



Mathematical Modeling to Estimate The Growth Of Guandu Bean Pods And Seeds

Rafael Ruy Gouvea¹, Vinicius de Souza Oliveira², Lucas Corrêa Souza¹, Alba Nise Merícia Rocha Santos³, Enilton Nascimento de Santana¹, Karin Tesch Kuhlcamp¹, Omar Schmildt⁴, Edilson Romais Schmildt*⁴, Sara Dousseau Arantes¹

¹Capixaba Institute for Research, Technical Assistance and Rural Extension (INCAPER), Linhares, Espírito Santo, Brazil; ²Federal University of Espírito Santo, Center for Agricultural Sciences and Engineering, Alegre, Espírito Santo, Brazil; ³State University of Santa Catarina (UDESC), Lages, Santa Catarina, Brazil; ⁴Federal University of Espírito Santo, North University Center of Espírito Santo, São Mateus, Espírito Santo, Brazil.

Abstract: The objective of this study was to adjust mathematical equations that accurately and non-destructively estimate the growth of pods and seeds of pigeon pea cultivar Fava Larga from the average length and width. For this, the maximum width (W), maximum length (L), and observed mass (OM) of pods and seeds were measured. Linear models of the first degree, quadratic, power, and exponential were adjusted based on the value of 400 pods and 1938 seeds, in which the OM was used as a dependent variable as a function of L and W as independent variables. For the validation of the models, the root means square of the error was calculated from all equations. Thus, in the estimation of the growth in mg of the pods and seeds of the 'Fava Larga' pigeon pea, the quadratic model equations represented by $y = 594.4994 - 1964.9471W + 2224.2070W^2$ and $y = 24.3003 - 221.91271W + 617.3084W^2$, respectively, obtained using the width.

Keywords: Statistical analysis; mathematics equation; modeling; non-destructive method; adjustment of equations; growth.

Adherence to the BJEDIS' scope: This work is based on the statistical analysis of a sample of pigeon pea pods and seeds, which through mathematical modeling allows the adjustment of equations that accurately and non-destructively estimates their growth.

*Address correspondence to this author at the Department of Agricultural and Biological Sciences/Plant Breeding Laboratory, North University Center of Espírito Santo, Federal University of Espírito Santo, Rodovia BR 101 Norte, Km 60, São Mateus, Brasil; Tel: +55-27-3312-1697; E-mails: edilson.schmildt@ufes.br; e.romais.s@gmail.com



1. INTRODUCTION

Pigeon pea or pigeon pea (*Cajanus cajan* (L.) Millspaugh) is characteristic of being an erect and shrubby plant. The species features a pair of trifoliate leaves with lanceolate or elliptical leaflets 4 to 10 cm in length. The flowers appear in terminal racemes 1.5 to 1.8 cm long, yellow or orange-yellow. The indehiscent pods of green, brown or purple, or even green flecked with brown, oblong. The seeds are presented between two and nine per pod, depending on the development and nutritional characteristics that can directly influence, of round shape, green or purple color when immature and white, yellow, brown to black when mature, being able to show light colors speckled with brown and purple (1, 2).

Pigeon pea is a drought-tolerant subsistence crop grown under rainfed conditions in the semi-arid tropics, including Asia, Africa and the Caribbean (3). The grains have a high protein content and are used after cooking in human food in arid and semi-arid regions, highlighting their nutritional, medicinal and functional properties (4). In addition, the whole plant has several bioactive flavonoids, stilbenoids and coumarins, with antioxidant and anti-inflammatory properties, being traditionally used in folk medicine and with cosmetic/cosmeceutical potential (5). In Brazil, pigeon pea is considered an unconventional food plant, widely cultivated in family farming (6), recommended for animal feed, as a cutting plant, silage or pasture; human food, such as dried grains or green vegetables; recovery of degraded pasture areas; and as a forage alternative for areas with scarce rainfall (7).

Research aimed at the development of mathematical models that allow estimating the volume and mass of fruits (8, 9) and mathematical equations for the estimation of leaf area (10) have been developed because through equations generated with the measurements of the length, width or combination of these variables, it allows to estimate accurately and with low cost, the growth of the studied species (11). Such mathematical equations for estimation of attributes of plants were developed in search of an easy and quick method to be executed and for being non-destructive.

These methods are useful both for small populations of plants, and in experiments with plants conducted in pots, and are important for being easily adapted to use in the field. Modeling and simulation have heuristic value, helping the researcher to formulate hypotheses about relevant processes and interactions, to quantify the impact of simple variables on system performance, and to suggest new needs for experimentation (12).

Thus, the objective of this study was to adjust mathematical model equations that accurately and non-destructively estimate the growth of pods and seeds of pigeon pea cultivar Fava Larga from the average length and width.

2. MATERIALS AND METHOD

To estimate the growth of pods and seeds of pigeon pea (*Cajanus cajan* (L.) Millspaugh) cultivar Fava Larga, 400 green pods were used, each containing 5 seeds, used for validation, collected from different plants with the same stage of development belonging to the Instituto Capixaba for Research, Technical Assistance and Rural Extension, located in the municipality of Linhares, North of the State of Espírito Santo, Brazil, with geographic coordinates 19° 25' 03" South latitude and 40° 04' 50 " West longitude. The climate is Aw according to the Köppen classification, being a humid tropical climate with dry winter and the soil is classified as fulvic neosol. The results of the soil analysis can be seen in table 1.

Table 1. Result of the chemical analysis of the soil in the experimental area.

Parameter analyzed	Extractor	Quantity
P (mg dm ³)	Mehlich: HCl 0.05 mol L ⁻¹ + H ₂ SO ₄ 0.025 mol L ⁻¹	152
K (mg dm ³)	Mehlich: HCl 0.05 mol L ⁻¹ + H ₂ SO ₄ 0.025 mol L ⁻¹	230
S (mg dm ³)	Ca(H ₂ PO ₄) ₂ 0.01 mol L ⁻¹	2,1
Ca (cmolc dm ⁻³)	KCl 1 mol L ⁻¹	7,5
Mg (cmolc dm ⁻³)	KCl 1 mol L ⁻¹	1,7

Al (cmol _c dm ⁻³)	KCl 1 mol L ⁻¹	0
H+Al (cmol _c dm ⁻³)	SMP Buffer Solution	1,5
pH in H ₂ O	pH in H ₂ O 1:2.5	7
Organic matter (dag kg ⁻¹)	Oxidation: Na ₂ Cr ₂ O ₇ 2H ₂ O + 4 mol L ⁻¹ H ₂ SO ₄ 10 mol L ⁻¹	2,4
Fe (mg dm ³)	Mehlich: HCl 0.05 mol L ⁻¹ + H ₂ SO ₄ 0.025 mol L ⁻¹	68
Zn (mg dm ³)	Mehlich: HCl 0.05 mol L ⁻¹ + H ₂ SO ₄ 0.025 mol L ⁻¹	8,4
Cu (mg dm ³)	Mehlich: HCl 0.05 mol L ⁻¹ + H ₂ SO ₄ 0.025 mol L ⁻¹	3,8
Mn (mg dm ³)	Mehlich: HCl 0.05 mol L ⁻¹ + H ₂ SO ₄ 0.025 mol L ⁻¹	93
Bo (mg dm ³)	BaCl ₂ 2H ₂ O 0.125%	0,46
Sum of Bases (cmol _c dm ⁻³)		10
CTC at pH 7.0 (cmol _c dm ⁻³)		11,5
base saturation (%)		86,9

In the laboratory, using a 150 mm Digital Caliper and a Shimadzu BL 320h Semi-Analytical Scale, the maximum width (W, in cm), maximum length (L, in cm) and observed mass (OM, in mg) (Figure 1), of the pods and seeds contained in each pod. For each of the measured characteristics, descriptive statistics were determined, obtaining the minimum, maximum, mean, amplitude, standard deviation and coefficient of variation.

Equations were adjusted based on the value of 400 pods and 1938 seeds, in which OM was used as a dependent variable (y) in function of W and L as independent variables (x). The models linear first degree linear represented by $y = \hat{\beta}_0 + \hat{\beta}_1x$, quadratic represented by $y = \hat{\beta}_0 + \hat{\beta}_1x + \hat{\beta}_2x^2$, power represented by $y = \hat{\beta}_0x^{\hat{\beta}_1}$ and exponential represented by $y = \hat{\beta}_0e^{\hat{\beta}_1x}$. In this way, eight were adjusted for each of the characteristics, and its determination coefficient (R²) was also obtained according to Kvalseth (13).

The root mean square error (RMSE) was calculated from all equations by the expression: $RMSE = \sqrt{\frac{\sum_{i=1}^n (EV-OV)^2}{n}}$, where, EV are the estimated values of pod and seed mass by the equations; OV are the observed values of pod and seed mass; n is the number of pod and seed used in the validation. The best equation is defined as the a that presents the smallest error in the estimation represented by the smallest RMSE values.

For statistical analyses, the R software (14) was used, with the aid of the ExpDes.pt data package version 1.2 (15).

3. RESULTS AND DISCUSSION

Data from descriptive statistics of pods and seeds of *Cajanus cajan* (L.) Millspaugh 'Fava Larga' used for growth modeling is shown in Table 2. It is noted that there was a high amplitude of samples with values higher than the averages obtained for all variables under study. This shows that in the adjustment of the model, they had from the lowest values to the highest values for both pods and seeds. This finding can also be proven by the high values of the coefficient of variation (CV) higher than 27% for all characteristics (16). This high variation ensures that, in the adjustment of the models, there are small, medium, and large pods and seeds, which makes it possible to use the adjusted models at different stages of crop development, allowing their use during the crop cycle, always respecting the intervals of the values used in the modeling (17).

Table 2. Minimum, maximum, average, total amplitude (TA), standard deviation (SD) and coefficient of variation (CV) of length (L), width (W) and observed mass (OM) of pods and seeds of *Cajanus cajan* (L.) Millspaugh 'Fava Larga' used in growth modeling

Variable	Unit	Minimum	Maximum	Average	TA	SD	CV(%)
400 pods							
L	cm	2.0500	8.7600	6.1785	6.7100	1.6743	27.10
W	cm	0.2800	1.5800	1.0115	1.3000	0.4180	41.33
OM	mg	45,0000	3610.0000	1270.2275	3565.0000	1083.1565	85.27
1938 seeds							
L	cm	0.0500	1.1100	0.5554	1.0500	0.2578	46.42
W	cm	0.0800	1.0300	0.4654	0.9500	0.2706	58.14
OM	mg	1,0000	412.0000	99.9106	411.0000	113.1364	113.24

Figure 1 shows the scattering diagram of the mass of green and ripe pods about their length and width. Note that there was a reduction in mass with the advancement of pod maturation. This fact is associated with the growth pattern of the species that, when starting its maturation, ceases its growth and has a reduction in water content. According to Passos (18), the reduction in moisture in the pod and pigeon pea seeds is associated with the point of physiological maturation, where at this stage there is a significant reduction in moisture, thus, seeds from green pods have a higher degree of moisture, while in mature seeds the reduction of the moisture content happens because they have already reached the harvest time.

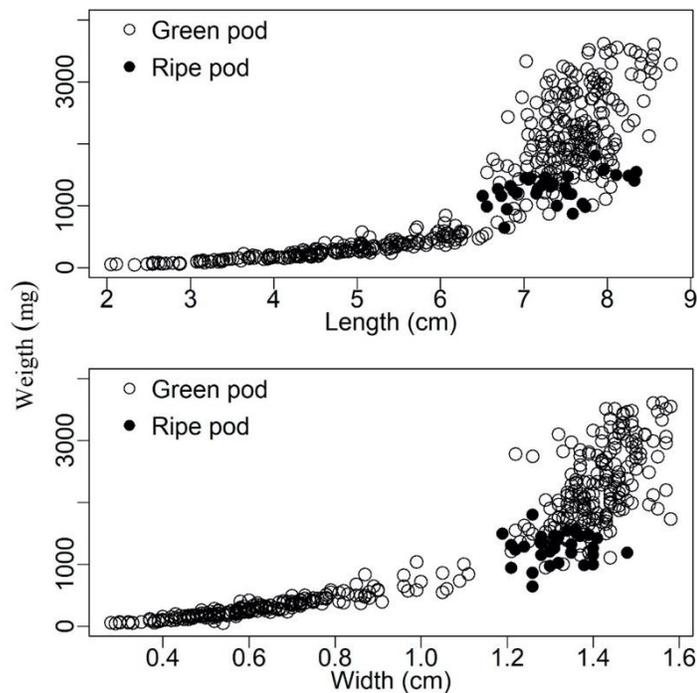


Figure 1. Scatter diagram for the weight data of green pods during growth and mature pods of *Cajanus cajan* (L.) Millspaugh 'Fava Larga' in function of pod length and width.

In figure 2, it was found that for all models, the equations adjusted from the width, best explained the growth of pods, thus, this variable is the most suitable. When individually analyzing the adjusted equations through the width, it can be seen that the quadratic model presented a value of the coefficient of determination (R^2) higher than the other models studied and a smaller error in the estimation of the growth of the pods with lower values of the mean square root of the error (RMSE). Thus, it is possible to say that this equation, in comparison with the others, is the one that best minimizes the errors generated in the estimation, therefore, it becomes more accurate in predicting the mass of pigeon pea pods.

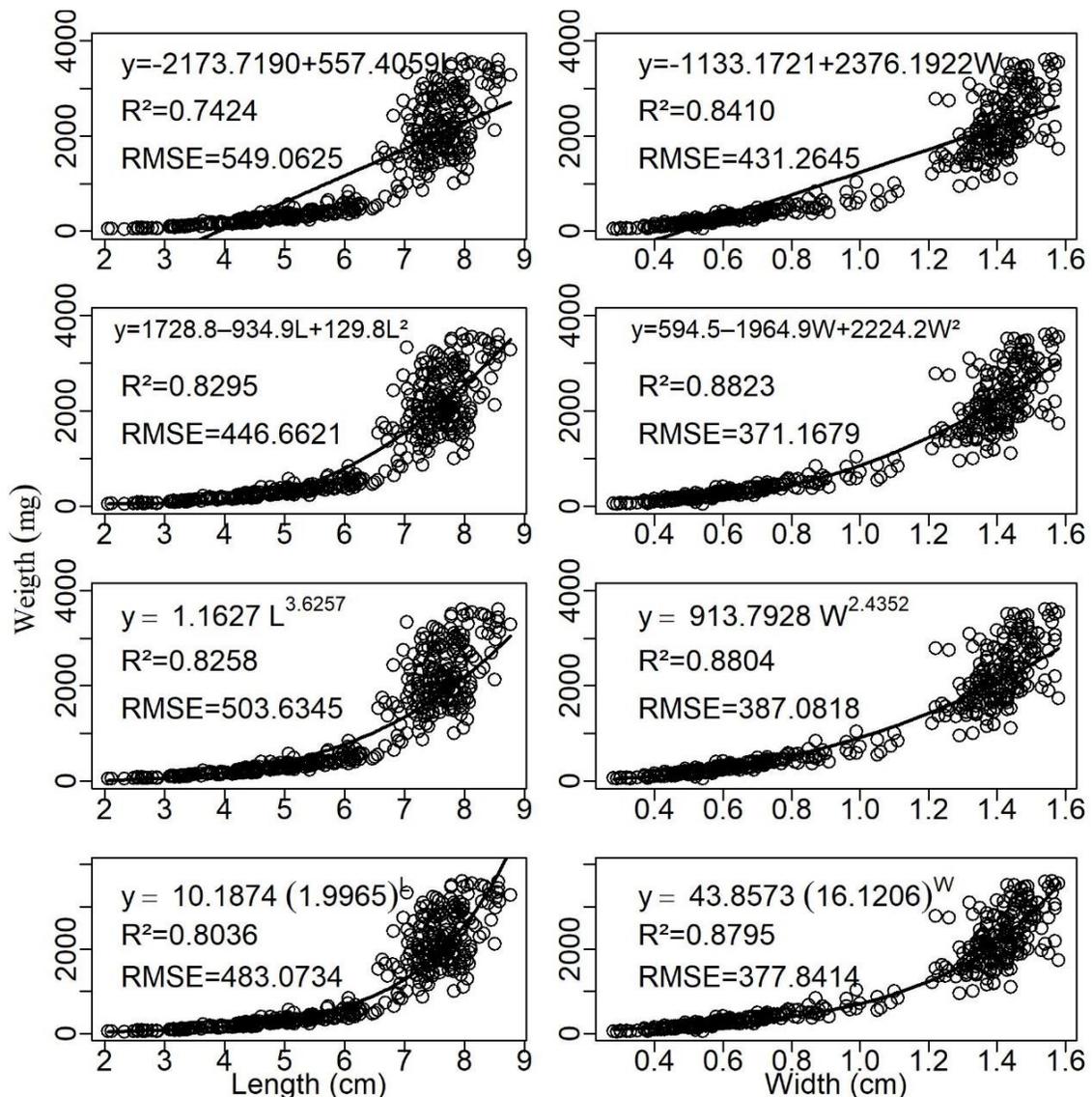


Figure 2. Growth curve of *Cajanus cajan* pods (L.) Millspaugh 'Fava Larga' for four statistical models with the representation of the estimated equation, the coefficient of determination and the root mean square error. Left and right column refer to the length and width, respectively, of the pods, as explanatory variables.



The eight equations adjusted to estimate the growth of pigeon pea seeds (Figure 3), five equations presented R^2 greater than 0.90, which is indicative of a good relationship between the variables under study. However, it is noted that the quadratic model equation adjusted from the width showed higher values of R^2 and was also considered more appropriate when analyzing the possible errors generated by the equation in the estimation of the growth of the pods since the RMSE value was the smallest in comparison with the others.

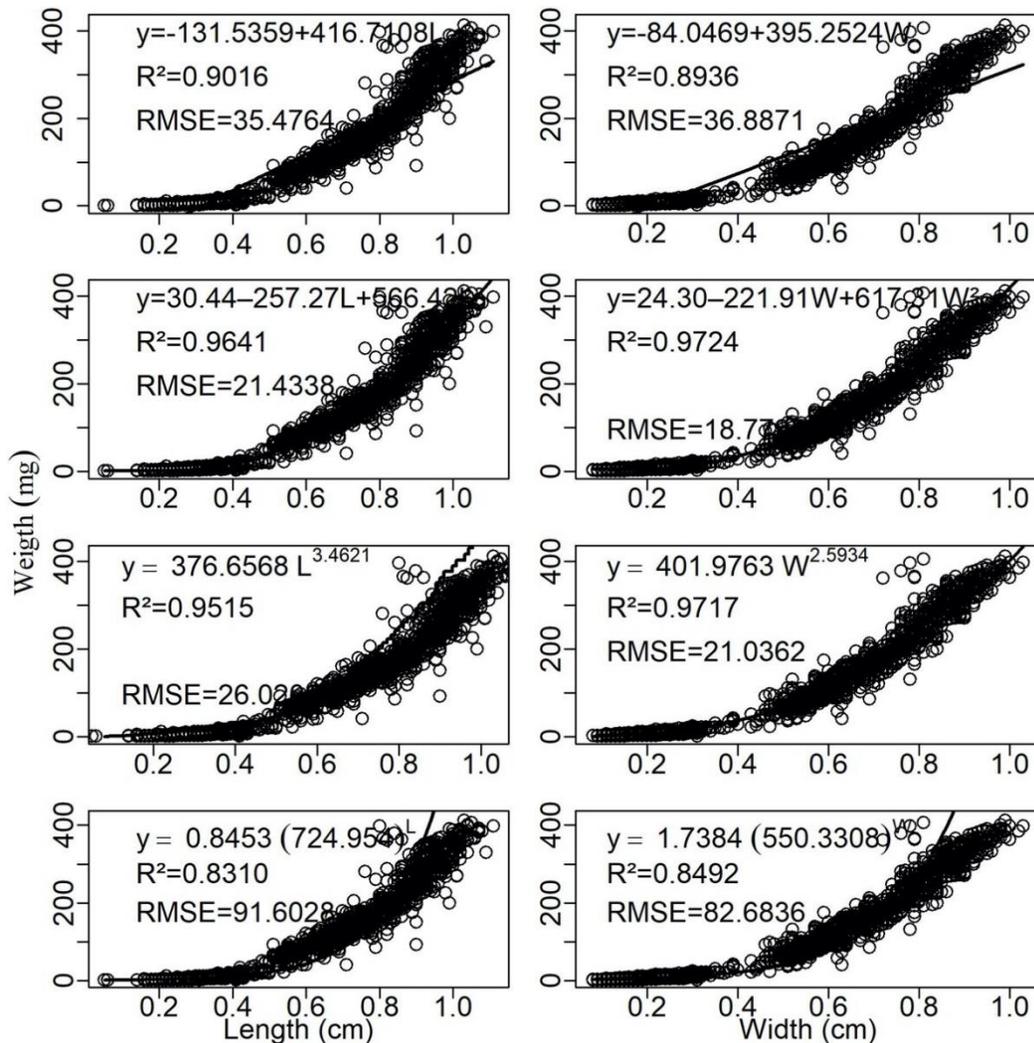


Figure 3. Growth curve seed *Cajanus cajan* (L.) Millspaugh 'Fava Larga' for four statistical models with the representation of the estimated equation, the coefficient of determination and the root mean square error. Left and right column refer to seed length and width, respectively, as explanatory variables.

However, to estimate the growth in mg of 'Fava Larga' pigeon pea pods and seeds, the quadratic model equations represented by $y = 594.4994 - 1964.9471W + 2224.2070W^2$ and $y = 24.3003 - 221.91271W + 617.3084W^2$, respectively, obtained using the width as an independent variable were more adequate because they were the ones that presented lower values of RMSE when compared to the others, a characteristic that translates into less error in the estimation of these variables, predicting more accurate values.

It is worth mentioning that the determination of the ideal harvest point for the pod and seeds of pigeon pea is a fundamental practice in the production system of this culture since this species can be used for various purposes such as human food and diet. animal. For human consumption, the green pods and seeds that are not yet hardened can be prepared like peas and/or processed in the form of preserves, the mature beans are softened and cooked like common beans (*Phaseolus vulgaris* L.), in addition to being milled and used as flour, as for animal feed, as they present a large number of proteins, the grains are ground and incorporated into rations, providing great nutritional value (19).

Thus, the use of modeling can be an important tool in decision making about crop treatments, especially in practices related to harvest (20, 21), therefore, it is possible to predict the mass of the pods in a non-destructive way, as the evaluations can be made with simple equipment such as a ruler without the need to remove the pods from the plant.

CONCLUSION

The growth, in mass, of pods of 'Fava Larga' pigeonpea pods is best estimated by a quadratic model equation using width as the explanatory variable, the same thing occurring with seed mass.

The equation that best estimates the growth of pods is $y = 594.4994 - 1964.9471W + 2224.2070W^2$, being the width (W) in cm and the mass in mg.

The equation that best estimates seed growth is $y = 24.3003 - 221.91271W + 617.3084W^2$, where the width (W) is in cm and the mass is in mg.

CONFLICT OF INTEREST

There is no conflict of interest.

ACKNOWLEDGEMENTS

This work was supported with the award of scholarships by the Coordination for the Improvement of Higher Education Personnel (Capes), the National Council for Scientific and Technological Development (CNPq) and Foundation for Support to Research and Innovation of Espírito Santo (FAPES). The authors thank Capixaba Institute for Research, Technical Assistance and Rural Extension (INCAPER) for technical support.

Author statement

Rafael Ruy Gouvea: conducted the experimente, and designed the study, **Vinicius de Souza Oliveira:** designed the study, performed the statistical analysis, and managed the writing of the manuscript, **Lucas Corrêa Souza:** conducted the experiment, **Alba Nise Merícia Rocha Santos:** conducted the experiment, **Enilton Nascimento de Santana:** conducted the experiment, **Karin Tesch Kuhlcamp:** conducted the experiment, **Omar Schmidt:** conducted the experiment, **Edilson Romais Schmidt:** designed the study, performed the statistical analysis, and managed the writing of the manuscript, **Sara Dousseau Arantes:** designed the study and conducted the experimente.



REFERENCES

1. SEIFFERT, Nelson F., THIAGO, Luiza, R. L. S. **Legumineira. Cultura forrageira para produção de proteína.** Campo Grande, MS: Embrapa Gado de Corte, novembro, 1983.
2. GODOY, Rodolfo, BATISTA, Luiz A. R., SOUZA, Francisco H. D., PRIMAVESI, Ana C. Caracterização de Linhagens Puras Seleccionadas de Guandu (*Cajanus cajan* (L.) Millsp). **Revista Brasileira de Zootecnia**, v.32, n.3, p. 546-555, 2003. DOI: 10.1590/S1516-35982003000300006. <https://www.scielo.br/j/rbz/a/TpD6VYmtmdzNrMPmwrMVtGy/?lang=pt>. Accessed 29 July 2022.
3. FULLER, Dorian Q., MURPHY, Charlene, KINGWELL-BANHAM, Eleanor, CASTILLO, Cristina Cobo, NAIK, Satish. *Cajanus cajan* (L.) Millsp. origins and domestication: the South and Southeast Asian archaeobotanical evidence. **Genet Resour Crop Evol**, v. 66, p. 1175-1188, 2019. DOI: 10.1007/s10722-019-00774-w. <https://link.springer.com/article/10.1007/s10722-019-00774-w>. Accessed 29 July 2022.
4. ARUNA, Talari, DEVINDRA, Shakappa. Role of pigeon pea (*Cajanus cajan* L.) in human nutrition and health: A review. **Asian Journal of Dairy and Food Research**, v. 37, n. 3, p. 212 - 220, 2018. DOI: 10.18805/ajdfr.DR-1379. <https://arccarticles.s3.amazonaws.com/arcc/Attachment-at-accept-article-DR-1379.pdf>. Accessed 29 July 2022.
5. TUNGMUNNITHUM, Duangjai, HANO, Christophe. Cosmetic Potential of *Cajanus cajan* (L.) Millsp: Botanical Data, Traditional Uses, Phytochemistry and Biological Activities. **Cosmetics**, v. 7, n. 4, p. 1-12, 2020. DOI: 10.3390/cosmetics7040084. <https://www.mdpi.com/2079-9284/7/4/84>. Accessed 29 July 2022.
6. BENEVIDES, Clicia Maria de Jesus, SANTOS, Adila de Jesus Silva, LIMA, Luciene Silva Dos Santos, TRINDADE, Bruna Almeida, LOPES, Mariângela Vieira, MONTES, Simone de Souza, SOUZA, Antonio Carlos dos Santos. Aspectos tecnológicos do subproduto de panc (farinhas de *Cajanus cajan* e *Phaseolus lunatus*): fortalecimento da agricultura familiar. **Brazilian Journal of Development**, Curitiba, v. 5, n. 11, p. 23221-23233, 2019. DOI: 10.34117/bjdv5n11-043. <https://ojs.brazilianjournals.com.br/ojs/index.php/BRJD/article/view/4342>. Accessed 29 July 2022.
7. EMBRAPA PECUÁRIA SUDESTE. **Guandu BRS Mandarin.** 2016. <https://www.embrapa.br/busca-de-publicacoes/-/publicacao/1047592/guandu-brs-mandarim>.
8. MACHADO, R.C.R.; ALMEIDA, H.A. de. Estimativa do volume do fruto de cacau. **Revista Brasileira de Fisiologia Vegetal**. v. 1, p. 115-117, 1989.
9. GATTWARD, J. N., SOUZA, I. V., CAMPOS, V. P., SOUZA, M. G. A., BARRETTO, W. S., SACRAMENTO, C. K., FARIA, J. C. 2006. Estimativa do peso da graviola tipo Morada em função do perímetro longitudinal. **In: Congresso Brasileiro De Fruticultura, Cabo Frio.** 2006.
10. SILVA, Natan F., FERREIRA, Francisco, A., FONTES, Paulo C. R., CARDOSO, Antônio A. Modelos para estimar a área foliar de abóbora por meio de medidas lineares. **Revista Ceres**. v.45, p.287-291, 1998. <http://www.ceres.ufv.br/ojs/index.php/ceres/article/view/2488>. Accessed 29 July 2022.



11. OLIVEIRA, Vinicius S., GOUVEA, Rafael R., SANTOS, Alba N. M. R., SOUZA, Lucas C., SANTANA, Enilton N., KUHLKAMP, Karin T., SCHMILDT, Omar., ARANTES, Sara D., SCHMILDT, Edilson R. Uso de dimensões lineares para estimar a área das folhas de *Cajanus cajan* (L.) Millspaugh através de método não destrutivo. **International Journal of Development Research**, v. 9, n. 10, p. 30747-30750, 2019. <https://www.journalijdr.com/uso-de-dimensões-lineares-para-estimar-área-das-folhas-de-cajanus-cajan-l-millspaugh-atraves-de>. Accessed 29 July 2022.
12. WULLSCHLEGER, Stan D., LYNCH, Jonathan P., BERNTSON, Glenn M. Modeling the below ground response of plants and soil biota to edaphic and climatic change – What can we expect to gain?. **Plant and Soil**, v.165, p.149-160, 1994. <https://www.jstor.org/stable/42939656>. Accessed 29 July 2022.
13. KVALSETH, Tarald, O. Cautionary note about R². **The American Statistician**, v.39, n.4, p.279-285, 1985. <https://www.jstor.org/stable/2683704>. Accessed 29 July 2022.
14. R CORE TEAM. **R: a language and environment for statistical computing**. Vienna: R Foundation for Statistical Computing, 2022.
15. FERREIRA, Eric B., CAVALCANTI, Portya P., NOGUEIRA, Denismar. A. **Package ‘ExpDes.pt’**. 2018.
16. PIMENTEL-GOMES, Frederico. **Curso de estatística experimental**. 15. Ed., Piracicaba: Fealq, 2009. 451p.
17. CASTELAN-ESTRADA, Mepivoseh, VIVIN, Philippe, GAUDILLERE, Jean-Pierre. Allometric relationships to estimate seasonal above ground vegetative and reproductive biomass of *Vitis vinifera* L. **Annals of Botany**, v. 89, n. 4, p. 401–408, 2002. DOI: 10.1093/aob/mcf059. <https://academic.oup.com/aob/article/89/4/401/200308?login=false>. Accessed 29 July 2022.
18. PASSOS, Asélio. V. **Estudo de épocas de colheita e desenvolvimento de vagens de feijão guandu (*Cajanus cajan* (L.) Millsp.), para obtenção de grãos e sementes não comerciais em pequenas unidades de produção família**. 2012. 32 f. Dissertação (mestrado) – Universidade Federal Rural do Rio de Janeiro, Curso de Pós-Graduação em Agricultura Orgânica, 2012.
19. PEREIRA, João. **O feijão guandu: uma opção para a agropecuária brasileira**. Planaltina, EMBRAPA – CPAC. Circular técnica. 1985.
20. CUNHA, Antônio R.; VOLPE, Clóvis. A. Curvas de crescimento do fruto de cafeeiro cv. Obatã IAC 1669-20 em diferentes alinhamentos de plantio. **Semina: Ciências Agrárias**, v. 32, n. 1, p. 49-62, 2011. <https://www.redalyc.org/pdf/4457/445744100005.pdf>. Accessed 29 July 2022.
21. OLIVEIRA, Vinicius S. **Modelagem estatística de características biométricas para cultivares de macieira pouco exigentes ao frio cultivadas no norte do Estado do Espírito Santo**. Dissertação (Mestrado em Agricultura Tropical) - Universidade Federal do Espírito Santo, Centro Universitário Norte do Espírito Santo. 2020, 52 f.

