



Use of Mathematical Equations to Estimate Leaf Area of *Psidium cattleianum*

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Abstract: The objective of this study was to adjust a mathematical equation to estimate the leaf area of *Psidium cattleianum* from the linear dimensions of the leaves. For that, 260 leaves were digitized, and the length (L), width (W), product of the multiplication of length, and width (LW), and observed leaf area (OLA) were obtained. First degree linear, quadratic, and power model equations were fitted, where the OLA was used with the response variable as a function of L, W or LW as an explanatory variable. All models were validated with a sample of 50 separate leaves for this purpose. Thus, the first-degree linear model equation $AFE = 0.473815 + 0.688891(LW)$ obtained from the product of multiplying the leaf length, and width can be used accurately, and without the need to destroy the leaves to estimate the leaf area of *Psidium cattleianum*.

Keywords: Modeling; linear measurements; non-destructive method; statistical analysis; validated; mathematical models.

Adherence to the BJEDIS' scope: This work is based on the statistical analysis of a sample of *Psidium cattleianum* leaves, which through mathematical modeling allows the adjustment of equations to determine the leaf area of the population of this specie in a non-destructive way.

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1. INTRODUCTION

The specie *Psidium cattleianum*, belonging to the Myrtaceae family, presents easy adaptation, , and its cultivation can be disseminated to different regions. This specie produces fruits that can be consumed, with an exotic flavor, and high amount of vitamin C, which guarantees good acceptance by the consumer market (1). Its flowers are white, hermaphroditic, with numerous stamens, and an inferior ovary with three or four locules containing more than 100 ovules (2). In addition to consumption in the fresh, and industrial form of the fruits, this specie contains pharmacological characteristics, having vitamins, antioxidants, and essential oils (1).

The study of the leaf area is essential to underst, anding the development of plants because through the leaves we can carry out studies that will give us answers about their growth, performance, irrigation efficiency, or even the fertilizers used, since the leaf area itself is associated with evapotranspiration, CO₂ fluxes, amount of water, and light interception of the plant (3).

A way to determine the leaf area in a simplified way is through the use of mathematical equations, which, from known, and easy to dimension measures of the leaf surface, accurately estimate the leaf area. Several authors have proposed mathematical equations to estimate the leaf area of several species (4, 5, 6, 7). However, it was not identified in the literature fitted equations to estimate the area of the leaves of *Psidium cattleianum*. How each specie has particular anatomy of the leaves, distinguishing one from the others, it is necessary to use specific equations that estimate the leaf area of each of them.

Thus, the objective of this study was to adjust, and validate mathematical equations that accurately estimate the leaf area of *Psidium cattleianum* from the linear dimensions of the length, and width of leaves.

2. MATERIALS AND METHOD

To carry out the present study, 310 leaves of *Psidium cattleianum* were collected in the municipality of São Mateus, North of Espírito Santo State, Brazil, with geographic coordinates of 18° 40' 36" South latitude, and 39° 51' 35" East longitude. The region is characterized according to the Köppen classification by the tropical climate Aw with dry winter, and predominance of rain in summer (8). The leaves were packed in paper bags, and transported to the laboratory.

All leaves were scanned with HP Deskjet F4280® Scanner in Tag Image File Format (TIFF) with 75 dpi resolution. Subsequently, with the help of ImageJ® software (9), the images were processed, and the leaves had their length (L, in cm) along the midrib, maximum width (W, in cm) (Figure 1), and observed leaf area (OLA, in cm²) determined. The product of multiplying the length by the width (LW, in cm²) was also obtained.

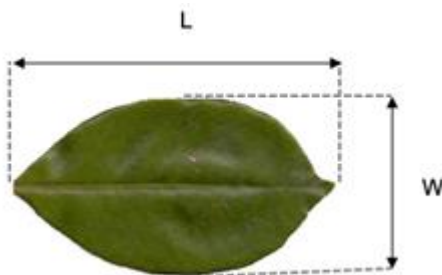


Figure 1. Representation of the length (L) along the midrib, and the maximum width (W) of leaves of *Psidium cattleianum*.

The calibration was carried out with a sample of 260 sheets where the OLA was used with the response variable (y) as a function of L, W or LW as an explanatory variable (x) , and the equations of the first degree linear, quadratic , and power model were adjusted as shown in Table 1, totaling nine equations , and their respective coefficient of determination (R²). The parameters $\hat{\beta}_0$, and $\hat{\beta}_1$ were estimated by the method of least squares.

Table 1. Denomination, and representation of equation models adjusted for *Psidium cattleianum* leaf area estimation

Denomination	Representation
Linear	$ELA = \hat{\beta}_0 + \hat{\beta}_1 x$
quadratic	$ELA = \hat{\beta}_0 + \hat{\beta}_1 x + \hat{\beta}_2 x^2$
power	$ELA = \hat{\beta}_0 x^{\hat{\beta}_1}$

In the validation, the values of L, W, and LW of 50 leaves separated for this purpose were substituted in the adjusted equations in the calibration, obtaining the estimated leaf area (ELA) in cm² for each model. The mean values of OLA, and ELA of the 50 validation leaves were compared by Student's t test at 5% probability. The mean absolute error (MAE), the root mean square error (RMSE), and the Willmott d index were also calculated (10), through the following equations.

$$MAE = \frac{\sum_{i=1}^n |ELA - OLA|}{n}$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (ELA - OLA)^2}{n}}$$

$$d = 1 - \left[\frac{\sum_{i=1}^n (ELA - OLA)^2}{\sum_{i=1}^n (|ELA - \overline{OLA}| + |OLA - \overline{OLA}|)^2} \right]$$

Where, ELA are the estimated values of leaf area; OLA are the observed values of leaf area; \overline{OLA} is the mean of the observed values; n is the leaves number used in the validation (n = 50).

The selection of the best models to estimate the area of *Psidium cattleianum* was by: non-significant values comparing the OLA, and ELA means, MAE, and RMSE closer to zero, and Willmott's d index (10) closest to one. For the statistical analysis, the R software was used (11), with the ExpDes.pt data package version 1.2 (12).

3. RESULTS AND DISCUSSION

Table 2 shows the descriptive statistics of the sampled data for modeling, and validation. The high coefficient of variation (CV) found can be explained by the high amplitude of the sample data with the presence of leaves used in the modeling with an area ranging from 11.94 to 70.23 cm², this characteristic becomes fundamental in mathematical modeling studies, since in this interval there are small, medium, and large leaves, covering most of the possible number of phenological stages (13, 4). An important fact to be mentioned is that the values used in the validation should not be smaller or larger than those used for the modeling, this should also be respected in future estimations, as extrapolated values can generate inaccurate information on the leaf area (14).

Table 2. Value minimum, maximum, average, amplitude, standard deviation (SD), and coefficient of variation (CV) of the variables: length (L); width (W); the product of length, and width (LW), and observed leaf area (OLA) of leaves of *Psidium cattleianum* used for modeling, and validation

Variable	unit	Minimum	Maximum	Average	Amplitude	SD	CV
260 leaves used for modeling							
L	cm	4.65	13.01	9.25	8.36	1.43	15.46
W	cm	3.34	7.90	5.35	4.56	0.76	14.17
LW	cm ²	16.77	102.15	50.42	85.38	8.14	27.92
OLA	cm ²	11.94	70.23	35.21	58.29	9.76	27.73
50 leaves used for validation							
L	cm	5.39	13.71	9.11	8.32	1.58	17.39
W	cm	4.06	7.24	5.27	3.18	0.82	15.63
LW	cm ²	22.33	99.33	48.96	77.00	15.67	32.01
OLA	cm ²	16.38	67.47	34.18	51.09	10.55	30.85

The adjusted equations to estimate the area of *Psidium cattleianum* are represented in Table 3. It was verified that the highest values of the coefficient of determination (R^2) are found in the adjusted models through the product of the multiplication of the length by the width (LW), being this is indicative according to Bianco et al. (15), of greater precision of the models to estimate the leaf area, since the combination of these measures implies in larger values of R^2 . However, the use of R^2 as the only way of selection is not recommended practice since it can lead to incorrect estimation of the leaf area (16). Therefore, the adoption of appropriate validation criteria allows to accurately indicate the best equation to estimate the leaf area.

Table 3. Equation linear first degree, quadratic, and power, its respective coefficient of determination (R^2) using the observed leaf area (OLA) as a dependent variable, as a function of the length (L), width (W), and product of the length to width (LW), of leaves of *Psidium cattleianum*

Model	Equation	R^2
Linear	$ELA = -23.715 + 6.370(L)$	0.8705
Linear	$ELA = -30.3237 + 12.2378(W)$	0.9051
Linear	$ELA = 0.473815 + 0.688891(LW)$	0.9867
Quadratic	$ELA = 7.75487 - 0.67418(L) + 0.38456(L)^2$	0.8861
Quadratic	$ELA = -5.7200 + 2.9404(W) + 0.8610(W)^2$	0.9106
Quadratic	$ELA = 1.0374335 + 0.6662990(LW) + 0.0002100(LW)^2$	0.9867
power	$ELA = 0.7217(L)^{1.7403}$	0.8862
power	$ELA = 1.4919(W)^{1.8743}$	0.9102
power	$ELA = 0.7315(LW)^{0.9882}$	0.9867

The criteria adopted for the validation, and selection of the adjusted equations to estimate the leaf area of *Psidium cattleianum* are shown in Table 4. When analyzing the values of the observed leaf area (OLA) in comparison with the values of the estimated leaf area (ELA), in all models, note that there is no significance between OLA, and ELA,

this implies that all equations obtained in this study can estimate leaf area values similar to the real values of leaf area.

Table 4. Observed leaf area (OLA), and estimated leaf area (ELA) from equations of linear first degree, quadratic, and power for the independent variables length (L), width (W), and product of length, and width (LW), in addition to the p-value, mean absolute error (MAE), the root mean square error (RMSE), and Willmott d index of leaves of *Psidium cattleianum* used for validation

Model	Variable	OLA	ELA	Value p*	MAE	RMSE	d
Linear	L		34.3251	0.9450	3.5769	4.3790	0.9533
Linear	W		34.1621	0.9915	2.6650	3.6281	0.9681
Linear	LW		34.2032	0.9927	0.8936	1.1157	0.9974
Quadratic	L		34.4855	0.8855	3.2367	4.1342	0.9605
Quadratic	W	34.1837	34.2536	0.9731	2.6247	3.4871	0.9713
Quadratic	LW		34.2147	0.9884	0.8975	1.1277	0.9972
power	L		34.3972	0.9187	3.2685	4.1812	0.9594
power	W		34.2721	0.9660	2.6284	3.4745	0.9715
power	LW		34.1896	0.9978	0.8982	1.1174	0.9972

Note. p* values higher than 0.05 indicate that the observed leaf area (OLA), and the estimated leaf area (ELA) do not differ by Student t-test.

Although all equations estimating the leaf area of *Psidium cattleianum* have aptitude, the first degree linear model equations fitted with the product of multiplying the length, and width can more accurately estimate the area of the leaves of *Psidium cattleianum* since this model decreased the incidence of errors with mean absolute error (MAE), and root mean square error (RMSE) values closer to zero, in addition to the Willmott d agreement index closer to one.

It is worth mentioning that the combined use of more than one measure in the estimation of leaf area can be more laborious in practice because it is necessary to measure one more variable (17), however, as observed in the literature, several plant species have their leaf area best estimated from the union of two variables such as, for example, length, and width (18, 19, 20, 4, 21) demonstrating that these equations have greater acceptability in the prediction of leaf area.

It is noted that there was a good relationship between OLA, and LW by the linear equation, and that the residual distribution is homogeneous with 99.7% of the values situated between ± 3 standard deviations (22) (Figure 2) proving that this linear model (Figure 3) is suitable for estimating from the *Psidium cattleianum* area. It should be noted that for the use of this equation it is not necessary to use specific tools, and its leaf area can be determined with simple equipment such as a ruler (18). Another important fact to be overlooked is that, despite the destruction of leaves to adjust the equation, after this practice, it is unnecessary to adopt the destruction of new leaves, and this equation can be used on the same leaf throughout the plant's development cycle, helping in studies where the number of experimental plots is limited.

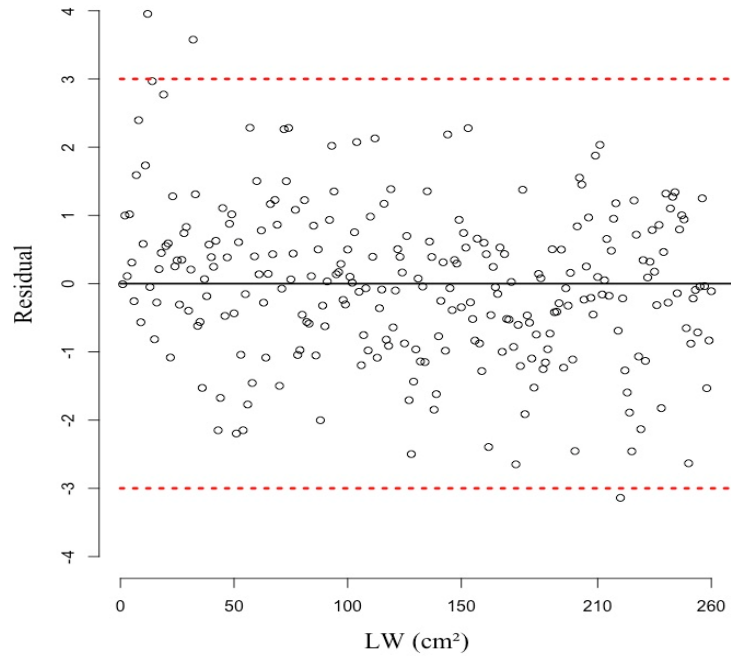


Figure 2. Residual distribution from the relationship between observed leaf area (OLA), and estimated leaf area (ELA) in *Psidium cattleyanum* leaf from the equation linear first degree.

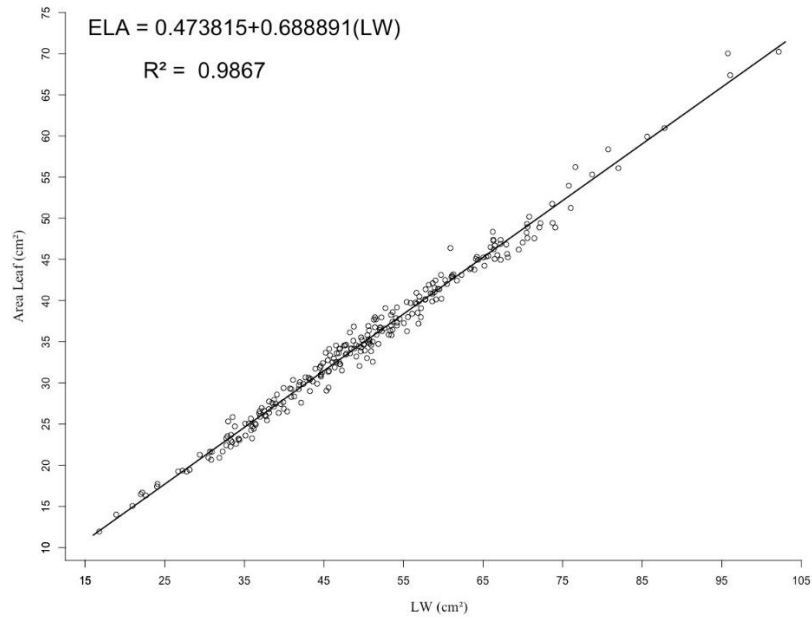


Figure 3. Equation linear first degree, and coefficient of determination (R^2) of *Psidium cattleyanum* leaf using the observed leaf area (OLA) as a dependent variable in function of the product of length, and width (LW).

CONCLUSION

The linear first degree model equation can $ELA = 0.473815 + 0.688891(LW)$ be used simply, and accurately, and without the need for destruction to estimate the leaf area of *Psidium cattleianum* from the product of multiplying the length, and width of the leaf blade.

CONFLICT OF INTEREST

There is no conflict of interest.

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Author statement

Caio Castro Abreu: conducted the experiment, **Lucas Almeida de Oliveira:** conducted the experiment, **Rafaela Fagioli Moreira:** conducted the experiment, **Rhayan Gabriel de Sousa Silva:** conducted the experiment, **Vinicius de Souza Oliveira:** designed the study, performed the statistical analysis, and managed the writing of the manuscript, **Ana Valéria Lima Santos:** conducted the experiment, **Ana Paula Braido Pinheiro:** conducted the experiment, **Omar Schmidt:** conducted the experiment, **Adriano Alves Fernandes:** conducted the experiment, **Edilson Romais Schmidt:** designed the study, performed the statistical analysis, and managed the writing of the manuscript.

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