Brazilian Journal of Experimental Design, Data Analysis and Inferential Statistics

Federal University of Rio de Janeiro
BJEDIS, Rio de Janeiro, Special Edition, v. 02, 2022
DOI: https://doi.org/10.55747/bjedis.v2i2.52685
E-ISSN: 2763-6925

Special Edition **Received 09/09/2022. Revised 10/27/2022. Accepted 11/032022. Published 11/23/2022.**

Mathematical Modeling to Estimate the Leaf Area of *Dalbergia frutescens* from Linear Dimensions

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Abstract: The objective of this work was to obtain mathematical equations and indicate the most appropriate estimation of the leaf area of *Dalbergia frutescens*, by a non-destructive method through the linear dimensions of the leaves. A total of 370 leaves of *D. frutescens* were collected, 270 of which were used to adjust the equations and 100 for validation. The length (L) along the main vein, the maximum width (W), the product of multiplying the length and width (LW), and the observed leaf area (OLA) was measured for all leaves. First degree, quadratic, and power linear equation models were fitted, where OLA was the dependent variable as a function of L, W and LW. From the 100 leaves destined for validation, and using the adjusted equations for each mathematical model, the estimated leaf area (ELA) was obtained. The means obtained from ELA and OLA were compared using Student's t test at 5% probability. The mean absolute error, the root mean square error, and the d Willmott index were also determined. The first-degree linear model represented $\mathrm{ELA} = 0.942619 + 0.706106(\mathrm{LW})$ based on the product of multiplying the length by the width is the most suitable to determine the leaf area of *D. frutescens* with high precision.

Keywords: Non-destructive method; adjustment of equations; validation; statistical analysis; linear measurements; modeling.

Adherence to the BJEDIS' scope: This work is based on the statistical analysis of a sample of *Dalbergia frutescens* leaves, which through mathematical modeling allows the adjustment of equations to determine the leaf area of the population of this species in a non-destructive way.

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1. INTRODUCTION

The species *Dalbergia frutescens* belongs to the Fabaceae family and is popularly known as Rabo-de-Bugio, Cipó-pau, Cipó-de-estribo, Jacarandá Branco or Canela-do-Brejo (1). Many species of *Dalbergia* have great economic importance due to the good quality in the production of wood for the construction of luxury furniture and civil construction (2), in addition, research relating to the anti-inflammatory, antirheumatic, antioxidant, and antibacterial activities of these species has been reported in recent years (3).

Knowledge of the leaf area is essential since the leaves assume very important functions in the plant, such as intercepting and absorbing light, carrying out photosynthesis, gas exchange, and transpiration (4). Thus, the determination of leaf area can be an important parameter in studies related to plant morphology, anatomy, and ecophysiology, as it allows obtaining a fundamental indicator for the understanding of plant responses to specific environmental factors (5). According to Monteiro et al. (6), leaf area is an indicator of great importance, being used to investigate ecological adaptation, competition with other species, and management effects, in addition to being used to determine the leaf area index, which can estimate the productivity of a plant ecosystem.

Leaf area estimation can be obtained using destructive and non-destructive methods. Although destructive methods are accurate, they have the inconvenience of demanding a lot of time and needing specific equipment, in addition to causing the destruction of the leaf, which makes it impossible to follow the crop cycle (5). Non-destructive methods are characterized by relationships between factors, such as leaf area and linear measurements performed on leaves, these relationships are generally expressed by mathematical equations that are used to estimate leaf area.

Thus, given the notorious importance of *D. frutescens* and the lack of studies that seek to adjust mathematical equations for this species, the objective was to determine the best equation that estimates the leaf area through the linear dimensions of the leaves.

2. MATERIALS AND METHOD

The study was carried out with 370 leaves of *Dalbergia frutescens* obtained in the municipality of São Mateus, North of Espírito Santo State, Brazil, located at 18º 40' 36'' South latitude and 39º 51' 35'' East longitude. All leaves had their petioles removed and were scanned using a Photosmart C4280® scanner with images saved in TIFF format (Tag Image File Format) and 75 dpi resolution. The images were submitted to the ImageJ® software (7), from which the length (L) along the midrib, the maximum width (W) of the leaf blade of each leaf were determined, both measurements were obtained in cm (Figure 1), the observed leaf area (OLA) in cm² was also determined. Subsequently, the product of multiplying the length and the width (LW) in cm^2 was obtained.

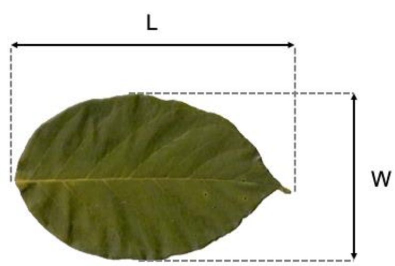

Figura **1.** Representation of the length (L) along the midrib and the maximum width (W) of leaves of *Dalbergia frutescens.*

The models linear first degree represented by $ELA = \hat{\beta}_0 + \hat{\beta}_1 x$ quadratic represented by $ELA = \hat{\beta}_0 + \hat{\beta}_1 x + \hat{\beta}_2 x^2$ and power represented by $ELA = \hat{\beta}_0 x^{\hat{\beta}_1}$ were tested. The equations were adjusted based on the measurements of 270

sheets, in which OLA was used as the dependent variable (y) in function of L, W and LW as independent variables (x). For each equation, its coefficient of determination (R^2) was also obtained.
All equations were validated. For this procedure, 100 separate leaves were used only in this practice, where, initially, the values of L, W, and LW of these leaves were substituted in the adjusted equations, thus obtaining the values of the estimated leaf area (ELA), in cm^2. The mean values of ELA and OLA were compared by Student's t test at 5% probability to know the similarity between the observed and estimated values by the equations. The mean absolute error (MAE), the root mean square error (RMSE), and the Willmott d index (8), was also calculated by the following equations:

$$\mathrm{MAE} = \frac{\sum_{i=1}^{n} |\mathrm{ELA} - \mathrm{OLA}|}{n}$$

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{i=1}^{n} (\mathrm{ELA} - \mathrm{OLA})^2}{n}}$$

$$d = 1 - \left[\frac{\sum_{i=1}^{n} (\mathrm{ELA} - \mathrm{OLA})^2}{\sum_{i=1}^{n} (|\mathrm{ELA} - \overline{\mathrm{OLA}}| + |\mathrm{OLA} - \overline{\mathrm{OLA}}|)^2}\right]$$

Where, ELA corresponds to the estimated values of the leaf area; OLA, are the observed values of leaf area; $\overline{\mathrm{OLA}}$ is the mean of the observed leaf area values; n, is the number of sheets sampled for validation (n=100).

As a criterion for selecting the best model to predict the leaf area of *D. frutescens*, the following were used: No significant difference between the compared means of OLA and ELA by Student's t test at 5% probability; MAE and RMSE as closer to zero and Willmott's d index (8) with values closer to unity. For statistical analyses, the R software (9) was used, with the aid of the ExpDes.pt data package version 1.2 (10).

3. RESULTS AND DISCUSSION

The values of descriptive statistics presented in Table 1 show that for the measurements of length (L), width (W), the product of length and width (LW), and observed leaf area (OLA) of *Dalbergia frutescens* leaves, there was a high amplitude of the sample data, is proven by the high values of the coefficient of variation (CV) for these characteristics. This fact demonstrates that the sample of 270 leaves used for modeling and 100 leaves used for validation are appropriate because it includes leaves with different sizes, which indicates different stages of development, a characteristic that is fundamental for the adjustment of mathematical equations since the use of these equations can be done throughout the crop cycle.

Table **1.** Value minimum, maximum and average, amplitude, standard deviation (SD) and coefficient of variation (CV) of the variables: length (L); width (W); product of length and width (LW) and observed leaf area (OLA) of *Dalbergia frutescens* leaves used for modeling and validation

Variable	und	Minimum	Maximum	Average	Amplitude	SD	CV
270 leaves used for modeling							
L	cm	3.05	16.63	9.16	13.58	2.39	26.05
W	cm	1.47	9.23	4.87	7.76	1.37	28.18
LW	cm^2	4.49	153.48	47.74	148.99	25.34	53.07
OLA	cm^2	3.68	108.28	34.65	104.60	17.92	51.72

100 leaves used for validation							
L	cm	3.53	14.59	10.08	11.06	2.55	25.36
W	cm	1.87	7.89	5.15	6.02	1.39	27.12
LW	cm^2	6.62	114.70	55.35	108.08	26.56	47.98
OLA	cm^2	4.85	82.42	39.98	77.57	18.99	47.51

The nine equations adjusted to estimate the leaf area of *D. frutescens* from the linear dimensions of the leaf blade are shown in Table 2. All models showed a high relationship between the dependent variable (OLA) and the independent variables (L, W or LW) with a coefficient of determination higher 0.93, showing that over 93% of the OLA can be explained by linear dimensions. In all models, LW showed greater ability to predict leaf area with R^2 values higher than those found when the independent variable was an individual measure (L or W). The fact that the use of a single linear measure has a lower relationship with the OLA can be explained because L and W are discrepant measures and when used singly they are less accurate than when used in combination (11).

Table **2.** Equation linear first degree, quadratic and power, its respective coefficient of determination (R^2) using the observed leaf area (OLA) as a dependent variable, as a function of the length (L), width (W) and product of the length to width (LW), of leaves of *Dalbergia frutescens*

Model	Equation	R^2
Linear	$\mathrm{ELA} = -31.8089 + 7.2536(\mathrm{L})$	0.9337
Linear	$\mathrm{ELA} = -27.7226 + 12.8121(\mathrm{W})$	0.9623
Linear	$\mathrm{ELA} = 0.942619 + 0.706106(\mathrm{LW})$	0.9966
Quadratic	$\mathrm{ELA} = 0.39393 - 0.05812(\mathrm{L}) + 0.38816(\mathrm{L})^2$	0.9667
Quadratic	$\mathrm{ELA} = -4.56451 + 3.12478(\mathrm{W}) + 0.93848\,(\mathrm{W})^2$	0.9817
Quadratic	$\mathrm{ELA} = 0.12760591 + 0.73990777\,(\mathrm{LW}) - 0.00027364(\mathrm{LW})^2$	0.9968
Power	$\mathrm{ELA} = 0.3924(\mathrm{L})^{1.9935}$	0.9667
Power	$\mathrm{ELA} = 1.8547(\mathrm{W})^{1.8145}$	0.9814
Power	$\mathrm{ELA} = 0.8136(\mathrm{LW})^{0.9715}$	0.9968

When analyzing the validation criteria (Table 3), it is possible to observe that all the adjusted equations presented an estimate of the leaf area statistically equal to the leaf area observed in the leaves of *D. frutescens* with p value > 0.05, however, despite this finding, note that the highest value was found in the equation of the model linear first degree fitted with LW (0.9896) showing that this equation presents more similarity between the leaf area estimated by the model and the real leaf area. In addition, this same equation presented a lower error in the prediction of leaf area verified by the values of the mean absolute error (MAE) and root mean square error (RMSE) closer to zero, in addition to the value of the d index as the closest to the unit.

Table **3.** Observed leaf area (OLA) and estimated leaf area (ELA) from equations of linear first degree, quadratic and power for the independent variables length (L), width (W) and product of length and width (LW), in addition to the p value, absolute error of error (MAE), root mean square error (RMSE) and Willmott's d index of *Dalbergia frutescens* leaves used for validation

Model	Variable	OLA	ELA	p value	MAE	RMSE	d
Linear	L		41.3282	0.6839	4.2537	4.9312	0.9825
Linear	W		38.3214	0.6085	2.9193	3.9552	0.9884
Linear	LW		40.0276	0.9896	0.8874	1.1353	0.9992
Quadratic	L		41.7692	0.596	3.2638	3.8162	0.9900
Quadratic	W	39.9842	38.2872	0.6006	2.5021	3.3578	0.9916
Quadratic	LW		40.0551	0.9830	0.9034	1.1389	0.9991
power	L		41.7638	0.5971	3.2697	3.8181	0.9900
power	W		38.3130	0.6055	2.5312	3.3757	0.9915
power	LW		40.0351	0.9878	0.8964	1.1425	0.9991

Note. p* values higher than 0.05 indicate that the observed leaf area (OLA) and the estimated leaf area (ELA) do not differ by Student t-test.

The results found in this study indicate that the first degree linear model equation fitted with the product of multiplying the length with the width $\mathrm{ELA} = 0.942619 + 0.706106(\mathrm{LW})$ whose behavior can be seen in Figure 2A and residual distribution can be seen in Figure 2B is the most suitable to estimate the leaf area of *D. frutescens* accurately and quickly. It should be noted that for the use of this equation, the LW range comprising from 4.49 to 153.48 cm^2 used for the modeling must be respected in future evaluations. Another relevant fact regarding the use of this equation is that it is particularly important in studies whose objective is to analyze plants without destroying them since this methodology does not require the removal of leaves to determine their area (12).

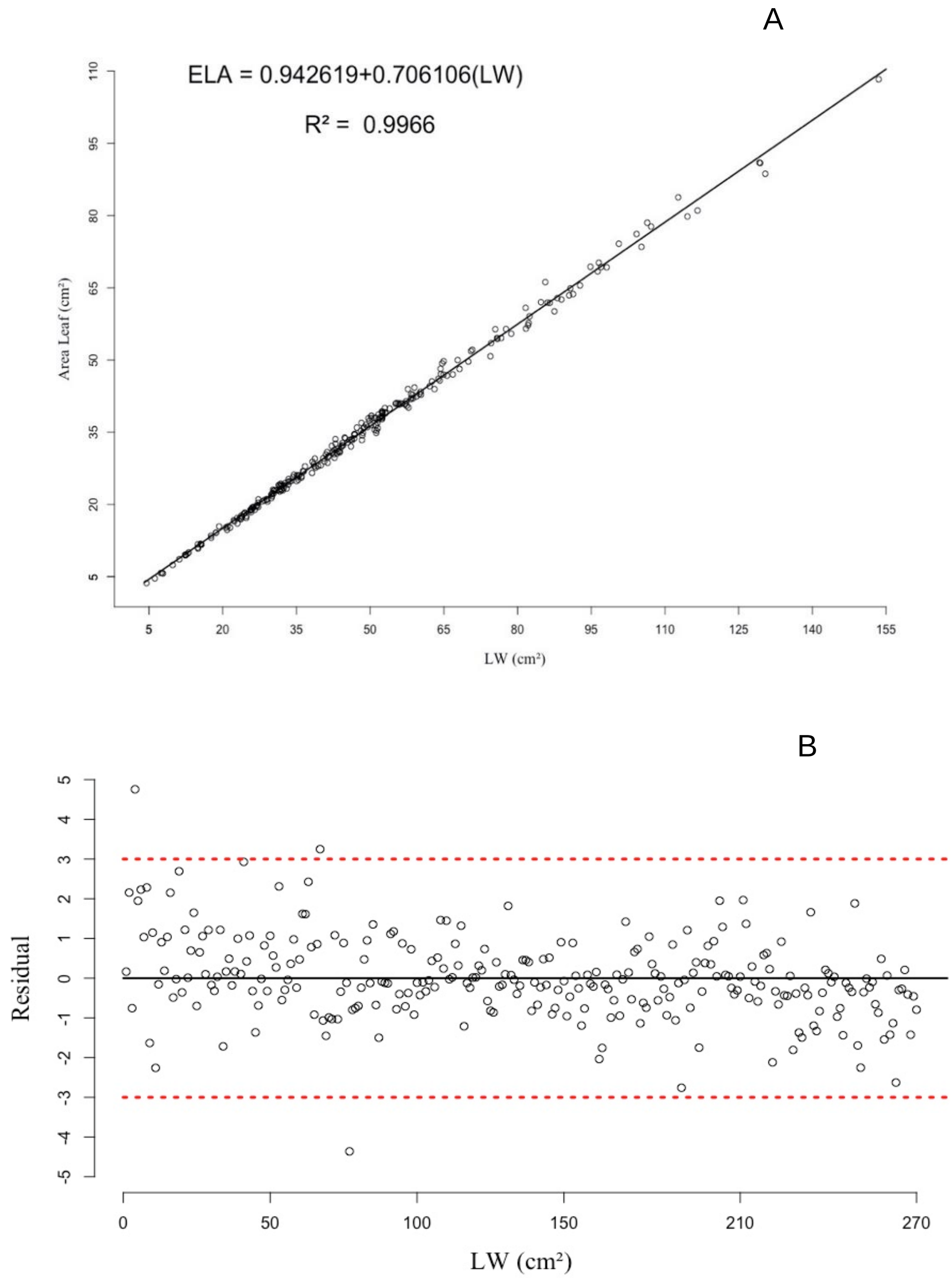

Figure **2.** A) Equation linear first-degree and coefficient of determination (R^2) and B) residual distribution of *Dalbergia frutescens* leaf using the observed leaf area (OLA) as a dependent variable in function of the product of length and width (LW).

CONCLUSION

The models that use the product of the multiplication between the linear measures of the length and width (LW) of the leaf blade satisfactorily estimate the leaf area of *Dalbergia frutescens*. The use of this input variable in the model equation linear first degree $\mathrm{ELA} = 0.942619 + 0.706106(\mathrm{LW})$ resulted in the best estimates of leaf area, being possible to determine with a high degree of precision, safely, without the need to destroy the leaves, and with low-cost equipment.

CONFLICT OF INTEREST

There is no conflict of interest.

ACKNOWLEDGEMENTS

This work was supported with the award of scholarships by the Coordination for the Improvement of Higher Education Personnel (CAPES), the National Council for Scientific and Technological Development (CNPq), Foundation for Support to Research and Innovation of Espírito Santo (FAPES) and Espírito Santo Technology Foundation (FEST).

Author statement

Bárbara Colombo Ramos: conducted the experiment, **Fabricio Lopes dos Santos:** conducted the experiment, **Lidiane Rodrigues Lima:** conducted the experiment, **Marta Gabriela Rossiter:** conducted the experiment, **Nayane dos Santos Bernardo:** conducted the experiment, **Vinicius de Souza Oliveira:** designed the study, performed the statistical analysis, and managed the writing of the manuscript, **Ana Paula Braido Pinheiro:** conducted the experiment, **Omar Schmildt:** conducted the experiment, **Adriano Alves Fernandes:** conducted the experiment, **Edilson Romais Schmildt:** designed the study, performed the statistical analysis, and managed the writing of the manuscript.

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