittgenstein turn. He had not embraced an extensional theory like Frege's in the *Tractatus*. According to our reconstruction, he has rather tried to maintain an *alternative mereological account that went down further until one reaches the level of "atomic facts"*. So, if GA was extended as Wittgenstein believed it should be and if it has reached the foundational level, there one would find just aggregates of atoms. Theories that accept aggregates at this level would benefit from the transitive character of this kind of relationship and don't have to lean on a scientific theory to explain physical bodies and their contribution to predicates' meanings. As we had illustrated in Vesuvius's example.

Despite the advantage of taking the part and whole relationship as primitive, Wittgenstein's approach still had to include an explanation of aphorism 2.01 "an atomic fact is a combination of objects" in order to avoid its apparent conflict with aphorisms 1.1 and 1.2. So, let's move ahead and try to explain in detail what Wittgenstein thought about "complexes and its parts".

In this other passage from the same source, the *Philosophical Grammar*, Wittgenstein returned once more to this topic.

A complex is composed of its parts, the things of a kind which go to make it up. (This is of course a grammatical proposition concerning the words 'complex', 'part' and 'compose'.) [...] And a complex is a spatial object, composed of spatial objects. (The concept 'spatial' admitting of a certain extension.) (WITTGENSTEIN, *Philosophical Grammar*, 1074, p. 200)

In this quote, he states that the relationship between physical bodies and their constituent parts is that of parts of a whole. So, at the level of bodies and their parts, one surely has transitive relations going on. It is reasonable to propose, also, that for Wittgenstein in the *Tractatus*, complexes, or physical bodies, were already clearly distinguished from facts.

44 Cf. (COOK, 2013, pp. A-4, A15-16, A-29)

So, for him, physical bodies could be divided in parts. But what about facts? Were they also physical bodies? Could they also be divided into parts or be themselves a part of the world? In the following quote Wittgenstein clarified what a fact should be for him from an ontological point of view.

[...] A chain, too, is composed of its links, not of these and their spatial relations. *The fact that these links are so concatenated, isn't 'composed' of anything at all.* (WITTGENSTEIN, 1974, p. 201)

Wittgenstein was not just trying here to avoid the confusion between facts and complexes; he was also clearly stating that complexes are physical objects, although facts are not. He remarked that, although a complex could be broken down into parts, a fact cannot, because a fact does not have physical parts. Facts were just the very situation that there were physical objects disposed in some way or another and not an additional physical thing. However, we still have 1.2 which states that "The world divides into facts". If facts were not like complexes, then it appears they could not be separated in parts either. But then how could they be "a combination of objects" and also be the divided parts of the world? In the same later quotes, Wittgenstein explained how a fact could also be decomposed in other more atomic facts by a further analytical step, without being divided into parts. In this later passage, According to him, the GA divides only the complexes and not the facts. For retrieving the original fact from an analyzed situation, one would have to point out just where the complexes' parts were localized and then recover back the whole situation:

That this constellation is located here, can of course be described by a proposition in which only its stars are mentioned and neither the word 'constellation' nor its name occurs.

Each part of the complex would be in a location at a time and that description would represent the original fact, the original complex's existence.

Comparing now Frege's comment about the Vesuvius with Wittgenstein's ideas about atomic facts, we can actually imagine Wittgenstein's answer to the elder's objection. The assertion that "Vesuvius erupts" would be an assertion of a fact. It is also reasonable to affirm that this fact was, neither the propositional content, nor the complex, but the occurrence of Vesuvius' eruption. The description of the actual localization and the route of each drop of lava would, in Wittgenstein's account, described that same fact in a more accurate way. But there was a proviso, the part and whole relationship holds only between the complex and its parts and not between the eruptive fact and the lavas' drops.

We believe that for the Wittgenstein of the *Tractatus*, the latter way of asserting facts was the only acceptable one. This would mean also that the uniqueness of the GA is the only path connecting the more generalized assertion of the Vesuvius' eruption and the more detailed assertion of the occurrence of each drop of lava which constituted Vesuvius' eruption.

1912, Page 121. I believe that our problems can be traced down to the atomic propositions. This you will see if you try to explain precisely in what way the Copula in such a proposition has meaning.

[...]

IV Alleegasse 16. Wien. 26.12.12. I had a long discussion with Frege about our theory of symbolism of which, I think, he roughly understood the general outline.

He said he would think the matter over. The complex-problem is now clearer to me and I hope very much that I may solve it. APPENDIX III EXTRACTS FROM WITTGENSTEIN'S LETTERS TO RUSSELL, 1912-20, p. 86

[...] every proposition which seems to be about a complex can be analyzed into a proposition about its constituents and about the proposition which describes the complex perfectly; i.e., that proposition which is *equivalent* to saying the complex exists. [my emphasis] (WITTGENSTEIN, Notes of Logic, 1957, p. 238)

As we said above, and can be attested in those quotes, his idea was that one should be able to translate the propositions which assert the occurrence of some event as a product of EPs containing the names of all the possible constituent elements of the complex, plus the localization of each of them, so that this product would imply the same as asserting the existence of the complex in a certain location and at a certain time.

Concluding our arguments in this section, we believe that even in the earlier period of his work Wittgenstein has never mixed-up facts and complexes. Quite the contrary, we do think that he was trying to accommodate both of them in his presentation of the more basic ontological ground in the *Tractatus*. We can even take a step further here. We think it is at least reasonable to suggest that the "*Tractatus's* Great analysis" is an attempt to explore and, perhaps, criticize Frege's idea of taking extensions as the ultimate building blocks at zero level. As Russell realized in his "introduction to *Tractatus*", the *Tractatus'* ultimate logical ground has no facts, only combinations of simple objects.

In this way the naming of simples is shown to be what is logically first in logic. (WITTGENSTEIN, *Tractatus Lógico-Philosophicus*, 1961, p. xiv)

## FINAL REMARKS

At the beginning of this paper, we have claimed that in the *Tractatus*, Wittgenstein struggled to deal with some difficulties inherited from Frege's ideas. The crucial one was Frege's axiom V failure and, with it, the failure of Frege's notion of "extension". We've proposed to see in the *Tractatus* a dual approach towards logic. We contended furthermore that this duality came from Wittgenstein's intent to solve Frege's difficulties with his extensionalist theory of language. We encountered this same idea of two distinct logical levels on Chateaubriand's book *Logical Form*. In section III, we've explored the possibility of reading Wittgenstein in the *Tractatus* as a philosopher who has explored precisely this same distinction. This meant splitting the task of logic in two: (1) one dealing with laws of truth; and (2) the other providing a metaphysical or grammatical explanation of reality. The first task should be accomplished by the truth-functional part of language. The second was supposed to be handled by the predicative logic, the one which finally provided instruments for building the bridge between language and reality.

We have claimed that Wittgenstein modified some of Frege's ideas to make them more suitable to his proposal: the construal of "sense" as "truth-condition", plus the complete determination of every proposition's sense. In order to realize his intends, the younger philosopher refuse Frege's generalization of a the predicative sub-propositional structure to all propositions including the bottom level ones. In its place, Wittgenstein suggested another kind of explanation, a mereological kind. This alternative should provide the right way to analyze the level of EPs, thus providing the sense needed by the rest of the linguistic structure.



Now, if Wittgenstein's strategy at EP's sub-propositional level was what could really provide sense to our language, then this was indeed something lacking in Frege's account of the matter. This "missing" level should be the one in which we find the actual "building blocks of sense", which forms the basic metaphysical ground. As we have promised at the beginning of the paper, we hope to have shown that Wittgenstein's approach to this basic metaphysical level in the *Tractatus* was not a predicative one and did not involve the subject/predicate form. We have suggested that we can find complexes instead of extensions at the bottom level and that these structures are more reminiscent of a mereological kind of analysis.

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### Abstract

This paper examines the concept of "fact" as presented in the following six opening aphorisms from Wittgenstein's *Tractatus Logico-Philosophicus* (1., 1.1, 1.2, 2., 2.01, 2.011). To address the apparent paradoxical nature of these aphorisms when read together, we will argue that, although Wittgenstein adopts a classical, truth-functional approach to truth for non-elementary propositions, he diverges regarding the semantic analysis proposed by this approach when it comes to elementary propositions. He employs instead a mereological framework at this foundational level demanding an entire reconstruction of the classical notion of "fact". We argue that this mixed approach to logic and semantics adopted by Wittgenstein in the *Tractatus* serves a dual purpose: both to avoid Russell's paradox as well as preserving Frege's notion of "propositional sense" as "truth-conditions." The paper also aims to evaluate the explanatory power of this proposal by applying it to a specific epistolary exchange that took place between Frege and Wittgenstein concerning the tractarian notion of "fact" published in 1919.

**Keywords:** Wittgenstein, *Tractatus Logico-Philosophicus*, Facts and Complexes, Mereology in the *Tractatus*, Sense as truth-conditions, Extensionality in the *Tractatus*.

### Resumo

*Este artigo examina o conceito de "fato" conforme apresentado nos seguintes seis aforismos iniciais do livro *Tractatus Logico-Philosophicus* de Wittgenstein (1., 1.1, 1.2, 2., 2.01, 2.011). Para lidar com o suposto caráter paradoxal da leitura conjunta desses seis aforismos, argumentaremos que, embora Wittgenstein adote uma abordagem clássica e verofuncional da verdade para proposições não elementares, ele diverge quanto à análise semântica proposta por essa abordagem com relação às proposições elementares. Ele emprega em vez disso uma estrutura mereológica nesse nível fundamental, decisão essa que o leva a propor uma reconceitualização da noção clássica de "fato". Nossa proposta é a de que essa abordagem mista com relação à lógica e à semântica adotada por Wittgenstein no *Tractatus* sirva a um propósito duplo: evitar o paradoxo de Russell, mas ao mesmo tempo preservar a noção fregiana de "sentido proposicional" como "condições de verdade". O artigo visa também avaliar o poder explicativo desta abordagem aplicando-a a um debate epistolar específico que ocorreu entre Frege e Wittgenstein sobre a noção tractariana de "fato" publicado em 1919.*

**Palavras-chave:** Wittgenstein, *Tractatus Logico-Philosophicus*, Fatos e Complexos, Mereologia no *Tractatus*, Sentido como Condições de Verdade, Extensionalidade no *Tractatus*.