"Ad dissonantiam per consonantiam": the scope and limits of Darius Milhaud´s system of "*Polytonalité harmonique*": the immanent and poietic levels (Part 1)*

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Abstract
A frequent feature in early twentieth-century music, which gained enormous impulse from Stravinsky´s introduction of complex polyharmonies in the *Rite of Spring*, has been the utilization of "dissonant" harmonic aggregates, presenting the peculiarity of being decomposable into traditional consonant units, such as perfect triads and "dominant 7th" chords. In a 1923 text, Darius Milhaud attempted to provide both a *rationale* and a taxonomy for these "new chords" through a system of "harmonic polytonality". It will be argued along this paper that Milhaud´s approach would be particularly enriched if seen, on the one hand, as a method for generating a particular "family" of unordered pitch-class sets, which are found not only in Stravinsky´s *fase russe* works but also among as different composers as Ravel, Ives, Villa-Lobos Britten or Messiaen; and, on the other hand, in the perspective of Molino & Nattiez´s tripartition theory. The first part of this article discusses the "immanent" and "poietic" levels.
Keywords
Harmonic polytonality – polyharmony – set theory – tripartition – Carnatic modes – Darius Milhaud – Igor Stravinsky.

Resumo
A partir da *Sagração da Primavera* de Igor Stravinsky, tornou-se frequente na música do início do século XX a utilização de poliharmonias complexas que, apesar de seu efeito dissonante, apresentam a peculiaridade de serem redutíveis à superposição de acordes consonantes tradicionais, como tríades e "acordes de 7ª de dominante". Em artigo datado de 1923, Darius Milhaud tentou fornecer uma *rationale* e uma taxonomia para esses "novos acordes" por um sistema de "politonalidade harmônica". Será argumentado, ao longo deste estudo, que a abordagem de Milhaud poderia ser significativamente enriquecida, por um lado, se entendida como um método com capacidade de gerar "séries não ordenadas" (*unordered pitch-class sets)* com características muito específicas e que são encontradas com frequência em obras da "fase russa" de Stravinsky, assim como nas de compositores tão diversos quanto Ravel, Ives, Villa-Lobos, Britten ou Messiaen; e por outro, se colocada na perspectiva da teoria da "tripartição" de Molino & Nattiez. A primeira parte deste artigo discute os níveis "imanente" e "poiético".
Palavras –chave
Politonalidade harmônica – poliharmonia – teoria dos conjuntos – tripartição – modos carnáticos – Darius Milhaud – Igor Stravinsky.

* A second part, completing this text, follows in a next issue of this journal. I would like to express my warmest thanks to Professors Vincenzo Caporaletti, Jacynthe Harbec, Malena Kuss, Ricardo Tacuchian, Jocy de Oliveira, Rami Levin, Ilza Nogueira, Manuel Pedro Ferreira, Sonia Rubinsky, and Ricardo Kubrusly for their attentive reading of this text, when still in progress; in a very special way, to Professor Régis Duprat for his decisive encouragement and important suggestions since its very beginnings; to Professor Luiz Paulo Sampaio for precious insights and guidance through the intricacies of "tripartition" theory; to Professor Maria Clara Barbosa, who – while elaborating the scalar tables presented in this text – elucidated for the first time the scope and limits of Milhaud´s "3-keys" hypothesis; and Professor Maria Alice Volpe for her unfailing support, and unvaluable advice all along the elaboration of this text.
Proofreading and editing by Maria Alice Volpe and Mário Alexandre Dantas Barbosa.

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Artigo recebido em 20 de abril de 2015 e aprovado em 20 de maio de 2015.

In 1923, the same year in which Arnold Schoenberg would reveal his "Twelve-tone technique", Darius Milhaud exposed in the *Revue Musicale* – in a famous article entitled "Polytonalité et Atonalité" – a "neo-tonal" theory in which Polytonality was presented as a powerful device for expanding the musical vocabulary, in his view "as bold as Atonality" in the exploration of new harmonic combinations, and often arriving at similar results, though with different means. An early admirer of Schoenberg, whose *Pierrot Lunaire* he had introduced and championed in France in 1922,[1] Milhaud does not confront one system against the other. He views both as equally valid techniques, and as the logical result of the historical development of two important, though distinct, musical traditions: one, his own, which he denominates "Latin",[2] associating it with Diatonicism; and the other, which he denominates "German", associating it with Chromaticism. He considers these two traditions the determinant factor in the choice between either system, recognizing no superiority of one over the other, but operating within each composer´s cultural background[3]:

> Polytonality and Atonality are not arbitrary systems. One results from the development of diatonic harmony and counterpoint, while the other from chromaticism, and both should thus be the object of complementary technical studies. […] A work´s inner life will wholly derive from the composer´s melodic invention, and thus polytonality and atonality will supply him with a wider scope, a richer technique, and more complex means for expressing his sensibility, his imagination and fantasy.

The present study is not concerned with such "culturalistic" views[4] belonging very specifically to the context of the 1920s,[5] but will instead focus on two standpoints:

Firstly, it proposes that the controversies surrounding Polytonality – subject of a vast literature[6] – **would be better understood if put in the perspective of Jean**

[1] Milhaud would also give the French première of *Herzgewachse*. Concerning his visit, in the company of Poulenc, to Schoenberg in Vienna in 1921, see Drake (1982, p. 132-133).

[2] Milhaud ("L'Évolution..." 1923[1982], p. 201): "Le diatonisme et le chromatisme sont les deux pôles de la'expression musicale". A concept also adopted by Bela Bartok who wrote in 1938 (*Revue Musicale*): "Nos élites intellectuelles [...] ont reconnu que l'esprit latin est infiniment plus proche du genie hongrois que l'esprit germanique."

[3] According to Milhaud (1923[1982], p. 194), "Ce qui déterminera le caractère polytonal ou atonal d'une oeuvre, ce sera bien moins le procédé d'écriture que la mélodie essentielle qui en sera la source, et qui vient du 'coeur' seul du musician;or on ne s'invente pas une tradition, on la subit et on la travaille."

[4] That such views were widespread in the cultural atmosphere of the 1920s, in the most different corners, is reflected in Schoenberg´s well known reply to Alexander Zemlinski, concerning Milhaud: "As to Milhaud being 'insignificant' I do not agree: Milhaud strikes me as the most important representative of the contemporary movement in all Latin countries: Polytonality […] Whether I like him is not to the point, but I consider him very much." See letter dated October, 26 (Schoenberg, 1922, p. 80).

[5] See André Coeuroy's *Panorama de la musique Contemporaine* (1928), who analysed the diverse manifestations of early twentieth-century music "sous le signe du National": he sees Atonality as culturally germane to Central Europe: "le Schönberguisme envoûte toute l'Europe centrale [...] L'on ne pénètre pas du premier coup dans l'étrange sorcellerie du *Pierrot*. Si contraire qu'il soit au génie latin, il s'affirme comme une de ses oeuvres les plus excitatrices du XXeme siècle."

6 See Barbara Kelly (2003), and François de Médicis (2005b) and the comprehensive works, dedicated to "Polytonality", by Danièle Pistone (2005) and Philippe Malhaire (2013).

Molino´s and Jean-Jacques Nattiez' "tripartition" method[7], which operates a distinction between three different (though complementary) layers of analysis: the "poietic" (the process that originates the work), the "immanent" (a dissection as "objective" and as "neutral" as possible of the work´s structure), and the "esthesic" (the reconstruction of the work by an interpreter or by a listener).

Secondly, it considers the "*Polytonalité Harmonique*" ("Harmonic Polytonality") system, as presented by Milhaud in his 1923 article, as a point of departure for discussing some compositional procedures which became widespread in early twentieth-century music, and then to examine its basic assumptions and *modus operandi*, **its capacity of generating pitch-class collections** – "new chords", in Milhaud´s terminology – which otherwise would not be obtainable within the limits of a diatonic framework, as well as some of the properties of such pc collections[8].

A remarkable feature in Milhaud´s system – considering the highly dissonant results which can sometimes be obtained – is that **its "building blocks" consist exclusively of the most consonant and "conventional"**[9] **among those materials**[10]: the perfect Major and Minor triads – as the sole basis for what he denominates "Harmonic Polytonality"[11] – and the Major and the Minor modes – as the basis for what he denominates "Contrapuntal Polytonality".

In this sense, Milhaud´s *Polytonalité Harmonique* system illustrates a very specific application of Richard Cohn (1988)´s notion of "transpositional combination" (TC),

[7] For Molino and Nattiez, only by examining these three levels in turn, will the analyst grasp the "musical fact" in all its multifaceted totality, "the essence of a work is at once its genesis, its organization, and the way it is perceived." For a comprehensive analysis of the tripartition model, see Sampaio (2014).

[8] Differently from previous studies that have already made use of the pitch-class set theory for analyzing works with "polytonal" textures, the present study uses the pc-set theory purely as a metalinguistic tool, focusing on the mechanics of the "P.H." system. With respect to the first approach, Keith Daniels (1982, p. 331) has pointed out that: "I am drawn inexorably to the conclusion that pc-set analysis is of quite limited usefulness in coming to terms with the polytonal music of Darius Milhaud. The linear nature of the polytonality, which results from Milhaud´s tendency to superpose lines in different keys, limits the possibilities of segmentation, since horizontal sets will, for the most part, be diatonic."

[9] This approach was well understood but negatively viewed by Alban Berg, who (though unnamed) probably had Milhaud in mind when he blamed: "those [composers] who write in two or more (major or minor) keys simultaneously, but the musical procedures within each one often betray a frightening poverty of invention. [...] Another [composer] may write in a bold harmonic style and not shrink from any combination of tones, but he has room only for melodies that hardly overstep homophony and are further characterized by the use of only two- or four-bar phrase; see Alban Berg´s article "Why is Schoenberg´s music so difficult to understand?" (1924). The paradox between the complexity of some Milhaud´s textures and the simplicity (at the limit of triviality) of the melodic and chordal building blocks is also echoed in the same year by Alfredo Casella in his article "Tone Problems of Today" (1924): "when I hear in his quartet or his symphonies, four, five or more instruments performing simultaneously as many melodies of an inoffensiveness quite inadequate to the end proposed I […] cannot conscientiously assert that the result is agreeable to my ear."

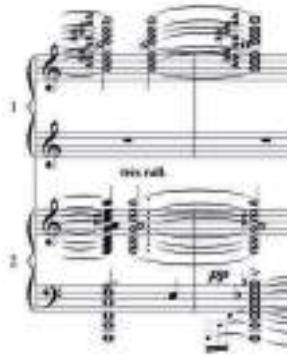
[10] According to Milhaud (1923): "Le diatonisme implique la croyance en l'accord parfait (composé de sa fondamentale de sa tierce majeur ou mineur, et de sa quinte) comme une réalité fixe reposant sur une gamme majeur ou mineur que le compositeur utilisera dans la composition de ses thèmes mélodiques."

[11] As precised by Médicis (2005a): "Sur le plan analytique, la distinction entre polytonalité harmonique et contrapuntique n'est pas absolue, et on peut distinguer entre les deux formes d'écriture une foule de stades intermédiaires". while Daniele Pistone (2005, p. 29) warns against "P.H." being a misnomer, and explains: "la polytonalité harmonique mérite à peine son nom, puisqu'elle oscille nécessairement entre une tendance à la polarisation (la ramenant à la tonalité unique) ou à un éparpillement des axes tonals qui conduit forcément à l´atonalité (ou à la porte de la pantonalité)" (Pistone, 2005, p. 26). Thus, in the present study, attempting to remain as close as possible to Milhaud´s own terminology, it is used either in quotation of the composer´s own views, but principally (associating the term "P.H." with the notion of "system") to denote to the "simplified model" of *accords classés* combinations (and the resulting pitch-class formations), which can be derived from his 1923 article.

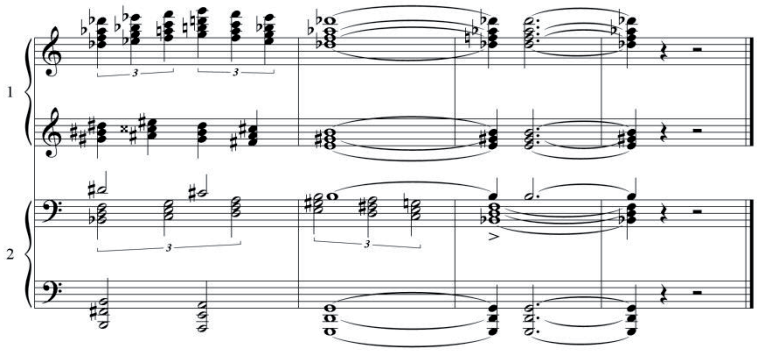
in which the "operands"/ lower-cardinality set-classes will exclusively consist of the two perfect triads, i.e. the (inversionally related) trichords ([0,4,7] and [0,3,7]), whose diverse sum operations will then result into higher-cardinality sets (=Milhaud´s *Accords*). It is interesting to note that this apparently limited and "conventional" point of departure has not prevented – in a number of Milhaud´s works – the making of musical textures of extreme complexity, and which often mobilize the chromatic total:

Example 1 – Mobilization of the chromatic total via triads

Piano Sonata (1916)



Les Euménides – Act I (1919)



The following points will be developed along this study:

a) the number of "new combinations" obtainable through the *Polytonalité Harmonique* system, is **significantly smaller than what had been envisaged by Milhaud**, as the combinatorial exercises of his article overlook the numerous situations of transpositional equivalency, mostly invariances and the frequent cases of homonymy with diatonic *accords classés*[12];

b) the characteristics of such pc sets, entirely derived from the superposition of triads, reinforce the view, already expressed in a number of important studies on Milhaud (Drake, 1989; Mauwer, 1997; Kelly, 2003) of **the central place of Modalism in his musical idiom**[13], which, "by construction", is necessarily implied by triadic summation (see Tymoczko, 2002);

c) the fact that *Polytonalité Harmonique* ("Harmonic Polytonality"), **does not constitute a source of "infinite" new possibilities** for expanding the ressources of the musical vocabulary [14], being much closer to Modality than imagined, does not

[12] French term currently used, in the tradition of the Paris Conservatoire, to denote those chords which are acceptable (i.e. "classified") under traditional tonal theory.

[13] "The concept and nature of modality seems a more productive way of approaching Milhaud´s music which is broadly modal rather than narrowly tonal" (Mawer, 1997, p. 18); "The true basis of his music is not tonality but modality" (Drake, 1989, p. 201).

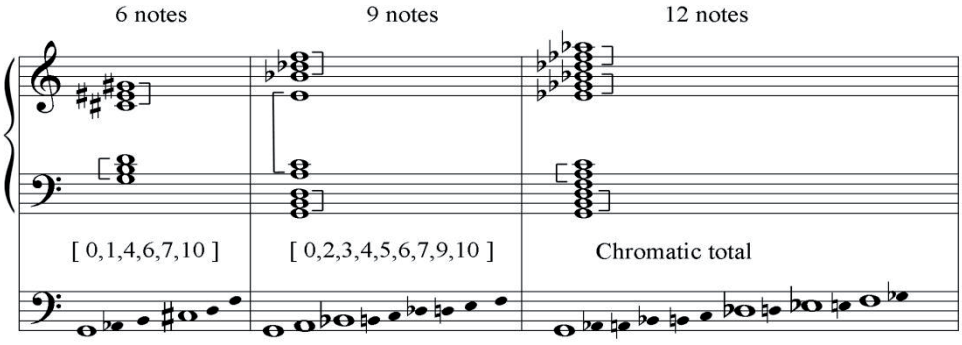
[14] As commented by Médicis (2005a, p. 99): "Dans l'article de Milhaud les tableaux traduisent une forme d'ivresse purement statistique qui semble gagner le compositeur devant l'abondance des ressources expressives qu´offre la polytonalité."

diminish its importance as an analytical tool, giving relevant insights for understanding Milhaud´s own "poiesis"[15], illuminating aspects not only of his own compositional approach, but also of some of his contemporaries, all belonging, in the first decades of the twentieth century, to a same "Zeitgeist".

SECTION I – THE "IMMANENT LEVEL": *POLYTONALITÉ HARMONIQUE* VIA TRIADIC SUPERPOSITION

Milhaud´s method for obtaining a new array of harmonic combinations (*Accords)*, for which we will be using the generic denomination "polyharmony"[16], is thoroughly explained in the section of his "Polytonalité et Atonalité*"* article[17], dedicated to the "P.H." system[18]. Through an exhaustive exploration of all possible cases of triadic superposition along the chromatic scale, he identifies a significant set of harmonic aggregates, which will often extrapolate the limits of the diatonic framework:

Example 2 – non-diatonic triadic superpositions



Obs.: whole notes indicating the triads fundamentals.

Looking from another perspective, **Milhaud´s *Polytonalité Harmonique* "system" can also be seen as a particular method for generating unordered pc sets** (with cardinal numbers ranging from 4 to 12) presenting in common two remarkable properties: their "mode of construction", consisting exclusively of the addition of transpositionally/inversionally-equivalent sub-sets; the limitation of "generative"

[15] See Médicis (2009, p. 246) "l' approche pré-compositionnelle qui sous-tendles oeuvres polyharmoniques de Milhaud".
[16] The term "polytriad" would better described Milhaud´s specific system, but would restrain its potential for generalization, if applied to other *accords classés.*
[17] The article has already been the object of some remarkable and comprehensive studies such as those by François de Médicis (2005), Deborah Mauwer (1997), and Barbara Kelly (2003).
[18] According to Milhaud: "Nous pouvons imaginer à l'infini l'étude des superpositions tonales qui doivent faire l'objet d'un supplément pour les traités d'harmonie (textbooks) [...] Les contrepoints d'accords se combinent à l'infini [...] On voit, par les différentes étapes qui s'echelonnent de la bitonalité jusqu'au maniement des 12 tons à la fois, combien les ressources de la polytonalité sont vastes et combien les possibilités d'expression sont étendues".

sub-sets (and thus of TC operations) to only two trichords [0, 4, 7] and [0, 3, 7] i.e. the major and minor triads[19],to the exclusion of other possible *accords classés*. Though foreseeing "infinite" possibilities with respect to the simultaneous combination of many "keys" (i.e. two or more triads with "roots/*fondamentales*" on different degrees of the chromatic scale), Milhaud seems to circumscribe the appropriate realm of triadic superposition[20] to what he denominates the "2 and 3 keys" cases[21], which could be generalized as follows:

Table 1 – Generalization of "2 and 3 Keys P.H." as transpositional combinations[22]

manières	**"2 - Keys"**	**"3 - Keys"**
"a": Major triads:	To [0, 4, 7] * Tn [0, 4, 7]	To [0, 4, 7] * Tn [0, 4, 7] * Tm [0, 4, 7]
"b": Minor triads:	To [0, 3, 7] * Tn [0, 3, 7]	To [0, 3, 7] * Tn [0, 3, 7] * Tm [0, 3, 7]
"c": Major / Minor triads	To [0, 4, 7] * Tn [0, 3, 7]	To [0, 4, 7] * Tn [0, 4, 7] * Tm [0, 3, 7] To [0, 4, 7] * Tn [0, 3, 7] * Tm [0, 4, 7] To [0, 3, 7] * Tn [0, 4, 7] * Tm [0, 4, 7]
"d": Minor / Major triads	To [1, 3, 7] * Tn [1, 4, 7]	To [0, 3, 7] * Tn [0, 3, 7] * Tm [0, 4, 7] To [0, 3, 7] * Tn [0, 4, 7] * Tm [0, 3, 7] To [0, 4, 7] * Tn [0, 3, 7] * Tm [0, 3, 7]

n=1 to 6 m= 1 to 6, but m=/n

A) <u>"2-Keys *Accords*"/scales: diatonic, minor, and octatonic collections</u>

We reproduce Milhaud´s well known exercise showing, for each chromatic degree, all the possible superpositions between two major triads, for which he gives the following directives:

> One ought to study very methodically the different harmonic combinations which result from the superposition of two keys, to study the inversions of the chords thus obtained, as well as the different progression which can be established between them (This

[19] As Milhaud postulates his system exclusively on perfect triads, *Polytonalité harmonique* could also be summarized as a system based on all possible combinations of the two Tn-type set classes [0, 4, 7] and [0, 3, 7].

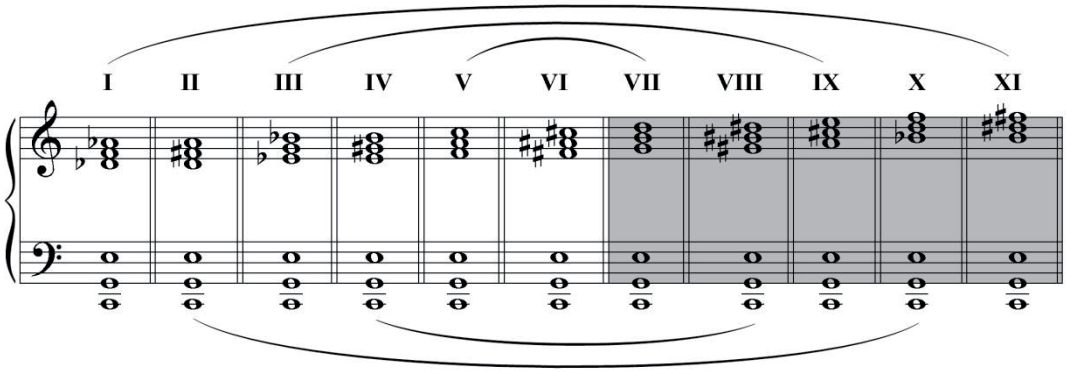
20 The present text limits itself to triadic polyharmony: for *polyharmonies non-triadiques*, see Médicis (2009, p. 261).

[21] Though acknowledging that 12 pc collections can also obtained with Major and Minor chords superposed over 4 or more "keys" (which he has also used), Milhaud seems to show a bias, in the case of superpositions > "3 keys", in favor of "quartal" harmony; while for the denser simultaneities of 4 or more "keys", he tends to assign them to the field of "Contrapuntal Polytonality", rather than that of "Harmonic Polytonality".

[22] See Cohn (1988), Forte (1973, p. 6-8), and Straus (2005, p. 38-44).

is multipliable by the fact that there are different ways of expressing two keys simultaneously: major/ major, minor/ minor, major/ minor, and minor/major.

Example 3 – Milhaud´s 1923 tabulation of the "bitonal" ("2 Keys" *Accords*) case[23]

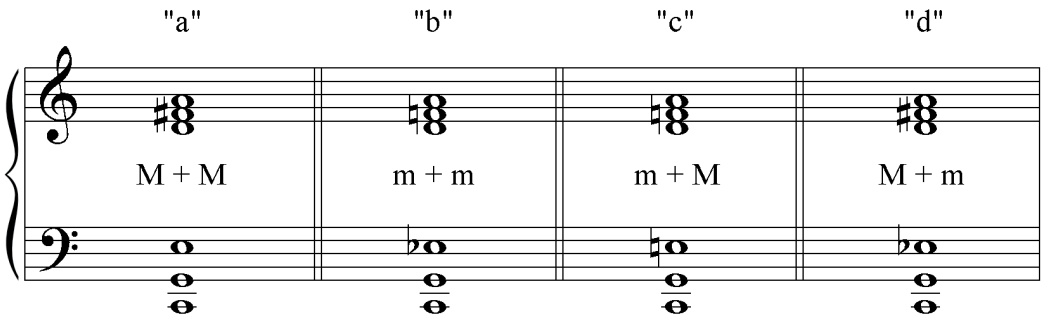


Obs.: the grey area is added to Milhaud´s original example, and corresponds to chord rotations (*renversements*).

From this exercise, he derives 11 *Accords* which "can be modally expressed" in 4 triadic combinations: "MM" (major + major), "mm" (minor + minor), "Mm" (major + minor), and "mM" (minor + major), which he denominates *manières* "a", "b", "c" and "d". Therefore, Milhaud provides a taxonomy for the identification of each "polyharmony" according to the transposition level ("I to XI") measured in semitones, and the "mode" ("a" to "b"): thus *Accord* II-"a" is the superposition of two major triads at the transpositional distance of two semitones, while *Accord* I-"b" will correspond to two minor triads at the distance of one semitone.

[23] Extracted from Milhaud´s article "Polytonalité et Atonalité" (1923).

Example 4 – *Accords´* "modal" qualifications (*maniéres*: "a", "b", "c", and "d")



From the 11 transpositional levels and the 4 possible combinations between Major and Minor triads, Milhaud estimates the existence of 44 "bitonal" *Accords*, overlooking the fact that those *Accords* built on basis of the same interval classes (V and VII, IV and VIII, III and IX, II and X and XI) actually consist of (transposed) *renversements*, and **are themselves transpositionally equivalent sets**.[24] Therefore, instead of 44, the (corrected) number of collections obtainable through Milhaud´s system add to 23, distributed as follows:

[24] Deborah Mauwer (1997) had already pointed out the fact that Milhaud´s "chord" I, II, III, IV, V, VI and "chords" XI, X, IX, VIII, VII are equivalent sets, and identified, in Allen Forte´s classification their corresponding prime sets. This has likewise been shown by Frank Jedrzejewski (2011).

Table 2a – Accords/sets obtainable via the "2-Keys"
***Polytonalité Harmonique* method**

Milhaud´s *Accords* **Forte´s supersets** **Pr. Forms**	*manière* **"a"** **M + M** **7-35 7-32 8-28**	*manière* **"b"** **m + m** **7-35 7-32 8-28**	*manière* **"c"** **M + m** **7-35 7-32 8-28**	*manière* **"d"** **m + M** **7-35 7-32 8-28 6-20**	**TOTAL** **7-35 7-32 8-28 6-20 T**
I (XI)	[6]	[6]***	[5]	[6]	1 2 1 - 4
II (X)	[6]	[6]	[6]	[6]	3 1 - - 4
III IX)	[5]*	[5]	[6]	[4]**	1 - 3 - 4
IV (VIII)	[5]***	[5]*	[4]	[6]	1 1 1 1 4
V (VII)	[5]	[5]**	[5]	[5]	3 1 - - 4
VI	[6]	[6]	[6]	[6]	- 1 4 - 4
TOTAL	2 2 2	2 1 3	2 2 2	3 1 1 1	9 5 9 1 24

Obs: a) "*" compatible with 6-20, "**" compatible with 7-32, "***" harmonic major; b) brackets indicate the (sub-) sets cardinal numbers.

On Table 2a, in which the total of 24 outcomes are obtained through the superposition, over interval classes 1 to 6 (*Accords* I to VI), between all perfect major and all perfect minor chords (*manières* "a" to "d"), it can be seen that:

a) **the number of polyharmonies resulting from "bitonal" triadic superpositions is significantly smaller than originally imagined by Milhaud**[25]: the 24 outcomes (instead of the evaluation of 44, in the "*P&A*" article) actually correspond to 23 sets/ *Accords* (*Accord* VI-"c" and *Accord* VI-"d" being identical), to which could be added the "polymodal" combination, at the same transposition level, of the "major / minor chord",which was not considered by Milhaud in his exercise[26], and for which François de Médicis (2005, p. 100) has suggested the denomination *Accord* "0", adopted since in the literature.

[25] Moreover, the small degree of differentiation between many of those sets, and/or their belonging to a common "super-set", often imply similarity relations (within – but also between – the referential collections, which share a number of identical adjacencies, an example being the "Dorian" tetracord common to the three) which reduce the relevance of individualizing their identity.
[26] See Corrêa do Lago (2002, p. 55).

b) when linearized, the majority of those 24 polyharmonies (the 23 bitonal *Accords* + *Accord* "0") fit as subsets, three major "modal"[27] referential collections: the diatonic and "harmonic minor" heptads and the octatonic, while, with one exception[28] (and abstracting from considerations related to centricity), **3 prime forms are sufficient to encompass (as supersets) the quasi-totality of "bitonal" combinations obtainable through Milhaud´s *Polytonalité Harmonique* method** (see Table 2b): Forte´s 7-35 [0, 1, 3, 5, 6, 8, 10], the diatonic referential collection containing the Major mode and its rotations (hence the pitch content of "Greek/Church Modes"); Forte´s 7-32 [0, 1, 3, 4, 6, 8, 9], the referential collection containing the "Harmonic minor" mode, as well as (its inversion) the "Harmonic major" mode[29], with 5 outcomes, to which should be added the "polymodal" *Accord* "0" [0,3,4,7]; Forte´s 8-28 [0, 3, 4, 6, 7, 9, 10], the octatonic referential collection, with 9 outcomes(through *Accords* III, IV and VI), concentrates most of "bitonality"'s additions to the traditional harmonic vocabulary; with one outcome, Milhaud´s *Accord* IV-"d" = Forte´s ("quasi-octatonic")[30] 6:20[31] [0,3,4,7,8,11] is the only exception of "bitonal"´ *Accord* not belonging to the 3 modal collections[32]. If, from the three major supersets, the focus is narrowed to those specific pitch-class sets which are generated in Milhaud´s bi-triadic exercise, the following prime forms (15 from the 23 *Accords*) will be obtained:

[27] Milhaud´s "P.H. system", such as proposed by in his "Polytonalité et Atonalité" article, does not contemplate diminished and augmented triads: thus, the chordal/scalar results correspond to a subset of the sets/scales obtained by triadic superimposition, mapped and identified by Tymoczko (1997).
[28] Milhaud´s *Accord* IV-"d" = Forte´s hexachord 6:20 (see note 31).
[29] This prime set includes as well, in the cases of *Accords* I-b and IV-a (IV-a being included in I-b), subsets of the "harmonic major" heptad.
[30] Milhaud´s *Accord* IV-"d" /Forte´s hexachord 6:20, being also a superset of the "octatonic subsets" *Accords* III-"a", IV-"b", and of *Accord* "0", corresponds to a "*mode à transposition limitée*" not inventoried by Messiaen. This condition (of limited transposition), and its frequent use in post-tonal music, is expressed as follows by Allen Forte (1978, p. 192-194): "to many students of early twentieth-century music, hexachord 6:20, which has only four distinct pitch class forms, will be familiar from the works of many composers".
[31] *Accord* IV-"d" / hexachord 6:20 is denominated "augmented" and/or "symmetrical-augmented", in Tymoczko´s identification of the 7 scales obeying the "two-consecutive semitones constraint".
[32] The many intersections between those collections illustrate Tymoczko´s observation that "we should not be surprised to find [such scales] serving as harmonic points of convergence for composers working in very different musical idioms".

Table 2b – correspondences between prime forms and Milhaud´s "bi-tonal *Accords*

Accords	"Masters"			
	"a" (MM)	"b" (mm)	"c" (Mm)	"d" (mM)
I	6z19	6z19	5:22	6x26
II	6:33	6:33	6:33	6:33
III	5:32	5:32	6z28	4:26
IV	5:21	5:21	4:20	6:20
V	5:27	5:27	5z17	5:34
VI	6:30	6:30	6z50	

On Table 2b all *Accords* are presented in both their vertical and linear forms, and expressed as unordered pc sets (with cardinal numbers 4 to 6) associated to a corresponding (superset) "modal" collection:

Table 2c – Milhaud's (bi-triadic) "2-Keys *Accords*": chordal and scalar formats

B) <u>The "3-Keys *Accords*"/scales: a rich modal spectrum</u>

In a subsequent section of his "Polytonalité et Atonalité" article (hereafter "P&A") Milhaud presents the tabulation in which the same *travail théorique* is applied to what he denominates the "3-Keys" case, attempting again to chart exhaustively all possible combinations between 3 roots of major and minor triads, superposed over each degree of the chromatic scale. In this new context, with the addition of a 3rd triad ("key"), the modal combinations increase accordingly from 4 to 8 ("MMM", "mmm", "MMm", "MmM", "mMM", "mmM", "mMm", "Mmm"), yielding 55 "*Accords* expressed through 8 *manières* and thus resulting, according to Milhaud, into 440 combinations which could be further enriched through chordal inversion.

Table 3 – Milhaud´s 1923 tabulation of "3 Keys" *Accords* [33]

Obs: the grey area is added to Milhaud´s original example, and corresponds to chord rotations (*renversements*).

Analogously to what was shown in the "2 keys" case, the grey area on the "3 keys" (Table 3) indicates (transposed) chordal rotation of those "chords" which appear in the white area: thus, by correcting the "double-counting" of those chordal inversions which are transpositionaly equivalent sets, the total number of "new" combinations in the 8 "modes" falls from 440 to 152, while the combinations between triadic roots fall from 55 to 19.

[33] Extracted from Milhaud´s article "Polytonalité et Atonalité" (1923).

Table 4 – *Accords/sets obtainable via "3-Keys" *Polytonalité Harmonique*****

Accords	*manières*"a" + "a"	*manières*"b" + "b"	*manières*"a" + "c"	*manières*"c" + "b"	*manières*"b"+"d"	manières"d" + "a"	*manières*"c" + "d"	*manières* "d" + "c"	Total
	(M+M+M)	(m+m+m)	(M+M+m)	(M+m+m)	(M+m+M)	(M+M+M)	(M+m+M)	(M+M+m)	
9 notes	1	1	1	1	3	3	1	1	12
8 notes	7	7	5	7	5	5	8	6	50
7 notes	9	9	8	6	7	7	5	7	58
6 notes	2	2	5	5	4	4	4	4	30
5 notes	0	0	0	0	0	0	1	1	2
	19	19	19	19	19	19	19	19	152

* number of outcomes (not including "0" *Accords*)
** duplications (by homonimy) not deducted

In addition to the fact that 152 outcomes[34] already corresponds to a reduction by 2/3^{rds} of the total number of *Accords* originally estimated by Milhaud (440)[35], this significantly lower figure still represents an overestimation of the actual number of different sets generated by the "3-keys"operation.[36] Besides the fact that both "similarity" and "inclusion" relations[37] would make irrelevant the effort of differentiation between some sets, the "3-keys" case also presents numerous (non–univocous) situations of "homonimy", i.e., of a same scale being obtainable through different operations of triadic superposition (different *Accords* and *manières* over different roots), for instance:

a) the major mode is obtainable both through the operations: "M+M+M" over roots (0, 5, 7), as *Accord* V-"a" + *Accord* II-"a", or M+m+m over roots (0, 2, 4), as *Accord* II-"c" + *Accord* II-"a";

b) the octatonic, through the superpositions "M+M+m" or "M+m+m" over roots (0, 3, 6), etc...

c) Forte´s 6:20: will occur 8 times on the 3-triadic case (all superpositions of *Accords* IV over the 4 *manières*), with same pc content of bi-triadic *Accord* IV-"d".

Though the Milhaud *Accords* – if grouped according to their prime forms – will actually occupy only a fraction (about 1/3rd) from the total space of Forte´s 220 pc sets, it should be noted, however, that **their linearization over 3 roots, allows a substantial enrichment of the modal *palette*,** with the incorporation of new "scales" (from 6 to 9 pitch classes), in addition to those previously obtained in the "two-keys" case.

C) Compatibility between "3-Keys" triadic superpositions and oriental scales:

Examining specifically the case of seven note scales, it can be noted that three triads superpositions – in addition to (in terms of pitch content) all "Greek/Church modes", and of the two forms of the Minor mode ("melodic" and "harmonic") – will also generate sets corresponding to scales usually associated with the periphery of Western musical traditions (such as the so–called "Gypsy" and "Andaluzan") and very particularly to a large number of Indian "ragas", among which the "Melakarta" scale types[38], referred to in the French musicology of the early twentieth century as the "Carnatic modes"[39], and which became so much

[34] To which should be added those (12) chordal combinations, containing Médici´s "0" *Accord*s, not considered in Milhaud´s "3-keys" tabulation.
[35] See Médici´s observation (2005, p. 99): "Dans l'article de Milhaud, **les tableaux traduisent une forme d'ivresse purement statistique** qui semble gagner le compositeur devant l'abondance des ressources expressives qu´offre la polytonalité".
[36] Milhaud´s "3-keys" (3 superposed triads) exercise increases from 12 to 38 the number of *Accords/*set-classes presenting "TC" properties.
[37] In the sense given by Allen Forte (1973, p. 25; 52-3).
[38] See Powers (1980, p. 97).
[39] In the 1920s, the "state of the art" of Western knowledge with respect to extra-European traditions was epitomized by

part of the idiom of composers between the generations of Albert Roussel and that of Olivier Messiaen.

The *modes carnatiques* are the result of the combination between 12 core tetrachords ("Chacrams"), 5 of which exactly coincide with those familiar tetrachords, in Western music, from which all diatonic and minor modes[40] are formed; they present thus a higher degree of generality for describing modal heptads characterized by an invariant interval of "perfect 5th" between the Ist and Vth degrees. The resulting 72 modes (which do not exhaust all the possible combinations between those tetrachords) are subdivided into two classes of 36 modes each:

> - the first ("Çuddala-madhyama") constructed exclusively on basis of two perfect-4th tetrachords, where [0,5,7] are invariant over degrees I, IV and V; and of which most of the Greek/Church modes are a subset.
> - the second, ("Prati-madhyama"),differing from the first only by the raised IVth degree, consists therefore of one tritone tetrachord + one perfect-4th tetrachord, where [0,6,7] are invariant on degrees I, IV, and V.

In fact, the majority of the *modes carnatiques* of both "Çuddala" (all, 1st "Chacram" excepted) and "Prati" classes, are implied by the "3-keys" operation, as are also present a number of ("non-carnatic") heptads not fitting the "two-consecutive semitone constraint", identified by Dmitry Timoczko, and containing either two augmented seconds, or two adjacent semitones, or their combinations: two-consecutive semitones+ one augmented second [prime forms 7-26, 7-27, 7-28, 7-29, 7-30]; and two-consecutive semitones+ two augmented seconds [prime forms 7-21, 7-22].[41] This is not to suggest that Milhaud would have thought of using Indian modes (as did Roussel and Messiaen), but that the linearization of a large number of "3 Keys" chords, shows a relation of homonimy with such modes, as happens, for instance, in the *Choephores* [42]: modes Vagadeçvar (Lavignac number 34, formed by tetrachords / "chacrams" n. 6 and n. 4), Kokilaprya (Lavignac n. 11, formed by" chacrams" n. 2 and

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the highly influential *Encyclopédie Lavignac´*s inventory of extra-european musical "systems" (Chinese, Hindu, Arab, etc): the 1907 essay by the French indianist Joanny Grosset, in *Encyclopédie Lavignac* (1913, p. 257-375), has been a major source for generations of French composers from Messiaen´s theorization – in both his *Technique de mon langage musical* (1944), and his *Traité du rythme, des couleurs et d´ornithologie* (1949-1992[1994-2002]) – to the *Soixante douze* études *karnatiques* (composed in 1957-1984) of his student Jacques Charpentier.

[40] Corresponding to our familiar diatonic tetrachords from which are formed all the church modes as well as the descending and ascending forms of the melodic minor (Dorian, Phrygian, Hypo-Lydian and Lydian, corresponding respectively to Chakrams 4, 2 and 3 of the first category, and 4 of the second), while the upper tetrachord of the harmonic minor corresponds to Chakram 3 of the second category.

[41] Among "non-carnatic" modes: scales with two-consecutive semitones + two augmented seconds [7- Z17]; 3 consecutive semitones [7-23, 7-25, 7-Z18), two-consecutive semitones + one minor third [7-Z37].

[42] In a way as fortuitous (not being part of Debussy´s poiesis) as in Messiaen´s analysis (2001, p. 64) of *La Mer,* the identification of the "second cyclic theme" in the first movement (rehearsal marking "4") as the Indian mode "Vachaspati" (n. 64, in Lavignac´s listing of Carnatic modes).

n. 5), Sarasangi (Lavignac n. 27, formed by "chacrams" n. 2 and n. 5) "Vociférations fúnebres" section; and mode Charukali (Lavignac n. 26, formed by"chacrams" n. 5 and n. 2) in the "Incantation" section.[43]

Timoczko´s observation with respect to Stravinsky that "scales are sometimes surface phenomena produced by underlying superpositions, that do not conform to any single collection"[44], seems to apply particularly well to Milhaud: though – through this mutual imbrication – the "immanent level" in many textures of his works can be described in terms of "inventoried" modes, those might be misleading, in Milhaud´s case, if used as a basis for "inductive *poiesis*".

[43] Corresponding, respectively, to rehearsal markings: D +5-11, F + 7-10, J + 1-6, and R.

[44] Tymoczko makes the point that "focusing on scales may sometimes hinder the real musical understanding", and with respect to the *Petrushka* and "Augures Printaniers" chords, "Stravinsky was not likely to have been thinking in scalar terms".

Table 5a – Correspondences: hexachordal prime forms/Milhaud´s "3-triadic" *Accords*

	Milhaud´s *Accords* / "manières"							
PC sets	**MMM (aa)**	**Mmm (bb)**	**MMm (cc)**	**mmM (bd)**	**mMM (da)**	**Mmm (cb)**	**MmM (cd)**	**mMm (dc)**
Occurences								
6 : 14 - 1				IV + III				
6 Z 19 - 4			I + IV IV + I					I + IV IV + 1
6 : 20 - 8	IV + IV	IV + IV	IV + IV	IV + IV	IV + IV	IV + IV	IV + IV	IV + IV
6 Z 25 - 1				II + III				
6 Z 26 – 2					IV + I	I + IV		
6 : 31 - 2	IV + III	III+ IV						
6 : 32 - 2					II + III	III + II		
6 : 33 - 1							III + II	
6 Z 44 - 4			III + I		I + III	IV + I		I + III
6 Z 46 - 2			III + IV					
6 Z 50 – 2				III + III			III + III	

Table 5b – Correspondences: heptadic prime forms/Milhaud´s "3-triadic" *Accords*

Milhaud´s *Accords* / "manières"								
Pc sets/ Occurences	**MMM (aa)**	**Mmm (bb)**	**MMm (cc)**	**mmM (bd)**	**mMM (da)**	**Mmm (cb)**	**MmM (cd)**	**mMm (dc)**
7:19 -1								II + IV
7:21-10	I + IV III + I	I + III V + I		I + III III + IV	III + I IV + III		I + IV	IV + I
7:22- 4	I + III	III + I	I + III			III + I		
7:23 -2	II + III	III + II						
7: 5 -2 7:26 -2			IV + III	III + II	II + III	III + IV		
7:27 -2 7:28 -3 7:29 -3	III + II	II + III	III + II VI + I II + IV			II + III IV + II		V + I
7:30- 2 7:31 -4	III + III	III + III	II + II	IV + I			III + III	III + III
7:32 -7				I + II V + II	II + I	V + II	I + II V + II	II + I
7:34 - 2 7:6 -6	V + II	V + II	V + II	II + I	I + II	II + II V + II		V + II
7Z17 -2 7Z18- 2	III + IV	IV + III	V + I					VI + I
7Z37-2	IV + I	I + IV						

It should be noted, as a specificity of this "sub-space", that being built on perfect triads (characterized by the preeminence of the 5th) **their interval vectors generate a strong bias in favor of non-symmetrical scales.**

As noted by Antokoletz (1985, p. 67-68), with respect to the specific properties of "interval class 5", in the context of symmetrical and non-symmetrical pitch relations:

> In traditional tonal music, composers worked according to a system in which the octave was divided into unequal parts. The fundamental division was derived from the perfect fifth, which served as [..] the primary structural interval of both major and minor triads, [in turn. ..unequally]divided into major and minor thirds. [… By contrast, all other interval classes…] generate a cycle that subdivides one octave symmetrically. [Thus] the perfect fourth, or its harmonic inversion the perfect fifth, is unique among the intervals: unlike the others it generates a cycle that does not divide one octave symmetrically.

Although this "P.H."-derived pc set "space" does not exclude symmetrical formations – an example is the presence of the octatonic, excluding however all other Messiaen´s "Modes à transposition limitée" (1944), as well as a number of pitch formations found in Bartok´s and Messiaen´s music – its applicability will become the more limited the more the symmetrical subdivision of the octave; in a given texture, it will play a major part (chromaticism as the extreme case), and this is the case in most works of the Viennese School.

Table 6.1 – Milhaud´s "3-triads" *Accords*: chordal and scalar formats

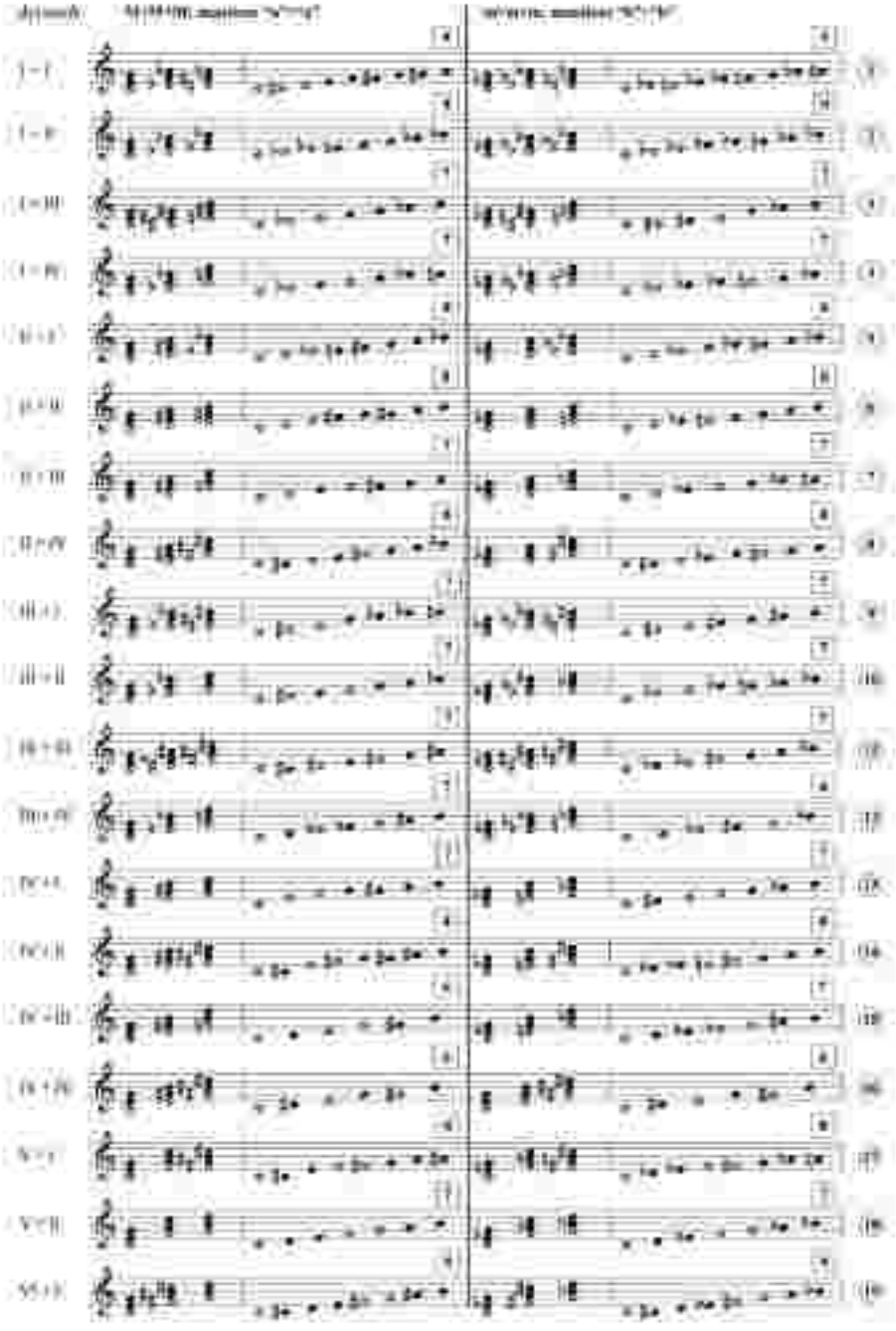


Table 6.2 – Milhaud´s "3-triads" *Accords:* chordal and scalar formats

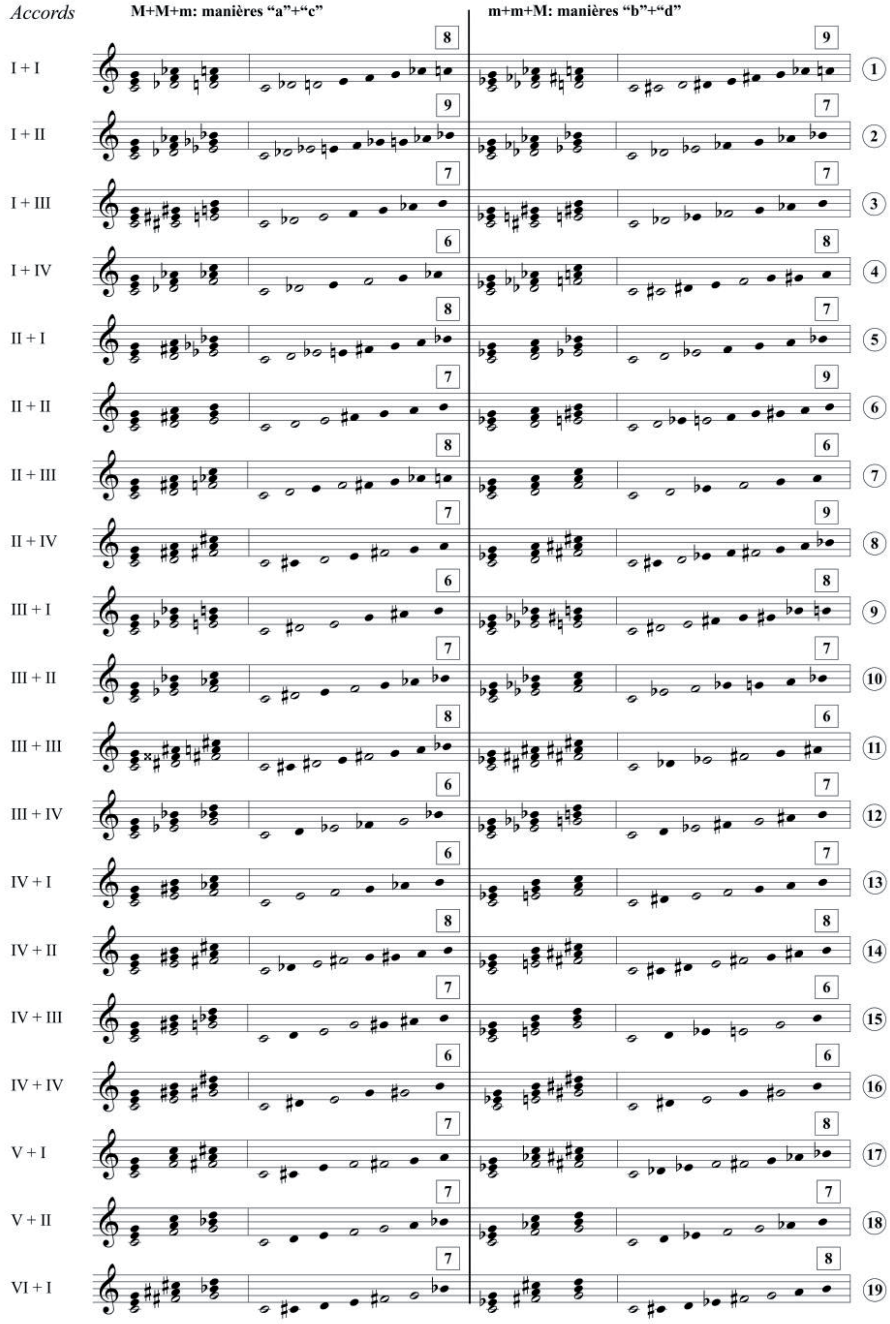


Table 6.3 – Milhaud´s "3-triads" *Accords:* chordal/scalar formats

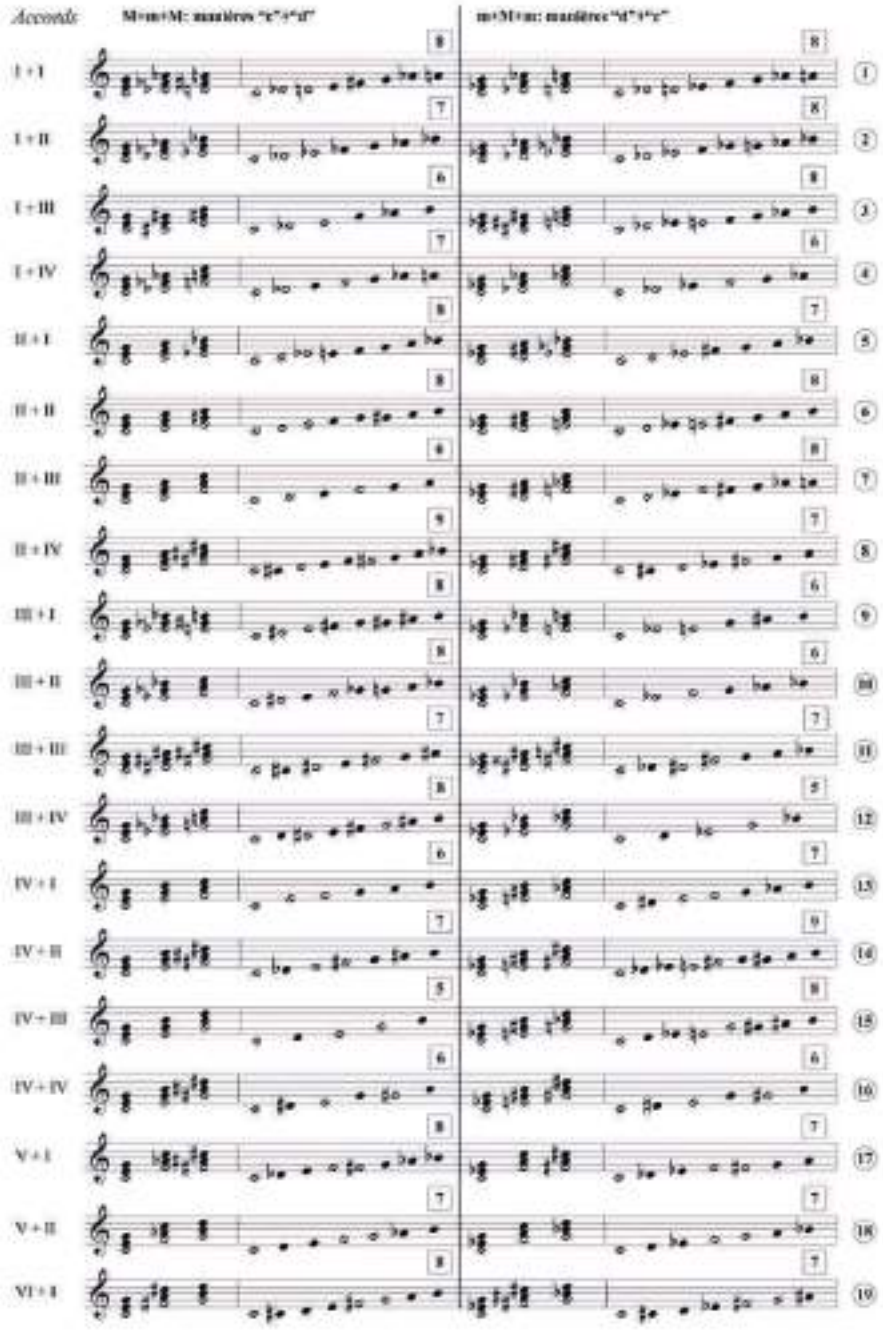
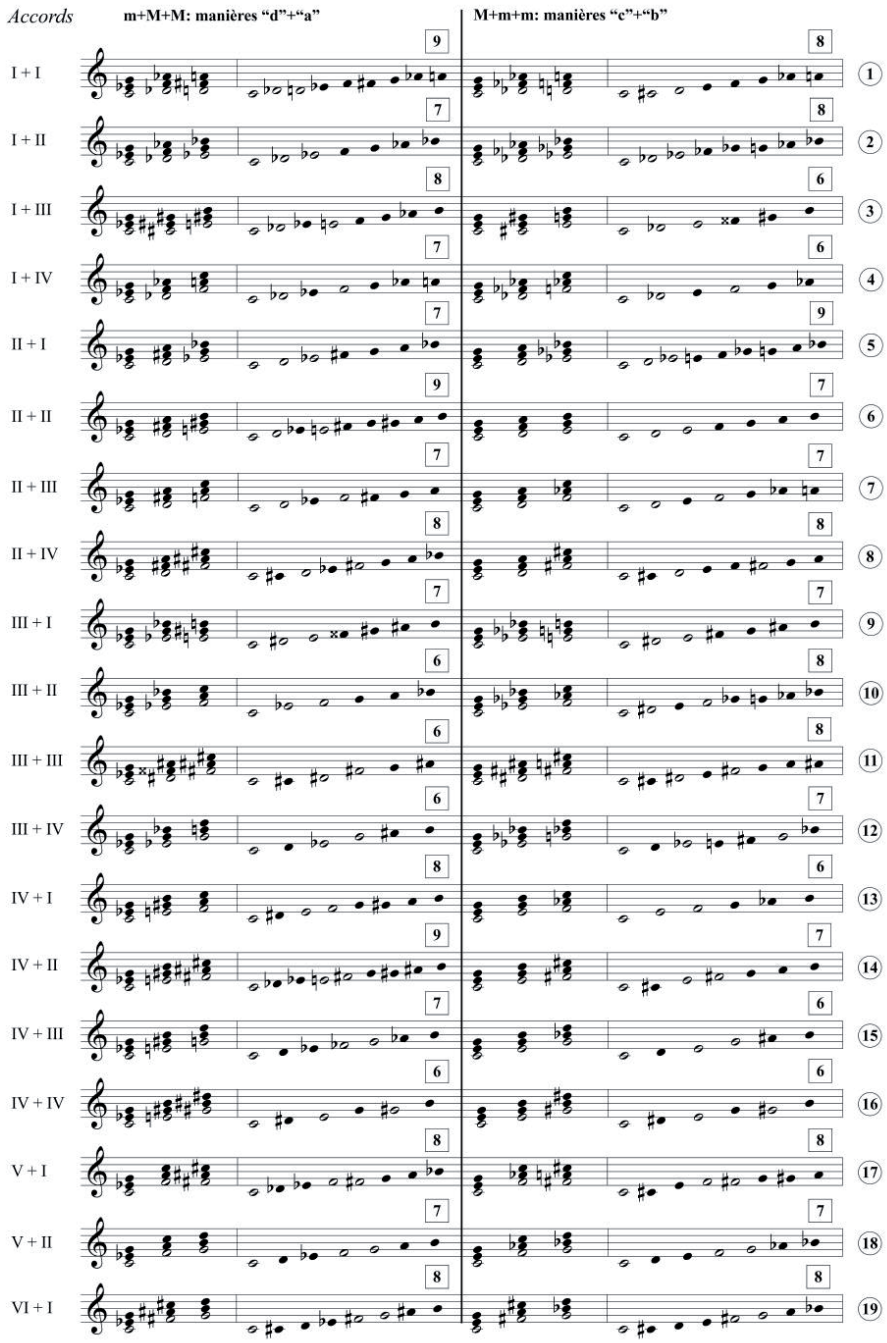


Table 6.4 – Milhaud´s "3-triads" *Accords:* chordal and scalar



SECTION II: THE POIETIC LEVEL: "*AD DISSONANTIAM PER CONSONANTIAM*"

The system of *Polytonalité Harmonique* exclusively based on triadic superposition, was of course a "simplified model", to which Milhaud did not adhere dogmatically: though perfect triads are the most frequent basis for his polyharmonies many instances of combinations between other *accords classés* (such as the "dominant seventh"), as well the use of quartal harmony are also frequently found in his work (see, for instance, *Choéphores*).

The superposition of *accords classés* became a very frequent procedure in early twentieth-century music, including important composers who did not necessarily identify themselves with the idea of Polytonality, and became part of the *lingua franca* in early twentieth-century music, whether or not composers associated the use of such aggregates to a "polytonal" approach. Thus, Milhaud´s theorization, rather than an influence was a reflection of common practice, a practice which had preceded Milhaud´s effort of theorization by a few years: in isolated experiments by the Italian Feruccio Busoni, the Polish Karol Szymanovski or the Brazilian Alberto Nepomuceno ; in a much more radical way, in the musical compositions of Charles Ives (starting with his 1890 *Variations on America*) and the works and theoretical writings of Charles Koechlin and Alfredo Casella; but specially in Stravinsky´s works along his *fase russe*, of which the *Sacre du Printemps* was to become the most emblematic and influential example.

In the discussion that follows, Milhaud´s classification system – while retaining the principle of intervallic measurement by semi-tones between triadic roots, and the "modal" qualification (*manière*s "a" for two major triads, "b" for two minor triads, "c" for a combination major/minor) – will be adapted to the 6 interval classes (*Accords* I to XI "becoming" *Accords* I to VI") and to the case of the "Major/Minor triad" case (Médicis´ "*Accord* 0") not considered in Milhaud´s text: the vast array of composers ranging from Ives and Stravinsky´s generation to that of Messiaen and Britten, give an illustration of how Milhaud´s personal system of classification of "bitonal" *Accords*, can be applied, along (though not in substitution to) other alternative analytical systems, to describe pc formations which were of an extremely frequent use during the first half of the twentieth century.

Table 7.1 – "Bitonal" *Accords* I

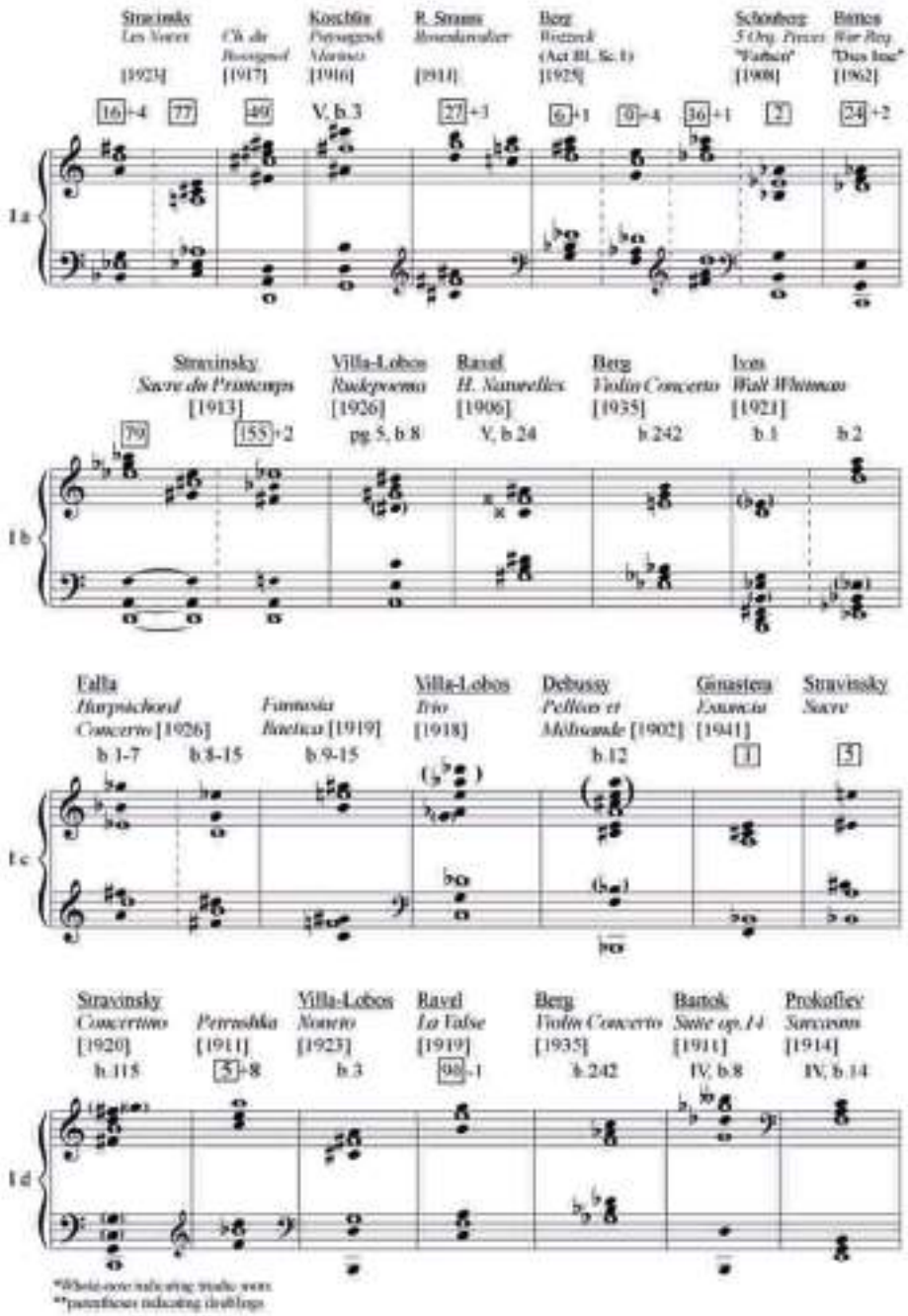


Table 7.2 – "Bitonal" *Accords* II

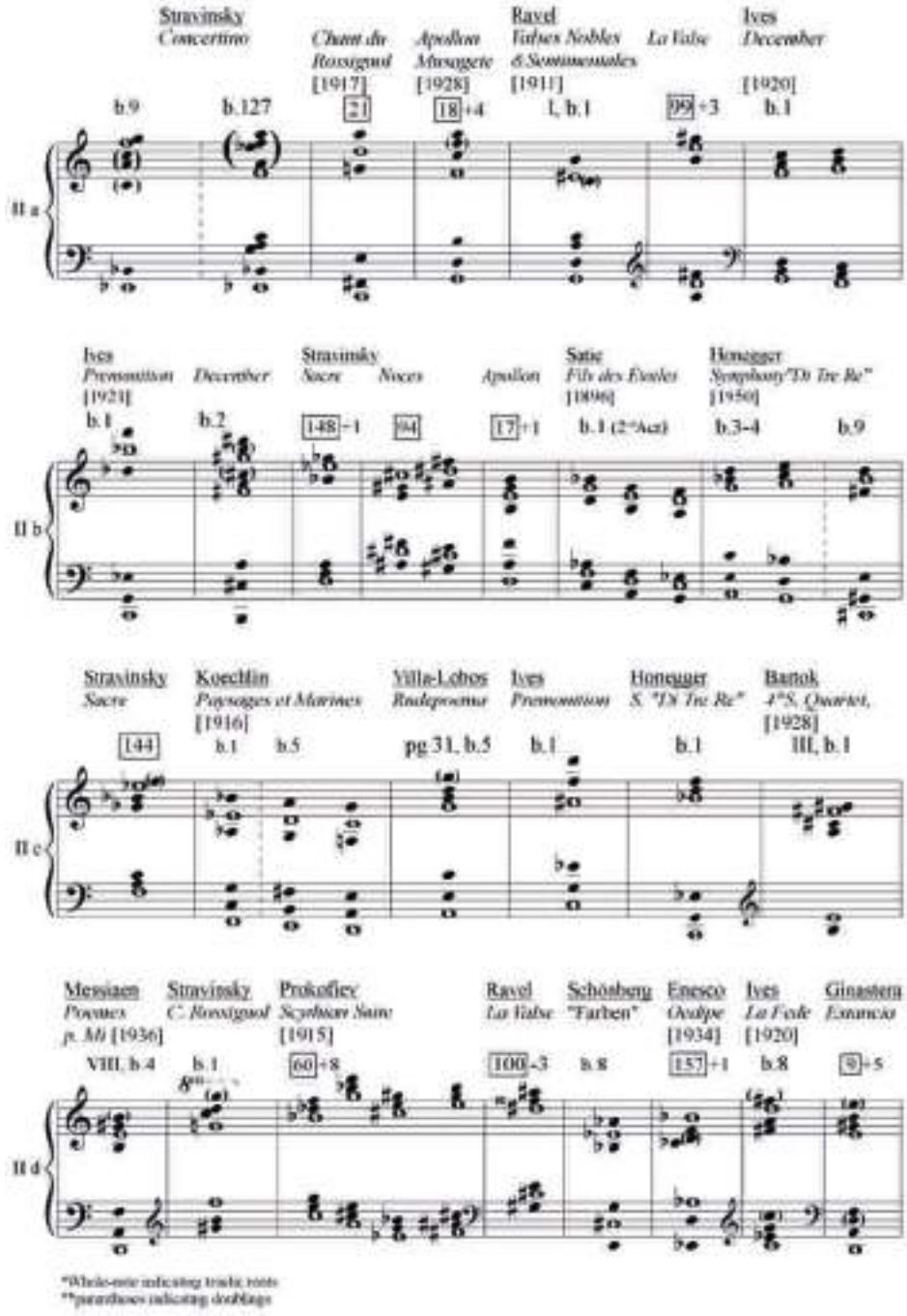


Table 7.3 – "Bitonal" *Accords* III

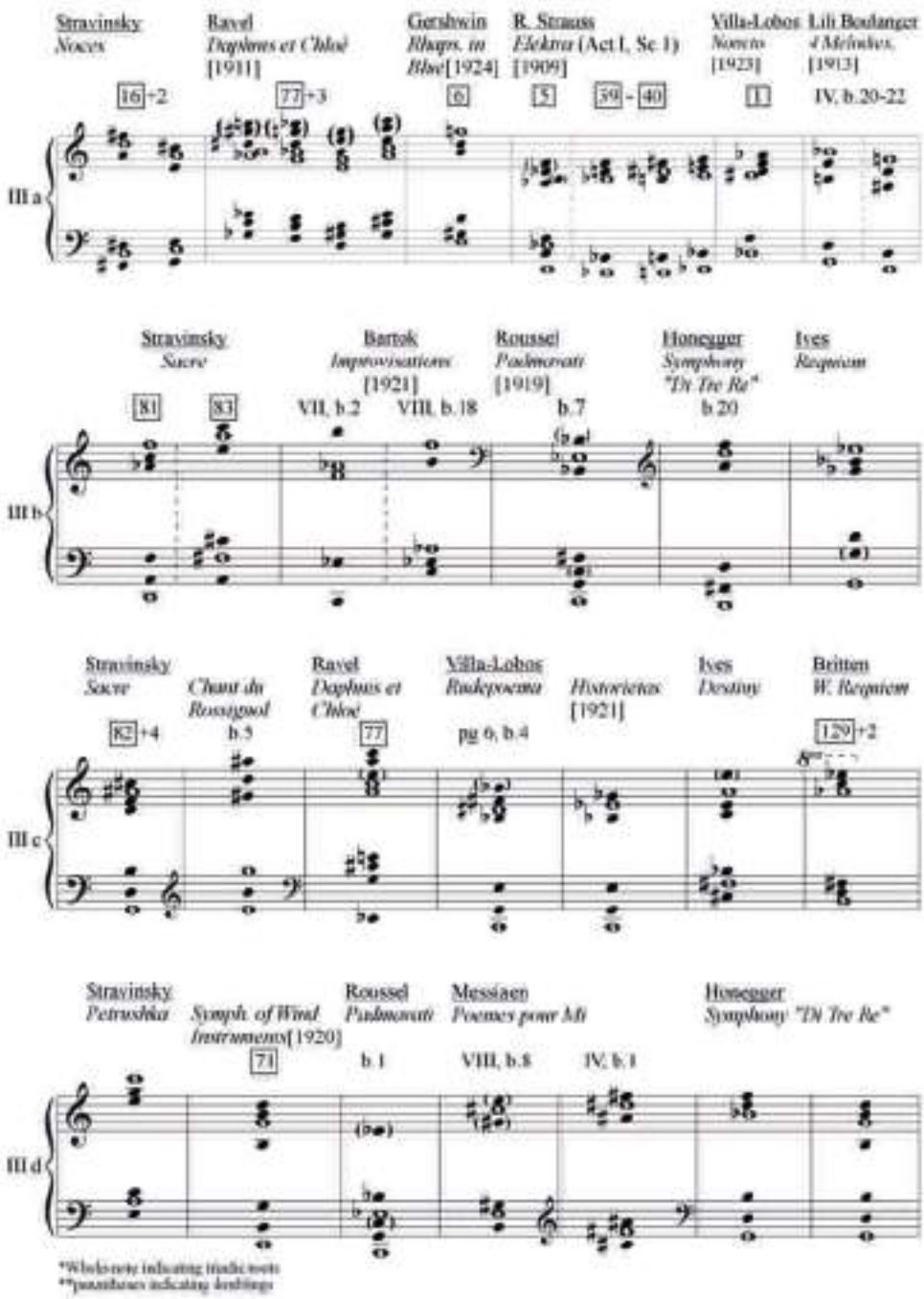


Table 7.4 – "Bitonal" *Accords* IV

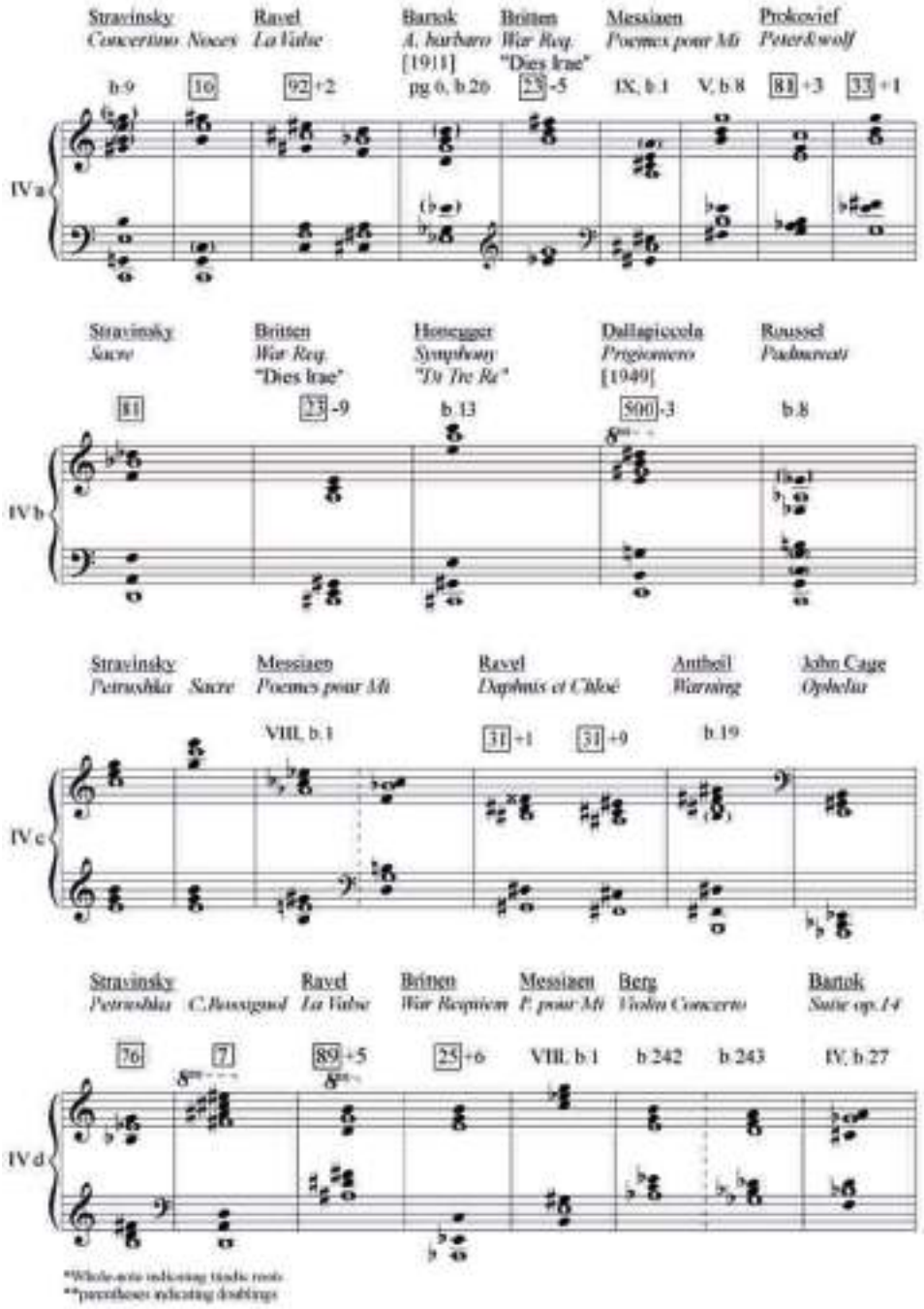
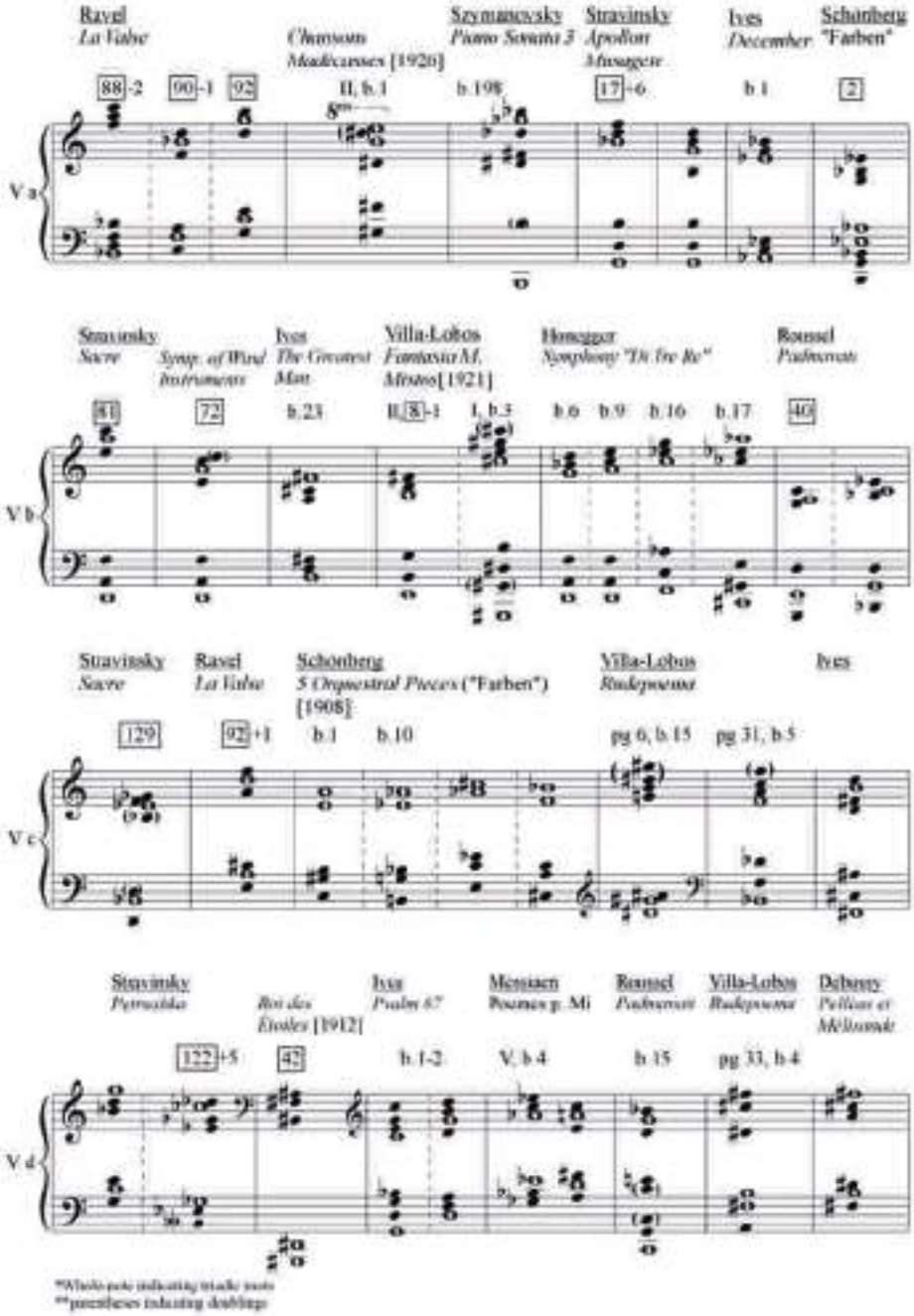
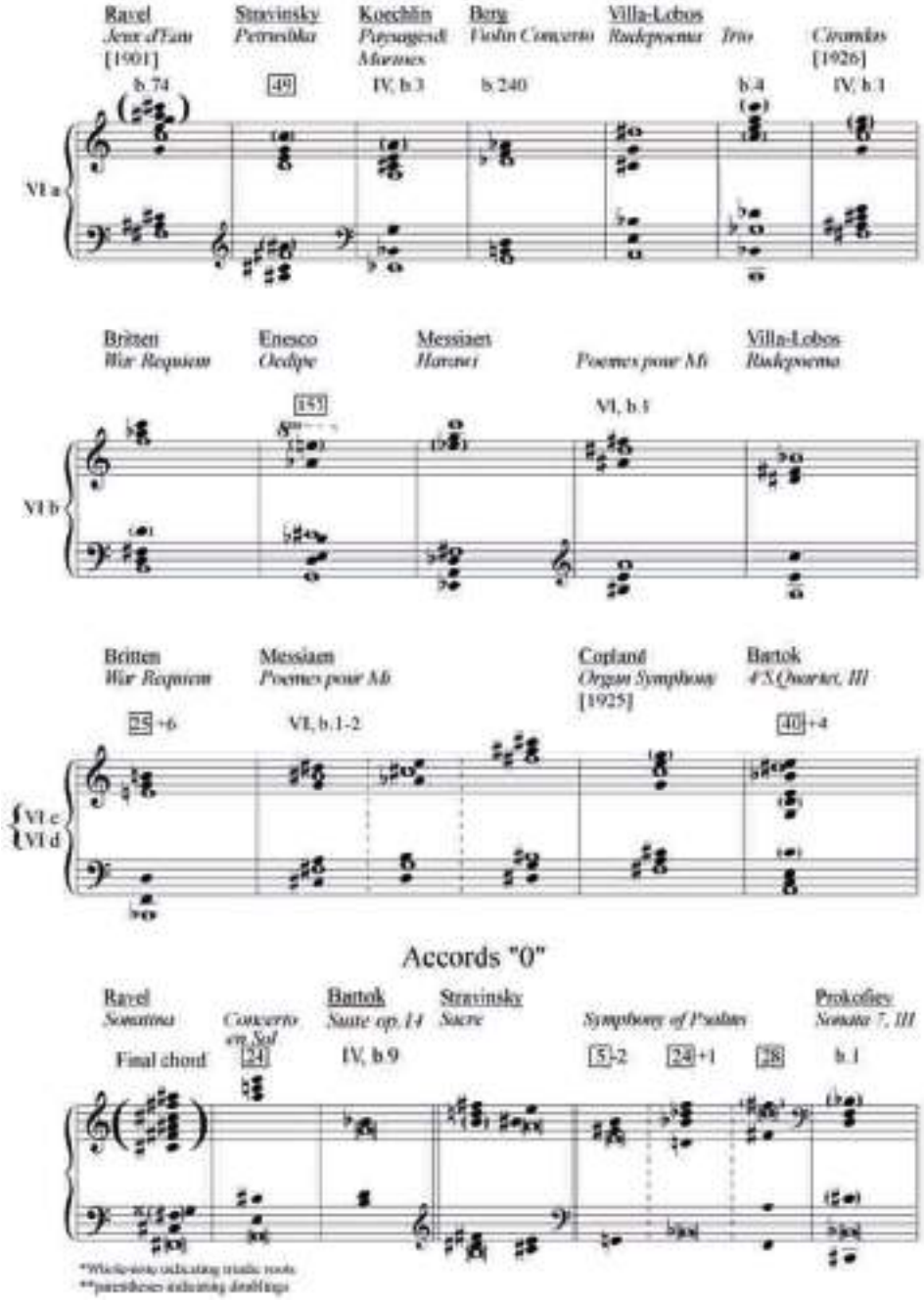


Table 7.5 – "Bitonal" *Accords* V



*Whole note indicating triadic roots
**parentheses indicating doublings

Table 7.6 – "Bitonal" *Accords* VI



It can be noted that sets reducible to the operation T0 [0, 3or4, 7] + Tn [0, 3or4, 7] were an extremely current feature in early twentieth-century music, and most of the examples presented above could be classified as "polychords", in Vincent Persichetti´s acception (for whom what defines a polychord is its segmentation into chordal units): "A clear grouping of chordal units is a requisite of polyharmony, and rearranging the tones of these units can destroy the polychordal organization".

This view, with close affinities with Milhaud´s poiesis, and that of many of his contemporaries, does not however exhaust **the "structural" virtualities of the "P.H. system" as a generator of pc sets.** In this sense, it should be noted that some of the examples above do not fit Persichetti´s conditions for characterizing as "polychords": though their pitch class content remains reducible to a combination between *accords classés*, the chord-units themselves having been "broken" through a non-triadic "rearrangement of tones". This is the case, in some of the above examples: in Ravel´s "Aoua" (C*hansons Madecasses) Accord* V-"a"; in Schoenberg´s "Farben" (*Five orchestral pieces*) initial- and predominant- *Accord* V-"c"; in Strauss´ leitmotivic chord (*Accord* III-"a") characterizing Elektra; in the Sphinx´s death scene in Enesco´s opera *Oedipe* (*Accord* VI-"b") etc.

This also illustrates the point made by Allen Forte who, while admitting the possibility of different descriptions for a same set, calls the attention to the "selection" problem whenever "a set is described in terms of a subset":

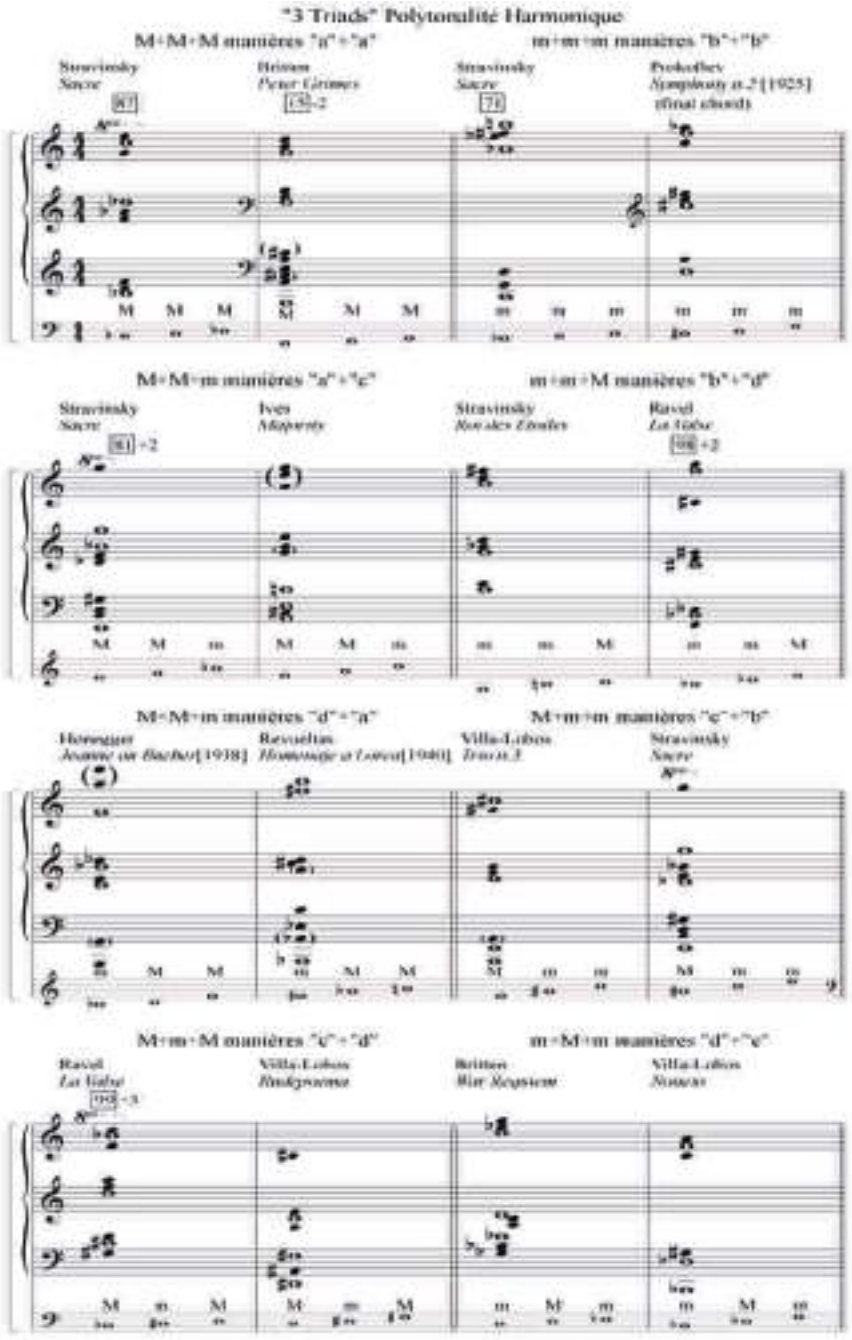
> The description [of sets] in terms of the combination of familiar triadic subsets […] is quite arbitrary as no evidence is adduced to show that those subsets are more significant than others". […] In the non-tonal repertory, the set 7-35 usually occurs in ways which do not readily associate with the familiar scale ordering […] or the traditional tonal context. Moreover, these sets occur in singular ways which tend to dissociate them from the traditional tonal context.

These considerations apply not only to cases where superpositions by 4^{ths} and 5^{ths} produce sets also obtainable through Milhaud´s triadic system triadic system – such as the beginnings of the third movement in Bartok´s *String Quartet Nº 4* or Koechlin´s "Chant du Chevrier" (*Paysages et Marines*), or the first of Ravel´s *Valses Nobles et Sentimentales* – or also when the set is compatible with unitonal harmonic traditional contexts – e.g. in Lili Boulanger´s *Dans l'immense tristesse*, where the simultaneous ascending and descending 7^{th} degree of the Melodic Minor mode coincide with "*Accord* 3-"a" – or simply cases of homonymy with *accords classes* – e.g. Major 9^{th} and *Accord* V-"d"; Minor 7^{th} and *Accord* III-"d"; Major 7^{th} and *Accord* IV-"d"). In this last case, what might "qualify" such sets as polychords in Milhaud´s (and/or Persichetti´s)

sense, is a combination of ordering and spatial distribution, in which the identity of each triadic unit is emphasized: this is the case, for instance, in *Petrushka´*s diatonic "chordal counterpoint" at the "Entrée des ivrognes" (*Accords* II-"d"; III-"d"; V-"c", and V-"d") or in Ives´ *Psalm* 67 (*Accord* V-"d").

Vincent Persichetti´s (1961) observation that "Polyharmony is seldom polytonal" – i.e., that in most cases, Polyharmony tends to occur in contexts which correspond to *polytonalité au sens large* rather than generating *polytonalité au sens strict* – applies to most of the examples above: *Accord* I-"c" which lasts along the first 24 measures of Falla´s *Harpsichord Concerto* over two transposition levels, are related closer to the tradition of Scarlattian *accaciaturas*; the frequent frictions producing *Accord* I-"a" in the *Rosenkavalier* appear in the character of *broderies*; the sonority of *Accord* I-"a" punctuating each variation of *Wozzeck´*s "Maria´s Scene", does not attempt to create concurring polarities; the "Golaud chord" (*Accord* I-"c") which Messiaen analyzes, unitonally, as a *triple broderie*, was, according to him, heard "bitonally" by Milhaud´s generation.

Table 8 – "3-Keys" *Accords* – different *manieres* combinations



The case of 3 superposed triads is also frequent, as can be seen in Table 8. Such aggregates, if we follow Milhaud´s terminology, can correspond to different combinations between *manières "a", "b", "c",* and "d": at the beginning of the *Rite´*s 2nd Section, the "3-Keys" *manière* "a*" Accord* (on rehearsal mark 87) resulting from the superposition of major-triads-only (over roots [0,2,3]) operates as a *chiaroscuro* in contrast to the *manière* "b" combinations of minor-triads-only in its first measures; the final chord of Prokofiev´s *Symphony* Nº *2* is entirely in *manière* "b" (over roots [0,1,4]) while composite combinations can be observed in the above examples, in works by composers as diverse as Villa-Lobos, Ives, Honegger, and Britten.

From those examples, it can be seen that the use (intentional or not) of "dissonant" aggregates decomposable into consonances – such as the polyharmonies which Milhaud denominated "2- and 3- Keys" *Accords* – has been an extremely frequent feature in early twentieth-century music: associated originally to composers such as Satie, Ravel, Stravinsky, Ives, Milhaud, and "neoclassicism" in general, it is present as a *procédé d´écriture* from the 1960s onwards, though in a very different spirit from the 1920s and 1930s, in works by composers such as Schnittke, Kurtag, Ligeti, Nancarrow, later Messiaen works (such as his opera *St François d'Assise),* Phillip Glass, John Adams, etc.

RECAPITULATION

Instead of embracing the vast themes which have been discussed, under the umbrella of the term "Polytonality" in the literature[45], the specific purpose of this paper has been the presentation of the "P.H. system", such as advanced by Milhaud in his 1923 article, as **a simplified model for generating chordal combinations (*Accords*, in Milhaud´s termininology) and/or unordered pc sets**.

The "P.H." vocabulary consists of a specific group of "dissonant" harmonic combinations, presenting the particularity of being decomposable into (and thus analizable as) superpositions of familiar consonances – notably the perfect triad – and for which Milhaud suggests a taxonomy based on:

a) the intervallic distance between "fundamentals";

b) the individual "mode" of each triadic unit;

c) and the number of superpositions.

With minor adaptations, Milhaud´s terminology has been found to be pertinent to a relevant part of the early twentieth-century music *repertoire*, which have been illustrated above with musical examples taken principally from works belonging to Stravinsky´s *fase russe*, but also from composers as diverse as Satie, Ravel, Roussel, Honegger, Enesco, Ives, Villa-Lobos, Copland, Britten, and Messiaen.

[45] For a wide and comprehensive review of different approaches related to Polytonality, see Noronha (1998), Pistone (2005), and Malhaire (2011; 2013).

In his seminal text "*La Polytonalité selon Darius Milhaud"* (2005a), Médicis had already called for the need of a "*théorisation flexible, ouverte à l'hybridation des approches méthodologiques*", and it has been found, along this study, that tools borrowed from both pc set and tripartition theories – as well as Richard Cohn´s notion of "Transpositional combination", and Timoczko´s demonstration of the mutual implication/imbrication between triadic superpositions and scalar/modal formation – could prove relevant auxiliaries to a better understanding and discussion of the issues raised by Milhaud.

In the case of Molino/Nattiez´s tripartition approach, it allows to isolate and dissociate questions pertaining to the "esthesic domain" – such as if polytonality can "exist" at all at the perceptive level– from those compositional decisions and strategies which are part of the "poietic" process, which may (or may not) happen to be "heard" as intended.

With respect to Cohn´s theorization, Milhaud´s "P.H." system can be described as a particular application of the "Transpositional combination" concept, in which *Accords* can be expressed as higher cardinality set-class products of two set-class "operands" [0, 4, 7] and [0, 3, 7].

The pc set theory apparatus brings important insights concerning the *modus operandi* of the Milhaud´s system: **while many *Accords* will correspond to set-classes presenting Cohn´s "TC" properties, all *Accords*/sets – however diverse or sometimes complex – can also be subsumed as outcomes of Forte´s transposition operator, over the six interval classes**, through an exhaustive exploration of all possible combinations between equivalent sets ("transpositionally equivalent" for *manières* "a" and "b"; "inversionally equivalent" for *manières* "c" and "d"), the generative set being, in the case of *Polytonalité harmonique*, the major and minor perfect triads (i.e. [0,3,7] and [0,4,7] two expressions, inversionally related, of Forte´s prime form "3:11").

If, for the analysis of the immanent level, the "P.H. system" cannot compete in terms of "neutrality"[46] with pc set analysis, whose tools allow to keep connotation to a minimum, Milhaud´s method – for the specific pc set space defined by those transpositional/inversional set classes to which it is applicable – provides a descriptively adequate model for a large *corpus* of early twentieth-century music, being attractive in many ways:

– simpler to memorize, as it depends only on the interaction of two "coordinates": the *Accords* I to VI (defined by the semi-tonal distance between chordal " roots") and *manières "a" to "d"* (defined by the modal identity of each units);

– from an intuitive angle: many "P.H." *Accords* can be associated with (and illustrated by) "sonorities" which have become familiar in the literature, with a

[46] See forceful point made by Nattiez (2000).

sometimes conspicuous presence in works ranging from *Pelléas, Elektra, Daphnis, Pétrouchka, Sacre*, to Schoenberg´s "Farben" or Berg´s *Violin Concerto*.

But specially, such a ("*ad dissonantiam per consonantiam*") **method of approaching dissonance in terms of consonant units** – of which the "P.H. system" represents both an illustration and a "simplified model" – appears as particularly revealing of a specific *poiesis*, which not only pervades most of Milhaud´s work, but is also recurrently found – as an *ad hoc* technique – in works by many among his most illustrious contemporaries – from Ravel and Stravinsky to Messiaen and Britten – and more recently in works by composers such as Ligeti, Kurtag, Schnittke, Nancarrow, or those associated with Minimalism, who, in different ways and degrees, have detached themselves from the Post-War serial tradition.

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