“Ad dissonantiam per consonantiam”: the scope and limits of Darius Milhaud’s system of “Polytonalité harmonique”: the esthetic level (Part 2)*

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Abstract
The first part of this study, published in a previous issue of this journal, discussed the Polytonalité Harmonique system, proposed by Darius Milhaud in 1923, from the perspective of the Molino & Nattiez’s tripartitional categories of the “immanent” and “poietical” levels. In continuation, this second part examines the system of “harmonic polytonality”, and some early controversies surrounding polytonality, from the perspective of the “esthesic level”. It is also suggested that the more general, and actually fundamental principle behind the system of “harmonic polytonality” – i.e., the compositional choice, among all possible “dissonant” vertical aggregate chords, of those which are decomposable into traditional “consonant” chords (accords classés) – applies well beyond the early twentieth-century répertoire, including later composers, such as Messiaen, Ligeti, Schnittke, Glass, Widmer, and Krieger.

Keywords

Resumo
Na primeira parte deste estudo, publicada no número anterior desta revista, o sistema de “politonalidade harmônica”, proposto em 1923 por Darius Milhaud, foi discutido no contexto da teoria da “tripartição” de Molino & Nattiez, segundo as categorias dos níveis “imanente” e “poiético”. Dando continuidade nesta segunda parte, o sistema “P. H.”, assim como as primeiras controvérsias em torno da politonalidade, são examinadas na perspectiva do “nível estésico”. Também é sugerido que o princípio geral, e, na realidade, fundamental, da “politonalidade harmônica” – i.e., o de decisões composicionais em favor de agregados harmônicos “dissonantes” que apresentam a característica de serem redutíveis à superposição de acordes tradicionais “consonantes” (accords classés) – dá-lhe uma abrangência além do repertório do início do século XX, permitindo entender procedimentos posteriores de escrita em compositores, tais como Messiaen, Ligeti, Schnittke, Glass, Widmer e Krieger.

Palavras-chave

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SECTION III – INTO THE “ESTHESIC” DIMENSION:
DIFFERENT PERCEPTIONS ON POLYTONALITY

The discussions around Polytonality reveal, among many of the main actors in the early twentieth-century musical scene, an extremely wide diversity of points of view and perceptions. While Milhaud conceived the possibility of different tonal polarities coexisting simultaneously, and operating on both vertical and horizontal levels – thus his denominations *Polytonalité Harmonique* and *Polytonalité Contrapuntique* – a view largely shared by composers such as Charles Koechlin and Alfredo Casella, no lesser musicians than his contemporaries and friends Paul Hindemith and Ernest Krenek would openly dismiss Polytonality, denying it any viability whatsoever. While the latter “heard” Milhaud’s music as a “heterophony [...] related to a basic key”,¹ for Hindemith:

> The ear perceives total sound [...] every simultaneous combination of sounds must have one root and only one [...] Polytonality is a catchword [...] entertaining for the composer, but the listener cannot follow the separate tonalities.

Between these extremes, a number of intermediate, not necessarily convergent opinions were then manifested. In her 1925 Rice Lectures, Nadia Boulanger expressed her scepticism with respect to *Polytonalité Harmonique* – preferring the term “polyharmonies” – endorsing Boris de Schloezer’s opinion² that Polytonality (understood as the simultaneous coexistence of different polarities) could only exist in a polyphonic context, i.e., under *Polytonalité Contrapuntique*. Surprisingly, this positive opinion with respect to *Polytonalité Contrapuntique* was not shared by the composer Alfredo Casella, a pioneer in the theorization on Polytonality, and who otherwise coincided with Milhaud on so many views, such as polytonality’s opening “new and limitless horizons of tone combinations to the composer”, as an “enhancement of diatonicism” and a “natural result of historical evolution”, its subdivision in “two species: harmonic and melodic”:

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¹ According to Krenek: “Milhaud’s technique follows two compositional principles: the tonally cadenced period of tradition, and a sort of heterophony tending towards polytonality [...] the heterophonous results of this technique are once again related to a basic key, giving a great deal of charming ambiguity in the notes of melodies and chords.”

² “Il faut distinguer entre la combinaison de tonalités différentes résultant de la projection sur une tonalité fondamentale de notes ou d’accords de passage appartenant à une tonalité différente, d’une part, et de la juxtaposition, d’autre part, de deux ou plusieurs lignes mélodiques, dont chacune possède sa propre tonalité et poursuit sa propre évolution. En se plaçant à ce point de vue, il n’y aurait polytonalité réelle que là où il y aurait polyphonie. En effet, dans le premier cas l’opposition des diverses tonalités aboutit nécessairement, soit à l’absorption par l’une des tonalités de ces rivales, et son affirmation dans une cadence, soit à la désagrégation de toutes les tonalités composantes et, par conséquent, à la tonalité. Seule l’écriture polyphonique permet le développement parallèle et entièrement autonome de deux, trois tonalités différentes” (Schloezer, 1923).
While harmonic polytonality has shown itself to be a musical agency of great expressive power [...] we cannot affirm the same of melodic polytonality. The systematic superposition of melodies has hitherto led to very few convincing results [...]. I myself have variously employed melodic polytonal superpositions. But in this, the several melodies acknowledged the leadership of one among the others.³

A similar scepticism to Casella’s with respect to the possibility of coexistence of different polarities was also expressed by Bela Bartok in his Harvard Lectures (1945), where he commented that “though much of my music and Stravinsky’s incidentally looks as if it is bitonal or polytonal, it is neither atonal nor polytonal”, and that his own technique should be described instead as “polymodal”: thus he retains, on the one hand, the notion of “simultaneism” (a feature which had also pervaded the plastic arts from Italian Futurism and Marcel Duchamp’s Femme descendant l’escalier to Analytical Cubism⁴) with the superposition of different scales/modes, but on the other, he would not challenge the principle of unipolarity.

The so called polytonal music exists only for the eye [...] but our mental hearing will select one key as fundamental key and we will project the tones of the other keys in relation to the one selected.⁵ [...] Our hearing cannot perceive two or more different keys with two or more fundamental tones [...] Much mischief was done in the worship of bitonality.⁶

Early commentators of Stravinsky, such as Ernest Ansermet, Nadia Boulanger, Boris de Schloezer or André Shaeffner, were probably reflecting the composer’s own views when reacting against the then widespread labeling as “polytonal” for some of his works of the “Russian period”, insisting instead on their “centric” character; while, thirty years later, a similar view would again be expressed by Pierre Boulez, for whom, even the particularly “daring” polyharmonies employed in works such as Le Sacre, Le Roi des Étoiles or the Concertino, did not generate “polytonality”, and corresponded in fact to procédés of “harmonic verticalisation” (“cumulative-additive” chords as per André Boucourechliev’s description):

³ Casella, 1924, p. 167.
⁴ For a comprehensive discussion of Simultanéisme as a general phenomenon in the early twentieth-century art, see Read (1980) and Noronha (1998).
⁵ A view close to Schloezer’s (1923) commenting Stravinsky’s music: “there is always a strongly affirmed fundamental tonality to which melodic lines and harmonic complexes belonging to a different key temporarily join themselves. But the foreign key is either in the end, abandoned or else it melts, in modulation, into the fundamental tonality [...]. Beneath the complexities of a harmonic tissue where 2 or 3 different keys are woven together, one always distinguishes the plane of the principal tonality, which finally absorbs the others, and disintegrates itself by a cadence which destroys all doubts”.
Stravinsky’s music is fundamentally tonal, i.e., it gravitates around a “home” key, which is usually affirmed, in no uncertain manner by a perfect cadence. Occasionally it is true there are equivocal measures [...], which at a first glance would seem to be polytonal. Yet in spite of such passages I do not think that in Stravinsky’s case one can properly speak of polytonality [...] [If] the frenzied “Dance of the Earth” contains melodic lines which suggest other keys, nevertheless from the harmonic standpoint, it is nothing but a gigantic cadence in C Major.⁷ (Boulanger, 1925)

Another approach is that of composers such as Bela Bartok, Albert Roussel or Olivier Messiaen, who keep their distances in applying the term “polytonality” to their compositions, insisting instead on the “polymodal” character of their music. In his Technique de mon Langage Musical, Olivier Messiaen describes his use of polymodality as a texture in which: “the polymodal effect happens in between those notes which differ from one mode to the other”, generally consisting of the combination and superposition between modes à transposition limitée notably Mode N° 2, i.e., the octatonic, which he distinguished from “the three great modal systems Chinese, Hindu, and Ancient Greek, all of which are transposable 12 times”.

Anticipating Arthur Berger’s seminal article, with respect to Rimsky-Korsakoff’s tone/semitone scale in Stravinsky’s music, Messiaen warns against what he considers to be the misleading analysis – as “polytonal” – of those harmonic combinations which result from his use of the modes à transposition limitée.⁸ At the same time, Messiaen acknowledges “polytonality” (chère à Milhaud), both as a technique and a “sonority”, and uses the word as a “proxy” for designating polyharmonies:⁹

⁷ In this respect (of unipolarity in Stravinsky), a similar point of view, would also be expressed by musicians as diverse as Ansermet and Boulez: “Le style de Stravinski est donc une polyphonie d’éléments simples, [...] c’est leur action polyphonique qui crée un sens harmonique plus complexe, auquel il arrive exceptionnellement de se concrétiser dans un accord. Mais ce sens harmonique plus complexe n’est pas polytonal, comme on l’a dit dans le tâtonnement des premières exégèses [...]. Cet élément de contact, qui a toute la vertue sensible et logique d’une tonique, agit comme un pôle d’ou rayonne le faisceau des harmonies.” (Ansermet, 1922); “Il est à remarquer [...] que au point de vue tonal, le langage de Stravinski consiste en des attractions puissantes créées autour de certains pôles, ces pôles étant les plus classiques qui soient à savoir la tonique, la dominante, la sous-dominante. Une tension plus ou moins grande s’obtient grace aux appogiatures non résolues, aux accords de passage à la superposition de plusieurs modalité sur une même note attractive” (Boulez, 1966, p. 15) and Boulanger – both Paris Conservatoire students – a similar écoute and analysis of the “Danse de la Terre”, as a gigantic C Major cadence.

⁸ According to Messiaen: “Les combinaisons de notes que [les modes à transposition limitée] suscitent peuvent faire équivoque avec des sonorités polytonales sont dans l’atmosphère de plusieurs tonalités à la fois, sans polytonalité, le compositeur étant libre de donner la prédominance à l’une de ces tonalités, ou de laisser l’impression tonale flottante [...] dans la polymodalité, nous assistons à l’éclosion d’aggrégats polytonaux complètement noyés dans la polymodalité.” A similar view, in favor of a “polymodal” écoute, was expressed by Boulez (in the“militant” tone of the 1950s), with respect to the Sacre: “parlerais-je de la polytonalité que l’on a tant remarquée dans le Sacre au point que l’on n’a plus vue qu’elle? [...] tout au plus signalera-t-on une polymodalité à partir des mêmes notes de polariété [...] C’est précisément à cette hiérarchie qui s’organise à partir des notes attractives que le Sacre doit sa physionomie harmonique. Nous sommes on ne peut plus loin de la gratuité polytonale” (p. 140-41).

⁹ The word has been used currently, in this looser sense, by many contemporaries of Milhaud (such as Britten, Copland, Honegger, etc). See Villa-Lobos’ notes on Choros N° 10, analyzing its section D as “a harmonic sequence of eight bitonal chords” (Museu Villa-Lobos, 1965, p. 158).
We cannot leave Tonality without speaking about Polytonality, which has been so wonderfully employed by Darius s Milhaud. Debussy had foreshadowed it in his famous “Golaud chord” (A Major X Bb Major). The Saudades do Brazil remain as remarkable examples of his science of color and of their superpositions.10

Such widely diverse views held by some of Milhaud’s most eminent contemporaries, illustrate the relevance of the categories proposed in Jean Molino and Jean Jacques Nattiez’s “tripatition” method, establishing a distinction between what they denominate the “poietic”, the “immanent”, and the “esthesic” levels.

François de Médicis has suggested a taxonomy, in order to disentangle definitional difficulties, establishing a distinction between what he denominates polytonalité au sens strict and polytonalité au sens large. The first case designates situations in which different polarities are simultaneously in presence, at least at the (intentional) “poietic” level: not only in those Milhaud’s works illustrating Polyonalité Contrapuntique – the four simultaneous different keys in the first movement of the String Quartet Nº 5, the superposition of 3 fugues in different tonalities in the Étude for piano and orchestra Nº 3, or in the canon alla setima over the 12 transpositions levels in l’Homme et son désir – but also in some experiments of multipolarity in the context of “Polyonalité Harmonique”,11 such as the superposition of harmonic functions and cadential formulae in the Saudades do Brazil.12 Médicis makes the observation that polyonalité stricte, even in Milhaud’s works, is rather the exception than the rule: not only is it present in few works (mostly concentrated in a short period: from 1917 to 1923), as its use is generally intermittent, an “important characteristic of polyonalité au sens strict being that it does not constitute a real idiom, differently from atonality and tonality.”13
Thus, *Polytonalité au sens large* should encompass not only the more frequent case of **“Polytonalité Harmonique”** (polyharmonies resulting from **“T.C.”** which, even when highly dissonant, remain decomposable into consonants units) and “polymodality”, but also include one particular case not adequately analyzable under the “P.H.” system: the “complement relation” to the chromatic total (combining sets 7-35 and 5-35), of which are examples:

Firstly, those textures often associated with the black&white “topography” of the piano keyboard – which from Debussy’s *Études* to Ligeti’s were originally displayed in Ravel’s *Jeux d’Eau* (1904) and *Petrushka*, and have found a particularly systematic and rich use in Villa-Lobos works (notably in *Prole do Bebê*’s “Polichinelo” and “Baratinha de papel”, in *Cirandas*’ “O Cravo brigou com a rosa”, and *Rudepoema*)

Secondly, Bartok’s “chromatic polymodality” in which, without abandoning the notion of a harmonic “center of gravity”, he obtains the chromatic total through the superposition of the Phrygian and Lydian modes:

As the result of superposing a Lydian and Phrygian pentachord with a common fundamental tone we get a diatonic pentachord filled out with all the possible flat and sharp degrees [...] In our polymodal chromaticism [...] the flat and sharp tones are not altered degrees at all; they are diatonic ingredients of a diatonic modal scale. [...] The essential difference between atonality, polytonality and polymodality [is] that atonal music offers no fundamental tone at all, polytonality offers or is supposed to offer several of them, and polymodality offers a single one.\(^{16}\)

Example 5 – Bartok’s chromatic complementation between Lydian and Phrygian modes

![Example 5 - Bartok's chromatic complementation between Lydian and Phrygian modes](image)

a) in the case of **polytonalité stricte**: for Milhaud, both poiesis and esthesis coincide, i.e., he “hears” what he had intended in the act of composition; while composers such as Bartok, Hindemith, Krenek, and Casella (each in a different

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\(^{15}\) The “black&white” technique had a precedent in the Brazilian repertoire with the 10th variation in Alberto Nepomuceno’s *Variações sobre um tema original* (1902-12).

way) would acknowledge “polytonality” as a poietic device (the exercise of writing in simultaneous different keys), but would, on the esthesic level, deny the aural viability of hearing simultaneous tonal centers. With respect to Nadia Boulanger and Boris de Schloezer who, writing in the 1920’s on the wake of Milhaud’s most daring experiments, recognized, at least at the poietic level, the theoretical possibility of a *Polytonalité Contrapuntique* – polyphony being in their opinion a *sine qua non* condition for the existence of polytonality – their view of *Polytonalité Harmonique* was one of enrichment of the harmonic vocabulary through the use of polyharmonies, but did not recognize the capability of generating tonal multipolarity (in the sense given by François de Médicis to *polytonalité au sens strict*). In this, they coincide with analysts (belonging to different generations) of Stravinsky’s *fase russe*, such as Ansermet, André Schaeffner, Boulez or André Boucourechliev, who identify the presence of a predominant “center of attraction”.

b) in the case of *polytonalité in the sens large*: In his article, Milhaud declared, undogmatically, that there were “as many polytonalities as there are composers”, and that different and equally valid forms of listening (and thus of analysis) for a same musical “object” were possible (thus acknowledging, implicitly, the extension and diversity of the esthesic field). At the same time, he argued forcefully in favor of his own “syntagmatic” form of écoute, anchored on the perfect triad, and the diatonic modes:

> The analysis of a chord is often arbitrary and a matter of convention, and there is no reason whatsoever, for instance, not to consider a major 9th chord, rooted on C, as the superposition of a G minor and a C major triads.17

While in the example he shows from the second movement of Ravel’s *Sonata for violin and cello*, Milhaud considers as equally valid both the “unitonal” analysis as a sequence of 11th chords, and the “bitonal” analysis as a superposition of triads (*Accord I – manière “d”*).18

Charles Koechlin expresses a similar point of view to Milhaud’s, with respect to the possibility of existence of different, and equally valid, forms of hearing, giving, for instance, a “counter-example” to the different key signatures (C# minor x F minor, in each hand) in the first bars of Bela Bartok’s *Bagatelle Nº 1*, “locus classicus” of “Contrapuntal Polytonality”: he supplies an alternative (unitonal) orthography (“au

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18 The composer Alois Haba reports the tradition, at the Prague Conservatory, originatinated in V. Novak’s teaching, of an analogous approach, i.e, admitting the analysis of 5/6 notes chords as the superposition of two triadic groups (Haba, [1927]1984).
début de cette phrase, la bi-tonalité n’est qu’apparente”), closer to his own form of hearing in D flat minor, while, in 1945 (independently of his original “gesture”, at the poietical level, of the writing in two different signatures), Bartok himself would define it as “a Phrygian colored C major”, thus analyzed by Antokoletz (1985, p. 52):

The upper melodic line ambiguously begins in either C sharp Aeolian or E major, and the lower line unfolds modal fragments in C Phrygian and F Aeolian. However, at almost all cadential points, both lines metrically focus on the C major tonic chord as the primary vertical sonority.

Example 6 – Bartók’s Bagatelle Nº 1: different orthographies

Koechlin also makes the comment, of an esthetic order, comparing the different perceptions of Honegger and Milhaud with respect to the triad: while the former would always hear it as “dominant seventh” (as the 5th note of the harmonic series) “asking for a resolution”, Milhaud would hear the triad always as the affirmation of a key.

Such a diversity of écoutes/analysis is also clearly illustrated in the case of the Petrushka and “Augures Printanières” (Sacre du Printemps) chords, each of which elicits different descriptions:– the Petrushka chord, at its most “neutral” level, as Forte’s set 6-30 [0,1,3,6,7,9]; or “bitonally”, as a superposition of two perfect Major chords, at a distance of a tritone (analyzable under Milhaud’s “P.H.” system as an

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19 An analogous unintonal interpretation is given by Prokofiev’s biographer Israel Nestiev: “in connection with bitonal Sarcasm Nº 3, ostensibly written in a simultaneous combination of the flat minor bass and F sharp minor melody, it is actually a complicated B flat minor” (Nestiev, 1946, p. 62).
20 “The tonality of the Bagatelle Nº 1 is, of course, not a mixture of C sharp minor and F minor, but simply a Phrygian colored C major” (Bartok, 1976, p. 433).
21 Which Koechlin associates to the tradition of Napoleon-Henri Reber (1807-1880), one of the most influential nineteenth-century teachers at the Paris Conservatoire, whose Traité d’Harmonie (1862) rivaled with Emile Durand’s and Theodore Dubois’.
22 Discussing the ostinato (“tolshok”) chord of the “Augures Printanières” Boucourechliev comments that “of the five, six or seven different explanations of this famous chord none is wrong, none is entirely right.”
Accord VI-“a”); unintonally, as part of the octatonic scale (since Arthur Berger´s article); or still – for André Schaeffner and Andre Boucourechliev – as an irreducible entity, as a “sonority” (sonorité irréductible)23.

The “Augures Printanières” ostinato chord: at the immanent level, it could be described as Forte´s set 7-32 [7,8,10,11,1,3,4]; “unintonally”, as a 13th chord and/or, or as a display of the pitches of the collection of the A flat harmonic minor mode, and/or the recurring interpretation as unresolved appoggiaturas, or as a “quasi”-octatonic; or “bitonally”, as the superposition between the 1st inversion of a “dominant 7th” and a perfect triad (over roots Fb and E flat); under Milhaud’s system it would be analyzable as a “3-keys” Accord, combining Accord II-“d”+Accord I-“a” over triadic roots C#, Eb and Fb; for Boucourechliev’s (in Schaeffner’s line):

What we are faced with is an overall sound-phenomenon – a complex sound, however simple in theory. Whatever its origin or behaviour this chord must be considered as an indissoluble unit of sound and perceived as such for its individual quality. In the last resort therefore it is a question of timbre [...] Indeed the ear actually hears harmonic phenomena of this kind as timbre [...] it is even possible to suppose that everything in Stravinsky’s music that appears to be polytonal, or pseudo polytonal all disturbances of the harmonic spectrum, lies in its actual sound acting on the specific colour of the structure.

If from an esthesic perspective, the “existence” of Polytonality – as a phenomenon of simultaneous perception of independent polarities, or even of “polyharmonies” – remains indeterminate (in the face of so many conflicting and often equally well qualified points of view), it cannot be denied that, at the poietic level, it does “exist”, as part of the composer’s arsenal and intentions, notably the utilization of techniques of “Transpositional combination” on accords classés and traditional tetrachordal/scalar material. Moreover, Timoczko (2003) makes the point that “such a simultaneous combination produces a very particular type of musical experience”:

(Even if) we cannot hear the presence of multiple key-centers at once, it may still be that music that is polytonal in construction24 is perceptually distinctive, even though we cannot hear that this distinctive quality is due to the superposition of multiple keys.

23 According to Médicis (2009, p. 246], Milhaud’s system: “définit implicitement une forme de construction harmonique radicalement nouvelle, irréductible aux concepts familiers de l’ accord tonal (foncé sur des superpositions de tierces) ou de la pc set theory”.

24 Tymoczko (2003, p. 2) observes that “regardless of perceptual questions, polytonality may be perfectly used as a description of how some music is put together: if a composer combines an E major melody with a C major bass line, then we have a passage of music that is ‘polytonal’ in this merely constructional sense.”
“Ad dissonantiam per consonantiam” - Lago, M. A. C.

A number of relevant instances (most of which fitting the polytonalité stricte category) can be found in the literature: in Ives, Koechlin, and most spectacularly in various of Milhaud’s experiments between the 1915 Choephores and the 1920 String Quartet Nº 5 (a work in which four different transpositional levels are permanently displayed in simultaneity), with echoes, for instance, in the production of the young Ligeti during the 1940s.25

Example 7 – Ligeti (1943): example of polytonalité contrapuntique

**MILHAUD’S METHOD AS AN ANALYTICAL TOOL**

The exercise proposed by Milhaud in his P&A article is a highly “simplified” model26, based on extremely conventional assumptions and constraints:

a) in Polytonalité harmonique, the limitation of the constituent elements to the perfect Major and Minor triads;

b) the Major and Minor modes for Polytonalité contrapuntique.

In his own work, neither does he avoid combinations between other accords classés (notably the dominant seventh), nor limit his modal palette to the Major and Minor modes.

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25 See Cohen (2007, p. 16-17): “in four hands piano Polyphonic Etude (1943) the pianists recurrently repeat four folkloric melodies, each one differing with respect to tempo, length, and keys [...] the idea came from the reading of a Milhaud article on polytonality.”

26 In this highly simplified form, the model implicit in the Polytonalité & Atonalité article perfectly fits the notion of polytonalité par transposition parfaite put forward by Philippe Malhaire (2013, p. 139).
Example 8 – non-triadic superpositions in works by Milhaud

One particularly attractive feature in Milhaud’s system comes from the fact that its taxonomy is conceived semitonally, establishing a common ground with pc set analysis, and, thus, the compatibility between Accords I-VI and their corresponding interval classes 1-6. On the other hand, all manières “a” and “b” Accords, resulting from the superposition of 2 or more triads, are illustrations of “Transpositional combination”.

Another aspect also to be considered is that, if Milhaud’s stringent assumptions in his “P&A” article, were relaxed so as to incorporate other accords classés, and no longer exclusively rely on perfect triad combinations (where “polyharmonies” are always “polytriads”), its analytical scope would be significantly expanded, gaining in generality and thus becoming applicable to a much wider diversity of “harmonic verticalisation” cases: any two superposed accords classés, with their roots 1 to 6 interval classes apart, would remain describable (mutatis mutandi) as varieties of Accords I to VI. Such possibilities, no longer constrained beyond the “perfect triad-only” condition, were envisaged by Charles Koechlin, who outlined the following possibilities of expansion for Polytonalité Harmonique (which he vastly illustrated in his Heures Persanes): combinations of 2 or 3 perfect triads; combinations between a perfect triad and a 7th chord; combinations between a perfect triad, a 7th chord, and a chord with a different “formation” (e.g., superposed 4th).

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27 This common ground had been identified by Médicis (2009, p. 248).

28 According to Koechlin (1925): “Dans l’art polytonal nous nous trouvons en face de combinaisons extrêmement nombreuses, rien qu’en les écrivant sous la forme de divers accords superposés (et non entremêlés). Car nous avons déjà de la sorte: 1. les combinaisons de 2 ou 3 accords parfaits majeurs ou mineurs; 2. celles d’un accord parfait et d’un accord parfait de 7 ème ou de 7ème diminuée; 3. celles résultant de la simultanéité d’un accord parfait, d’un accord de 7ème, et d’un accord d’une autre formation (en quartes superposées par exemple).”

29 Another possibility of expansion of “P.H.” system, based on all 4 triads – thus including both diminished and augmented fifths – is (implicitly) given by Tymoczko who demonstrates, in the table “Classification of triadic progressions according to scale membership”, that “any superposition of two triads of any quality (major with major, augmented with diminished, etc) belongs to one of the seven scales” (the 7 scales obeying to his “consecutive semitone constraint” principle).
THE “P.H.” SYSTEM AS AN “ESTHESIC RECONSTRUCTION” OF THE SACRE

By relaxing just one single assumption of the “P.H.” system, allowing – instead of the “perfect triads only” condition – the substitution of one of the two superposed triads by a “dominant seventh” chord (which amounts to incorporate to the triad the next interval to the 5th in the harmonic series, i.e., the minor 7th), the resulting variant in the “2-keys case” – triad+“dominant 7th” – increases very significantly the number of polyharmonies which, in a work like the Sacre, become analyzable under Milhaud’s “P.H.” system.

Pieter van den Toorn has abundantly shown the extent to which, in Stravinsky’s works, the utilization and combination of those two accords classés i.e., the perfect triad and the “Dominant seventh” (as well as the “Dorian” tetrachord [0235]) are often derived from the modal context itself of the “wholetone/semitone scale” which, in the Rimski-Korsakov’s tradition, had become a frequent device in Russian music. However, there also are, in the Sacre, a number of Accords/pc sets which result from the sum of “operands” Triad ([047]) * Dominant seventh ([04710]), but do not fit the octatonic: if, on the one hand, this does not invalidate the importance of the octatonic as a chief analytical tool, on the other it highlights the relevance of Milhaud’s method, which is capable (from a higher degree of generality) of adequately describing such sets, as the examples below illustrate:

"Ad dissonantiam per consonantiam" - Lago, M. A. C.
Example 9 – Rite of Spring: (enlarged) polytonalité harmonique chords

**Interval Class [1]**

"Accord Ia": M + M (7th)

*Augures Primaniers: Danse Sacrale*

\[ \begin{array}{c}
13 \\
167 -1
\end{array} \]

**Interval Class [1]**

"Accord Ic": M (7th) + m

*Danse Sacrale*

\[ \begin{array}{c}
142 \\
167
\end{array} \]

**Interval Class [2]**

"Accord IIa": M (7th) + M

*Jeu du Rapt*

\[ \begin{array}{c}
31 +2
\end{array} \]

**Interval Class [3]**

"Accord IIIa": M + M (7th)

*Jeu du Rapt: Danse Sacrale*

\[ \begin{array}{c}
37 \\
38 \\
82 +4 \\
155 +2
\end{array} \]

**Interval Class [3]**

"Accord IIIc": M (7th) + M

*Jeu du Rapt*

\[ \begin{array}{c}
37 \\
38 \\
82 +4 \\
155 +2
\end{array} \]

**Interval Class [4]**

"Accord IVa": M (7th) + m

*Danse Sacrale*

\[ \begin{array}{c}
148 \\
192
\end{array} \]

**Interval Class [4]**

"Accord IVd": m + M (7th)

*Introduction II*

\[ \begin{array}{c}
81 +2
\end{array} \]

**Interval Class [5]**

"Accord Vd": m + M (7th)

*Rituel des Ancêtres*

\[ \begin{array}{c}
125 +1
\end{array} \]

**Interval Class [6]**

"Accord VIa": M (7th) + M (7th)

*Jeu du Rapt: Danse Sacrale*

\[ \begin{array}{c}
39 \\
40 -1 \\
44 +3 \\
186
\end{array} \]

**Interval Class [6]**

"Accord VIc": M (7th) + m

*Introduction II*

\[ \begin{array}{c}
79 +1
\end{array} \]
Example 10 – Rite of Spring: (enlarged) polytonalité harmonique progressions

In his writings, and notably in his autobiography,30 Milhaud makes many references to Charles Koechlin’s decisive role in the formation of his personal language31 and that it was from the precedent and study of his and Stravinsky’s works, that he developed his own systematic approach to Polytonality.32 It was not by chance, therefore, that he would become, in 1915, the dedicatee of Milhaud’s magnum opus on “Polytonalité Harmonique”,33 the Choéphores.

[In 1915] I had undertaken a thoroughgoing study of the problem of Polytonality […] The contemporary composers, Stravinsky or Koechlin, made use of chords containing several tonalities, often handled

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30 Milhaud (1953, p. 65-6).
32 This “poietic” dimension is highlighted by Richard Taruskin (1990): “The composer’s theoretical environment – his training, the theory books he knew, his ways of looking at his own music […], the music he heard, loved, hated, the books he read etc. [in an attempt] to find as many “external” corroborations, as one can, from the “internal evidence of the music”.
33 According to Médicis (2009, p. 246), “point culminant de son écriture polyharmonique”.

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contrapuntal or used as a pedal point. I set to work to examine every possible combination of two keys superposed and to study the chords thus produced [...] Then I did the same for three keys [...] I grew familiar with some of these chords: they satisfied my ear more than the normal ones, for a polytonal chord is more subtly sweet and more violently potent.

Referring to their frequent meetings in 1913-14, in Paris and to the occasion of a Koechlin visit to Milhaud’s home in Aix-en-Provence.

I often met him... I loved his music, his harmonic experiments and the marvelous range of his mind. He stopped for a few days in Aix [...] We talked together about music discussing the *Sacre du Printemps*[^34] which we had hailed with much enthusiasm at its first performance a year before. We not only admired its violent rythms, its harmonic discords and polytonality, all of which had been foreshadowed in *Petrushka* [...] but specially its novelty.

Pieter van den Toorn calls the attention to a novel aspect which makes its appearance in the *Sacre*, representing a radical turning-point in “harmonic” technique, and which could not fail to pass unnoticed:

The formulae [from the octatonic vocabulary] are as pertinent to the *Rite* as they are to the early *Scherzo Fantastique* and to the *Firebird*. Indeed, the *Rite* is one of the most thoroughly octatonic of Stravinsky’s works [...] and to an astonishing degree, the vocabulary that informs this referential commitment remains intact: triads, dominant seventh, and [0235] tetrachord. **The distinction rests primarily with the technique of superposition.** [...] These units are now superposed, played simultaneously no longer succeeding one another. And this is an invention from which starting implications accrue, not only in terms of pitch organization [...] and opens up a new dimension.[^35]

If Milhaud has not retained the octatonic as part of his *poiesis*, given his declared inclination for heptadic Major (and other diatonic scales) and Minor modes, he did adopt – to the point of devising a system of his own – the *Sacre*’s techique of

[^34]: While Milhaud in his conversations with Claude Ronstand would declare: “à l’époque du *Sacre* ce sont des agrégations harmoniques qui m’ont certainement aidés dans mes recherches d’alors.”

chordal superposition, mostly based on elements (triad, dominant seventh) which also belong to the octatonic vocabulary. It can therefore be noted that a very large number of harmonic aggregates which are found in the works of Stravinsky’s *fase russe*, notably in the *Sacre*, can be classified according to parameters of Milhaud’s “P.H.” system illustrating many instances of *Accords* I to VI (either purely triadic, or the variant superposing triad and “dominant 7th”), as well as superpositions of 3 triads under the various “modal” combinations (manières) “a” to “d”.

Such compatibilities render plausible the hypothesis, based on assumptions of the tripartition theory, that the “poietic construction” represented by the “P.H.” system might be an “esthesia reconstruction” by Milhaud resulting from his analysis of the *Sacre*, stimulated by his conversations with Koechlin in 1914, who, in 1932, would evoke the *Sacre* première as “the revelation of the polytonal language”.

**FROM ACCORDS TO RAPPORTS: “POLYTONALITÉ HARMONIQUE” AND “POLYTONALITÉ CONTRAPUNTIQUE”**

Another constraint, which can be made more flexible, originates from the fact that Milhaud’s method of analysis, in his *Polytonalité & Atonalité* article, is essentially “P.H.-oriented”, i.e., its taxonomy is entirely focused on those vertical aggregates generated by the operation (To [0, 3 or 4, 7] + Tn [0, 3 or 4, 7]). If by analogy, his method – based on interval-class relationships – were expanded to superpositions of melodic lines operating as ([7-35 or 7-32] + Tn [7-35 or 7-32]), the analysis of a number of polyphonic textures, frequent in early twentieth-century music, would then become possible.

While bitonal rapport’s have been illustrated previously, from excerpts of the *Saudades do Brasil*, the excerpt taken from Milhaud’s *String Quartet Nº 5* (example 11), in which the 4 voices move simultaneously over different transpositional levels of a same diatonic scale (C major collection 1st violin, D major 2nd violin; E major alto; and A flat major cello) –, illustrates the case of multiple *rapports*: *rapport* II-“a” between the 1st violin and both 2nd violin (D major x C major collections) and alto (D major x E major collections); *rapport* IV-“a” between the cello and both 2nd violin (A flat major x C major collections) and alto (A flat major x E major collections); *rapport* VI-“a” between cello and 1st violin (A flat major x D major collections).
However, it is interesting to note that transpositional and polarity relationships do not necessarily coincide, not only in the cases of those textures in which centricity is ambiguous (if not irrelevant), but specially in those – usually and appropriately – analyzed as polymodal. In Stravinsky’s *Berceuse du chat Nº 3* (example 12) there is a clear centricity in D (implying a Dorian mode listening) in the melody (voice and piano r.h.), while the l.h. accompaniment shows a certain ambiguity between F# and C#, but its pitch content is that of the F# major collection. There are, then, two concomitant *rapports*: a polarity relationship of major third (interval class 4, between D Dorian and F# major) between the tonics, within a transpositional distance of tritone (interval class 6) and thus a total pitch content given by the operation [To (7-35) + T6 (7-35)].
Example 12 – Polytonality & Polymodality

Stravinsky

*Berceuse du chat n.3*

[1915]

```
\begin{music}
\begin{staff}
C\flat\:\text{major}
\end{staff}
\quad\quad\quad\quad
\begin{staff}
C\#\:\text{minor}
\end{staff}
\end{music}
```

In the case of Bela Bartok’s *Bagatelle Nº 1*, in which the bi-polarity relationship may be perceived as a distance of one semitone (C Phrygian and C# Aeolian), the transpositional operator between the E major and A flat major collections corresponds to interval class 4, and thus the total pitch content is given by the operation [To (7-35) + T4 (7-35)].

Example 13 – non-coincidence between transposition and polarity relations

Transposition and Polarity Rapports

Stravinsky - Berceuse du Chat n.3
Rapports VI, 4

Bartok - Bagatelles n.1
Rapports IV, 2

```
\begin{music}
\begin{staff}
Dorian
\end{staff}
\quad\quad\quad\quad
\begin{staff}
Aeolian
\end{staff}
\end{music}
```

a) the whole-note indicates the pole/“tonic”

b) the first subscript (roman) indicating the transpositional, and the second (arabic) indicating the polarities *rapports*.
Thus, it should be possible to define one particular type of polymodality – basically applicable to transpositionally equivalent sets – in terms of 2 interval classes: the polarity relationship, and the rapport itself (defined by the transpositional operator: interval class 4 (between D Dorian and F# major) and 6 (rapport VI) in the Berceuse du chat; interval class 1 (between C Phrygian and C# Aeolian) and 4 (“Rapport”IV) in the Bagatelle.

CONCLUSION

In his seminal text “La Polytonalité selon Darius Milhaud” (2005), Médicis had already called for the need of a “théorisation flexible, ouverte à l’hybridation des approches méthodologiques”, and we would hope the discussion to be a step in this direction, as in fact it has been found, along this study, that tools borrowed from both pc set and tripartition theories – as well as Richard Cohn’s notion of “Transpositional combination”, and Timoczko’s demonstration of the mutual implication/imbrication between triadic superpositions and scalar/modal formation – could prove relevant auxiliaries to a better understanding and discussion of the issues raised by Milhaud.

In the case of Molino and Nattiez’s tripartition approach, it allows to isolate and dissociate questions pertaining to the “esthesic domain” – such as if polytonality can “exist” at all at the perceptive level (Hindemith, Bartok) – from those compositional decisions and strategies which are part of the “poietic” process, which may (or may not) be “heard” as intended: though Milhaud would consider the superposition of two (or more) triads as corresponding to “two (or more) keys”, rarely will such polyharmonies (Accords) imply polytonalité au sens strict, and will generally belong to textures fitting the case of polytonalité au sens large.

The purpose of this study has not been, therefore, that of embracing the vast themes which have been discussed in the specific literature under the umbrella of the term “Polytonality”, but only to present the “P.H. system”, such as advanced by Milhaud in his 1923 “Polytonalité et Atonalité” article, as a simplified model for generating chordal combinations (Accords, in Milhaud’s acception) and/or unordered pc sets. Its vocabulary consists of a specific group of composite – and often “dissonant” – harmonic combinations/sets, presenting the particularity of being separable into (and thus analizable as) superpositions of familiar consonances – notably the perfect triad – and for which Milhaud suggests a taxonomy based on:

a) the intervalical distance between “fundamentals”;

36 For a wide and comprehensive review of different approaches related to Polytonality, see Noronha (1998), Pistone (2005); for a historical account, see Koechlin (1925), and Malharie (2011; 2013), noting the remarkable precedent, in the XIX century, of the Italian composer Pietro Raimondi (1786-1853), considered by Fétis: “l’esprit de combinaison le plus extraordinaire qui ait jamais existé”; and who recorded in his Biographie universelle des musiciens (1870, p. 162) experiments such as: “six fugues à quatre voix écrites en tons différents, mais qui peuvent être reuines en une seule fugue à 24 voix.”
b) the individual “mode” of each triadic unit;
c) the number of superpositions.

The linearization of such Accords – in both “2 and 3 keys” cases – also shows that they are, “by construction”, compatible with a particularly rich “modality” from those familiar modes belonging to the diatonic and harmonic minor collections, to the octatonic and a large assortment of “oriental” scales (notably most of the Indian Carnatic modes).

It has been the intention, along this study, to preserve as much as possible\(^{37}\) (by introducing only minor adaptations) Milhaud’s own terminology, which has been found to be pertinent – as an analytical tool – to a relevant part of the early twentieth-century music répertoire: in the musical examples presented above, Milhaud’s Accords have principally been illustrated from works belonging to Stravinsky’s *fase russe*, but they also appear as plausible descriptions for a significant number of sets/chordal formations belonging to works by composers as varied as Satie, Ravel, Roussel, Honegger, Enesco, Ives, Villa-Lobos, Copland, Britten and Messiaen. At the same time, Milhaud’s Accords circumscribe a particular group of scales – a subset within Forte’s mapping of the prime forms space (interesting for what is included, as much as by what it tends to exclude, i.e., symmetrical scales) –, sharing properties derived from the interval vector of the triad, with a “sonority” (in the sense implied by Schaeffner and Boucourechliev) independent of their ordering as polyharmonies (in Persechetti’s sense). Thus, Milhaud’s “P.H.” system can be described, and summarized, as a particular application of Cohn’s “Transpositional combination” concept, in which Accords can be expressed as higher cardinality set-class products of only two set-class “operands” [0,4,7] and [0,3,7].

The pc set theory apparatus proves particularly illuminating in that it brings important insights concerning the *modus operandi* of the Milhaud’s system: while many Accords will correspond to set-classes presenting Cohn’s “TC” properties, all Accords/sets, however diverse or sometimes complex can be subsumed as outcomes of Forte’s transposition operator over the six interval classes, through an exhaustive exploration of all possible combinations between equivalent sets (“transpositionally equivalent” for manières “a” and “b”; “inversionally equivalent” for manières “c” and “d”), the generative set being, in the case of *Polyonalité harmonique*, the major and minor perfect triads (i.e., [0,3,7] and [0,4,7] two expressions, inversionally related, of Forte’s prime form “3:11”); and in the case of *Polyonalité contrapuntique*, the major and minor modes collections (Forte’s prime forms “7-35” and “7-32”).

\(^{37}\) Other terms such as “polytriad” (Charles Burkhart) if applied to strict “P.H.”, or the expressions “polyscalar superpositions” (Tymoczko), or “simultaneous diatonic harmonic contexts” (Cox), would be more precise.
If, for the analysis of the immanent level, the “P.H. system” cannot compete, in terms of “neutrality”, with pc set analysis, whose tools allow to keep connotation to a minimum, Milhaud’s method – for the specific pc set space defined by those transpositional/inversional set classes to which it is applicable – provides a descriptively adequate model for a large corpus of early twentieth-century music, being attractive in many ways:

- simpler to memorize, as it depends only on the interaction of two “coordinates”: the Accords I to VI (defined by the semi-tonal distance between chordal “roots”) and manières “a” to “d” (defined by the modal identity of each units);
- from an intuitive angle: many “P.H.” Accords can be associated with (and illustrated by) “sonorities” which have become familiar in the literature, with a sometimes conspicuous presence in works ranging from Pelléas, Elektra, Daphnis, Pétrouchka, Sacre, to Schoenberg’s Farben or Berg’s Violin Concerto.
- moreover, the “P.H. system” is susceptible to expansion and generalization, if its method is to be applied to a wider range of accords classés, by relaxing (in varying degrees) the rigid “perfect-triads-only” constraint (e.g., the Accords I to VI, in the Sacre, resulting from the combination not only of 2 triads, but also of the sonorities of triad + “dominant 7th”, or Koechlin’s suggestion incorporating both diminished and augmented triads etc).

But specially, such a (“ad dissonantiam per consonantiam”) method of approaching dissonance in terms of consonant units, – of which the “P.H. system” represents both an illustration and a “simplified model”, appears as particularly revealing of a specific poiesis, which not only pervades most of Milhaud’s work, but is also recurrently found – as an ad hoc technique – in works by many among his most illustrious contemporaries, from Ravel and Stravinsky to Messiaen and Britten, and more recently in works by composers such as Ligeti, Kurtag, Schnittke, Nancarrow, or those associated with Minimalism, who, in different ways and degrees, have detached themselves from the Post-War serial tradition.

ANNEX I – MILHAUD “P.H.” CATEGORIES APPLIED TO HARMONIC PROGRESSIONS

In the preceding examples, Milhaud’s “P.H.” categories were applied “locally”, as case-by-case morphological descriptions of specific harmonic aggregates. However, numerous instances can be found, in the literature of early twentieth-century music, in which, – as an analytical tool –, they may be applicable as well to progressions (enchaînements) as well as to longer contexts (notably in Milhaud’s and Messiaen’s works). Médicis (2005) had already convincingly demonstrated the systematic

38 See forceful point made by Nattiez (2000).
use by Milhaud, notably in the *Choephores* (see the “Libation” scene illustrated in example 16), of progressions consisting of a same “2-Keys” Accord in which the roots are interchangeable via chordal inversion\(^\text{39}\), a procedure which can also be found in works of other Milhaud contemporaries (see on example 14 quotations from Busoni, Stravinsky, and Ives). Other examples (by no means exhaustive) are presented in example 14, such as: one same Accord articulated over different roots (as in the example of Busoni’s *Sonatina*); different Accords over different roots (as in the example of Stravinsky’s *Concertino*), either in one same “mode” throughout (\textit{manièr e “a”} in Messiaen’s “Action de Grace” example) or many and mixed \textit{manières} (as in Messiaen’s “Épouvante”).

Example 14 – “2-Keys” Accords: examples of “P.H.” progressions

LONGER SECTIONS

The following examples range between two extreme cases: Ives’ *Psalm 67* first section, consisting almost exclusively of only one “colour” – that of *Accord* V-“d” –, and the Introduction of Honegger’s *Di Tre Re Symphony*, in which a wide assortment of *Accords* and *manières* are utilized: an intermediary case is the *Choephores*’ “Libation”\(^{40}\) scene, in which the “colour” of *Accord* VI–“a” strongly predominates, though a varied array of other *Accords* (notably *Accord* II–“d”) is also displayed.

As in the previous case with progressions, Milhaud’s categories (*manières* “a” to “d”) for qualifying each *Accord*, also allow us to establish useful distinctions between musical textures: those displaying exclusively major triads in superposition (*manière* “a”), which appears to be the most frequent case in the literature (see example 15, Stravinsky’s *Les Noces*, and Ernest Widmer’s “Concatenações”); others based exclusively on composite *manières* (“c” and “b”) Stravinsky’s *Petrushka*’s “Entrée des ivrognes”; or those which utilize the entire range of *Accords* and *manières* such as Messiaen’s “Épouvante” (*Poèmes pour Mi*)

Example 15 – “2-Keys” *Accords* textures, combining different *Accords* and *manières*

a) one *Accord* (“V”) and one *manière* (“d”):

![Example 15](image)

b) different *Accords* (“II-V”) and all *manières* (“a-d”):

![Example 15](image)

\(^{40}\) See Médicis (2009, p. 249-254) for remarkable analysis of this scene.
c) different Accords (“I-IV”) two manières (“c-d”):

Igor Stravinsky, Petrushka

```
IV   II – V   III   II – V   III   II – III V   III   I   V   III   V
  d   b   d   d   b   d   d   b   d   d   d   d   d   d
```

d) different Accords over one manière

Igor Stravinsky, Les Noces, 16 - 17

```
IV   III   I   II   VI   III   I   II   III   VI   III   V   VI   IV   I
```

e) the same

Ernest Widmer, Ludus Brasilienses (“Concatenações”)

```
III   I   III   III   – III V   III   I   – II   III   IV V   III   I   II   – II V
```

"Ad dissonantiam per consonantiam" - Lago, M. A. C.
f) composite Accords and manières

Messiaen, "Action de Grâces" (Poèmes pour Mi), b.16-19

g) predominantly one manière over different Accords

Messiaen, Epouvante
Example 16 – Milhaud’s *Choephores*, “Libation”

Milhaud, *Choephores* - "Libation"

*first chord of each bar*
ANNEX II – EXAMPLES OF “P.H.” CATEGORIES IN POST-1960 MUSIC

From the previous examples, it can be seen that the use (intentional or not) of “dissonant” aggregates decomposable into consonances – such as the polyharmonies which Milhaud denominated “2/and 3- Keys” Accords – has been an extremely frequent feature in early twentieth-century music: associated originally to composers such as Satie, Ravel, Stravinsky, Ives, Milhaud, and “neoclassicism” in general, it is present as a procédé d’écriture from the 1960s onwards, though in a very different spirit from the 1920s and 1930s, in works by composers such as Schnittke, Kurtag, Ligeti, Nancarrow, later Messiaen works (such as his opera St François d’ Assise), Phillip Glass, John Adams etc.

Though taking different forms (and not associated to a “revival polytonality”), an approach in common has been the compositional choice of textures – within a particular subset of the pc sets space – characterized by aggregates which are decomposable into consonant units (triads and/or other accords classés), as in the following examples.

The Brazilian composer Jocy de Oliveira (2014, p. 157) has provided an analysis of Luciano Berio’s Sequenza IV (1966), explaining its derivation from two basic chordal aggregates, the second consisting of the superposition of two perfect triads (major+minor). It is interesting to note that such chordal aggregates have in common the “sonority” of Milhaud’s manières “c” and “d” (see example 17).

Example 17 – Luciano Berio, Sequenza IV

![Example 17 – Luciano Berio, Sequenza IV](image)

In Ligeti’s work Ramifications (1969) the two groups of strings are tuned at a distance of a quarter tone: the following recurrent accumulation of triadic formations can be found (up to 6 different roots in “a” and “b” combinations).
“Ad dissonantiam per consonantiam” - Lago, M. A. C.

Example 18 – Ligeti, *Ramifications*

In Edino Krieger’s 12 tone *Ludus Symphonicus*’ “Intrata” section (1963), a number of triadic superpositions will also be found: Milhaud’s bitonal *Accords* I c (b. 81), II a (b 74), IV a (b 78), VI a (b 72); and triadic accumulations up to 4 and 5 chordal roots: such combinations are derived from the structure itself of the set which consists of (internally ordered) 3 perfect triads + a diatonic trichord.
Example 19 – Edino Krieger, *Ludus Symphonicus*

Edino Krieger - *Ludus Symphonicus* "Intrata"

“Ad dissonantiam per consonantiam” - Lago, M. A. C.

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