

# **WAGNER'S LAW FOR BRAZIL A DISAGGREGATED ANALYSIS**

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**ABSTRACT** The present paper aims to assess the validity of Wagner's law for the Brazilian economy during the period 1948-1993, by making use of cointegration techniques. The paper's main contribution relates to the consideration of different categories of government expenditure, namely consumption, investment, and transfer payments. The results indicate that there is no support for Wagner's law regarding any of those categories, nor can it be supported when public expenditures are considered as a whole.

**Key words:** Wagner's law, cointegration

## **A LEI DE WAGNER NO BRASIL: UMA ANÁLISE DESAGREGADA**

**RESUMO** O presente artigo pretende avaliar a validade da lei de Wagner para a economia brasileira no período de 1948 a 1993, por meio de técnicas de cointegração. A principal contribuição do artigo relaciona-se com a consideração de diferentes categorias de gastos governamentais, a saber, consumo, investimento e pagamentos de transferência. Os resultados indicam que não existem evidências favoráveis à lei de Wagner em nenhuma dessas categorias, nem tampouco quando se consideram os gastos governamentais como um todo.

**Palavras-chave:** lei de Wagner, cointegração

## 1. INTRODUCTION

The role of the public sector in facilitating economic development is still a controversial issue, especially with respect to particular activities in which it is believed that public ownership would induce inefficient resource allocation.<sup>1</sup> In any case, the importance of infrastructural government expenditure and an increasing need of public good provision as urbanization accelerates have long been recognized within the so-called 'Wagner's law' [see Bird (1970) and Gemmel (1993) for surveys]. A general statement of the 'law' embodies the notion that as industrialization proceeds, the relative importance of government expenditures would grow. At this level of generality the 'law' allows multiple versions [see Mann (1980)]. One such interpretation postulates that as real *per capita* income rises, the relative size of the government would grow. It is worth mentioning that revenue constraints to government expansion, as emphasized by Peacock and Wiseman (1961), had been acknowledged by Wagner himself while formulating his 'law'.

The present paper intends to provide a test of Wagner's law for the Brazilian economy, making use of different levels of aggregation for government expenditures within a time series framework, and it is organized as follows. Section 2 surveys the routes for testing Wagner's law. The third section briefly discusses the relevant time series concepts to be used in the paper. Section 4 outlines the data to be used and presents empirical results. Finally, the fifth section summarizes the paper and offers some conclusions.

## 2. TESTING WAGNER'S LAW

An influential version of Wagner's law [e.g. Musgrave (1969), Mann (1980), Nagarajan and Spears (1990), and Murthy (1993)] can be summarized in terms of the following equation:

$$\ln(G/GDP) = \beta_0 + \beta_1 \ln(GDP/POP) + \varepsilon \quad (1)$$

where  $G$  = government expenditure,  $GDP$  = real gross domestic product,  $POP$  = population,  $\varepsilon$  = random disturbance term. Wagner's law would be vindicated if  $\beta_1 > 1$ . Essentially, the literature is limited to bivariate regres-

sions, although some studies point out that additional explanatory variables should be considered.<sup>2</sup>

In broad terms, one could highlight three stages in the evolution of research on that topic:

- (a) early cross-section studies covering different countries;
- (b) a movement towards time series studies, and concern over the selection of stable sample periods;
- (c) explicit consideration of problems arising from the time series nature of the data, as related with the presence of stochastic trends.

The early emphasis on cross-section data was clearly unsatisfactory, as the law refers to an increasing importance of the government as industrialization develops and therefore relates rather to a particular economy than to a set of different economies at distinct stages of development. Bird (1970, p. 76) makes this point clear: “there is nothing in any conceivable formulation of Wagner’s ‘law’ which tells us country *A* must have a *higher* government expenditure than country *B* simply because the level of average *per capita* income is higher in *A* than in *B* at a particular point in time. The ‘law’ simply asserts that the ratio will *rise* in *A* (and *B*) as *per capita* income rises, and a rising ratio over time is quite different from a higher ratio at a point in time” (emphasis in the original).

The next natural step was the consideration of economies over time, which has been done either through time series studies for particular countries or in terms of panels of countries. In the case of economies frequently involved in wars, distortions would tend to originate in the path of government expenditure, a problem which has been addressed either by restricting the sample to peace periods or by introducing dummy variables.<sup>3</sup>

Finally, the literature began to stress the need to test for stochastic trends in the data; in fact, Granger and Newbold (1974) pointed out the possibility of spurious regressions when there are trends in the data, and as Engle and Granger (1987) would clarify, the existence of a long-run equilibrium between variables with stochastic trends requires that they ‘cointegrate’ in a sense to be explained in the next section.

The testing of Wagner’s law in terms of obtaining a significant positive coefficient  $\beta_1$  in expression (1) requires a previous test on whether  $\ln(GDP)$  and  $\ln(GDP/POP)$  cointegrate, otherwise there is no long-run equilib-

rium between the variables. Murthy (1993) and Henrekson (1993) took the first step in that direction by testing for cointegration in the context of Wagner's law considering the economies of Mexico and Sweden, respectively, and obtained mixed results: whereas the former confirmed it, the latter rejected it.

One problem with the aforementioned studies is their reliance on the Engle-Granger cointegration test procedure, whose low statistical power is well known by now [see, e.g., Banerjee *et al.* (1993)]. A more powerful procedure is the maximum-likelihood approach advanced by Johansen (1988). Ashworth (1994), Hayo (1994), and Murthy (1994) considered that technique in tests of Wagner's law using Mexican data, while Hondroyiannis and Papapetrou (1995) studied the Greek case; the overall evidence seems not to support Wagner's law.

The aim of the present paper is to contribute to this debate by further considering the relevance of different classes of government expenditures, since it seems reasonable to postulate that the arguments in Wagner's law may be relevant for some categories of government expenditure but not for all of them [see, e.g., Musgrave (1969) and Courakis *et al.* (1993)]. On the other hand, the empirical literature has traditionally focused either on total government expenditure or on government consumption.<sup>4</sup> In this article we consider Brazilian economy through three classes of government expenditure, namely consumption, investment, and transfer payments; at this level of aggregation, the conclusions from previous studies are at most tentative, as they have neglected possible spurious regression issues emphasized by the cointegration literature.

The rationale for considering different categories of public expenditure can be found in Musgrave (1969). In his model of 'efficient' consumer behavior, agents make choices between outlays on public goods  $G_1, (\dots), G_m$ , and private goods  $P_1, (\dots), P_m$ , such as to maximize utility from both types of goods,  $U(G_1, (\dots), G_m, P_1, (\dots), P_m)$ . In this framework, only when the conditions of Hicks' aggregation hold for  $G_1, (\dots), G_m$ , are public expenditures as a whole related to the level of *per capita* income.

Once one agrees that disaggregation is a relevant topic, a question arises as to whether a functional characterization of government expenditures (education, health, defense, etc.) as contrasted to economic categories

(public consumption, public investment and transfers) provides the most appropriate framework for analysis. Wagner developed his arguments in terms of functional expenditures. In this article, we follow Musgrave (1969) and Courakis *et al.* (1993) and concentrate instead on the division of public expenditures by economic categories.

### 3. COINTEGRATION TECHNIQUES

As mentioned before, the long-run equilibrium between economic variables can be identified with the notion of cointegration. Engle and Granger (1987) showed that a necessary condition for a pair of variables to be cointegrated is that they are integrated of the same order. A series  $y_t$  is said to be integrated of order  $k$  [say  $I(k)$ ] if it has a stationary, invertible ARMA representation after differencing  $k$  times.<sup>5</sup>

The existence of a long-run equilibrium between two variables (so that they are cointegrated) requires that, even though each variable is integrated (of the same order), a linear combination of them is integrated of some lower order. More formally: suppose that two series  $x_t$  and  $y_t$  are both  $I(k)$ ; if there is a constant  $A$ , such that  $(x_t - Ay_t)$  is  $I(k - b)$  for  $b > 0$ , we say that  $x_t$  and  $y_t$  are cointegrated of order  $k, b$  or  $(x_t, y_t) \sim CI(k, b)$ .

To check whether each series has the same order of integration we employ the Dickey and Pantula (1987) test. If the presence of multiple unit roots is at issue, the traditional sequential use of the Dickey and Fuller (*DF*) test would be problematic, as it assumes the absence of unit roots under the alternative hypothesis. In that sense, Dickey and Pantula propose to reverse the test sequence by starting with a maintained hypothesis that assumes the largest number of unit roots at first (in our case, two unit roots). The test may be described as follows. First, consider the model in second differences:

$$\Delta^2 y_t = \beta_0 + \beta_1 t + \beta_2 \Delta y_{t-1} + \beta_3 y_{t-1} + u_t \quad (2)$$

In the event of testing for the presence of two unit roots against a single unit root, the null hypothesis would be such that  $\beta_2 = \beta_3 = 0$ . However, notice that both under the null and alternative hypothesis one has  $\beta_3 = 0$ ; thus one can simply run the regression without  $y_{t-1}$  as a regressor and test the significance of  $\beta_2$  with the *DF* critical values.

Second, in the case of rejection of the null hypothesis, one would consider the full equation and test for the significance of  $\beta_3$  in order to test for the presence of a single unit root. In this paper we adopt this reversed sequential testing approach and use the relevant critical values of the *DF* statistic for small samples provided by MacKinnon (1991).

An underlying assumption is that the error term  $u_t$  presents no serial correlation. If this is not the case, one has to proceed in terms of augmenting the regression by including lagged terms of the dependent variable up to a maximum lag determined by the diagnosed absence of residual serial correlation; therefore, this procedure is analogous to the traditional Augmented Dickey-Fuller (*ADF*) case.

As for the cointegration test, we will make use of Johansen's procedure as outlined below. Suppose the following VAR representation for a vector  $X$  composed by  $I(1)$  variables:

$$X_t = \Pi_1 X_{t-1} + (\dots) + \Pi_k X_{t-k} + \varepsilon_t \quad (3)$$

The corresponding long-run response matrix is given by:

$$\Pi = I - \Pi_1 - (\dots) - \Pi_k \quad (4)$$

The test accounts for checking the rank of the matrix  $\Pi$ . Three possibilities can arise:

(a) Rank ( $\Pi$ ) = 0. In this case there is no cointegration and equation (3) can be rewritten as a traditional VAR in first-differences;

(b) Matrix  $\Pi$  has full rank. This case would contradict our assumption that the variables are  $I(1)$  and would be an indication of over-differencing, hence a model in levels would be more appropriate.

(c) Matrix  $\Pi$  has reduced rank. Here there is cointegration and its rank is equal to the number of cointegrating relationships. Under this assumption, matrix  $\Pi$  can be written as the product of two matrices,  $\Pi = \alpha\beta'$ , where  $\alpha$  is the 'loading' matrix and  $\beta$  is the matrix whose components are the cointegrating vectors.

If a long-run relationship between a particular class of government expenditures and real *per capita* income is detected, one has to identify whether the channel of influence is running from the latter to the former

(as suggested by Wagner's law) or the other way around (as emphasized by some Keynesian aggregate models).<sup>6</sup>

The literature has addressed this problem within a Granger causality framework, relying on results obtained by Granger (1988), who showed that if two variables are  $I(1)$  and cointegrated, then there exists Granger causality in at least one direction. Ram (1986) studied the question of causality for a group of 63 countries, obtaining mixed results. Ram did not tackle the issue of non-stationarity of the involved variables, however, therefore his results may be subject to spurious regression bias. Lin (1995) dealt with the non-stationarity issue and found evidence that the causality runs from real GDP *per capita* to government share for Mexico during the period 1950-1990 (confirming Wagner's law predictions), although no evidence was found when the sample period was shortened to 1950-1980.

#### 4. EMPIRICAL RESULTS

##### 4.1 Data Description

The variables used in the study are  $\ln(GC/GDP) \equiv LGC$ ,  $\ln(GI/GDP) \equiv LGI$ ,  $\ln(GT/GDP) \equiv LGT$  and  $\ln(GDP/POP) \equiv LGDP$ , which denote, respectively: the share of government consumption expenditure on gross domestic product, the share of government investment on gross domestic product, the share of government transfer payments on gross domestic product, and the real gross domestic product *per capita*.<sup>7</sup>

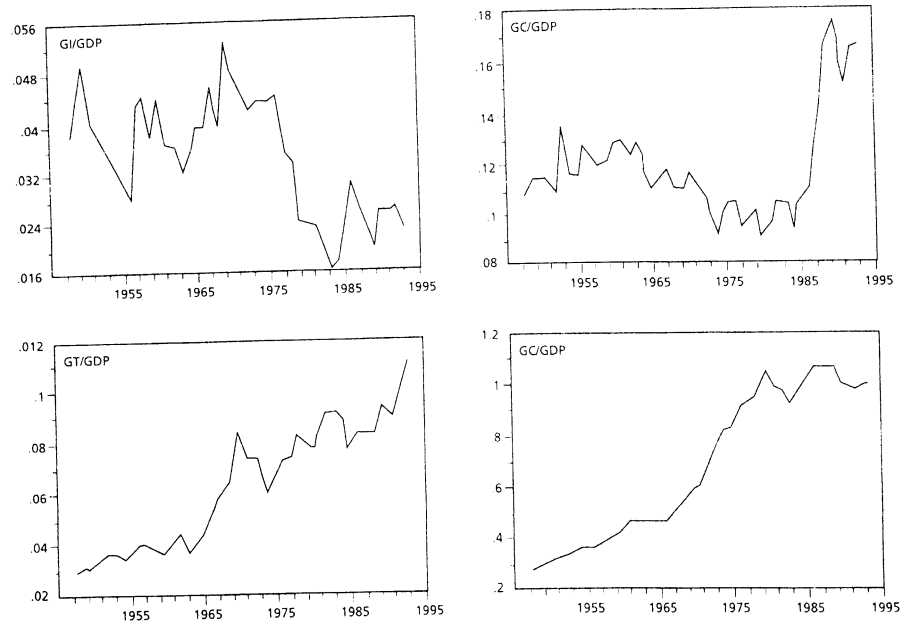
Figure 1 shows the relevant series for our study: public investment ( $GI/GDP$ ), public consumption expenditures ( $GC/GDP$ ) and transfers ( $GT/GDP$ ) as a share of real  $GDP$ , and real  $GDP$  *per capita* ( $GDP/POP$ ).

##### 4.2 Results

The results concerning the order of integration of each variable are presented in table 1 below, where we report the Dickey-Pantula ( $DP$ ) statistics and the Lagrange Multiplier test for serial correlation advanced by Godfrey (1978).

Perusal of table 1 leads to the conclusion that all variables involved are  $I(1)$ , and the test procedures did not require the augmented equations, as no evidence of serial correlation (up to order 2) was indicated by the  $LM$  sta-

Figure 1  
Government Expenditure Shares and Real GDP Per capita in Brazil (1948-1993)



tistics for the simple model. In principle one can be concerned with the issue of unit root testing when a structural break appears to be important. In fact, one would observe a stronger tendency towards acceptance of the null hypothesis of unit root. In this sense, Perron (1989) has devised a modified *DF* test in which one accounts for a single structural break in the series. An influential version of that test extends the usual *DF* formulation by including as an additional regressor an intercept dummy which assumes value 1 after the break, and 0 otherwise. We considered this version of Perron's unit root test for the series *LGC* and *LGI*, which displayed strong breaks in terms of the visual inspection of their graphs. The tabulated values of the statistics depend on the proportion of break time relative to sample size (henceforth  $\lambda$ ). In the present case, we considered a break in 1986 which corresponded to  $\lambda \approx 0.84$ . We therefore report the critical values for the closest tabulated values of  $\lambda$ . The existence of unit root is corroborated by Perron's test, as indicated in table 2.



Having obtained the same order of integration for the series, we can proceed by testing for cointegration. Since Johansen's test requires a well-specified system our starting point was an unrestricted second-order VAR in ( $LGC$ ,  $LGI$ ,  $LGT$ ,  $LGDP$ ), which was further simplified following the approach suggested by Hall (1991).<sup>8</sup> Apart from the lag length choice, another issue concerns the exact specification of the deterministic terms in the VAR. In this case, the strategy was to start from an unrestricted constant and a restricted trend and test for their significance.<sup>9</sup> Moreover, one could contend that the validity of Wagner's law would require the estimation of bivariate VARs of the form ( $LY$ ,  $LGDP$ ), where  $Y = GC, GI, GT$ , as well as linear combinations of them. However, in the presence of links among the endogenous variables, e.g. through government budget constraints, the estimation of bivariate VARs would imply the occurrence of a SUR problem. We therefore chose to pursue the analysis for the four-variable VAR and, if the hypothesis of cointegration could be rejected, test for the validity of Wagner's law through appropriate restrictions in the components of the cointegrating vector. The results displayed below refer to the first-order VAR involving a restricted trend.

**Table 1**  
The Dickey-Pantula test of integration

Variable		First step	Second step
$LGC$	$DF$	-7.204	-0.824
	$LM$	0.621	0.410
$LGI$	$DF$	-6.762	-2.241
	$LM$	0.261	0.513
$LGT$	$DF$	-6.573	-2.453
	$LM$	0.529	0.055
$LGDP$	$DF$	-4.322	-0.881
	$LM$	0.636	1.053

Critical values for the  $DF$  statistics are obtained from MacKinnon (1991). These values are -4.178 at 1% and -3.514 at 5%. The  $LM$  statistics are distributed as  $F(2,39)$  and  $F(2,38)$  in the first and second steps, respectively.

**Table 2**  
Perron's Unit Root Test

Variable	Test statistic	Critical values	
		$\lambda = 0.8$	$\lambda = 0.9$
$LGC$	-2.76	1%:	-4.33
		2.5%:	-3.99
		5%:	-3.75
$LGI$	-2.41		

Table 3 presents some diagnostic statistics for the VAR. In this table we display the following: single equation residual standard deviation ( $\sigma$ ); single equation statistical tests for serial correlation (*AR*), autoregressive heteroscedasticity (*ARCH*), heteroscedasticity (*H*), and normality (*N*); test statistics for: vector serial correlation (*vec AR*), vector heteroscedasticity (*vec H*), vector functional form (*vec FF*), and vector normality (*vec N*) [for details on these statistics, see Doornik and Hendry (1995)].

The results show that all the diagnoses are satisfactory for the usual significance levels. We can further pursue our analysis by undertaking Johansen's cointegration tests, which are summarized on table 4.

The evidence displayed in table 4 shows that there is no indication of cointegration for any of the categories of public expenditure, nor is there any when they are considered as a whole. Therefore, there is no support for Wagner's law concerning Brazilian economy. This result is in line with the ones obtained by Ashworth (1994) and Hayo (1994) for the Mexican

Table 3  
VAR Diagnostic Statistics  
Statistics

	<i>LGC</i>	<i>LGI</i>	<i>LGT</i>	<i>LGDP</i>
( $\sigma$ )	7.5%	14.6%	7.8%	3.2%
<i>AR</i>	0.65 [0.53]	0.10 [0.90]	0.17 [0.84]	0.23 [0.79]
<i>ARCH</i>	0.04 [0.85]	0.58 [0.45]	0.52 [0.47]	0.82 [0.37]
<i>H</i>	0.22 [0.99]	0.66 [0.75]	0.67 [0.74]	1.22 [0.32]
<i>N</i> (2)	2.91 [0.23]	2.09 [0.35]	2.64 [0.27]	0.30 [0.86]
<i>Vec AR</i>			0.93 [0.58]	
<i>Vec H</i>			0.97 [0.57]	
<i>Vec FF</i>			0.78 [0.94]	
<i>Vec N</i> (8)			7.78 [0.46]	

*N* and *Vec N* are distributed as chi-square with the appropriate degrees of freedom indicated in the table. All the remaining statistics have an *F* distribution with the degrees of freedom given as follows: *AR* (2, 37), *ARCH* (1, 37), *H* (10, 28), *Vec AR* (32, 104), *Vec H* (100, 147), *Vec FF* (200, 103).

The entries in square brackets indicate the tail probability associated with the value of the statistic.

**Table 4**  
**Johansen's Tests for Cointegration**  
**Sample: 1948-1993**

Eigenvalue	0.477	0.343	0.149	0.063
Null hypothesis	$r = 0$	$r \leq 1$	$r \leq 2$	$r \leq 3$
$\lambda_{\max}$	29.15	18.90	7.24	2.91
$\lambda_{\max\text{adj}}$	25.56	17.22	6.60	2.65
Critical value	31.5	25.5	19.0	12.2
$\lambda_{\text{trace}}$	58.19	29.05	10.14	2.91
$\lambda_{\text{traceadj}}$	53.02	26.46	9.24	2.65
Critical value	63.0	42.4	25.3	12.2

$\lambda_{\max}$ : Statistic for the likelihood ratio test based on maximal eigenvalue of the stochastic matrix.  
 $\lambda_{\max\text{adj}}$ : Same as  $\lambda_{\max}$ , but with a degrees-of-freedom adjustment as suggested by Reimers (1992).  
 $\lambda_{\text{trace}}$ : Statistic for the likelihood ratio test based on trace of the stochastic matrix.  
 $\lambda_{\text{traceadj}}$ : Same as  $\lambda_{\text{trace}}$ , but with a degrees-of-freedom adjustment.  
 Critical values for 5% significance level are obtained from Osterwald-Lenum (1992).

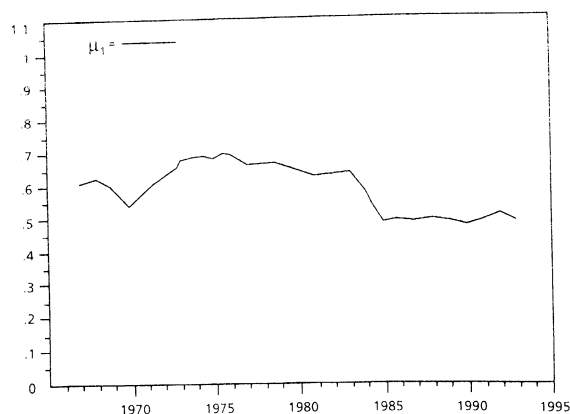
economy and by Hondroyiannis and Papapetrou (1995) for Greece. Our results are also close to those obtained by Courakis *et al.* (1993), who investigated the validity of Wagner's law for the same categories of public expenditure as our own in the Portuguese and Greek economies. They found support for Wagner's law only with regard to transfer expenditure in Greece and (at a lower significance level) consumption expenditures in Portugal.

Lin (1995) analyses the issue of sample stability, showing that evidence for Wagner's Law can be found for Mexico in the period 1950-1990, although no evidence is found for the shorter period 1950-1980. As shown by Gregory and Hansen (1996) and discussed by Maddala and Kim (1998), cointegration tests can erroneously reject the hypothesis of cointegration. A thorough investigation of Wagner's law under structural breaks is beyond the scope of the current paper and is left as a topic for future research. In what follows we study instead the robustness of our results with a shortening of the sample period considered.

Figure 2 shows the recursive estimate of Johansen's maximal eigenvalue for the system estimated above, and its inspection suggests a higher value for the maximal eigenvalue when the sample period is shortened to the early 1980's.

We therefore investigated the possible existence of cointegration in our VAR when the sample was shortened to the early 1980's. Table 5 reports the results for the cointegration tests for the period 1948-1983.

**Figure 2**  
**Johansen's Maximal Eigenvalue: Recursive Estimation**



There was then some evidence of one cointegrating vector for the VAR. We thus tested for the existence of a stationary Wagner's law kind of relation when appropriate restrictions were imposed on vector  $\beta$ .<sup>10</sup> In other words, we tested for the stationarity of vectors of the general form  $[\beta_1 LY_t + \beta_2 LGDP_t + \beta_3 \text{Trend}_t]$  where  $LY_t$  represents the log of the government expenditure component  $Y_t$ . In addition, validity of Wagner's law requires that  $\beta_1$  and  $\beta_2$  have opposite signs and that causality runs from  $LGDP$  to  $LY$ .

Table 6 shows the results for two cases: when the coefficient  $\beta_3$  is unrestricted and when it is restricted to zero. For all possible combinations of government expenditure components but one, there is no support for cointegration with *GDP per capita* (trend-adjusted or not). The only exception is a borderline  $p$ -value case for the aggregate government expenditure. Even in this case, support for Wagner's law can still be rejected since the estimated vector implies a negative relation between  $LY_t$  and  $LGDP_t$ . Therefore, there is no evidence that the stationary vector comprises a Wagner's law type of relation for the shortened period 1948-1983. One could finally consider a final robustness check. In fact, an anonymous referee suggested the introduction of the variable openness (*OPEN*) as measured by the share of exports plus imports in *GDP*, following the motivation from Rodrik (1998). This author showed that openness is positively related to the share of government spending in a cross section of countries and that this relation

