FROM COMMON LABOR TO QUANTUM OF KNOWLEDGE: THE RAMSEY MODEL[1]

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ABSTRACT The famous Ramsey model is reinterpreted according to the Quantum of Knowledge Theory. A new model of economic growth emerges. It explains the passage from a common labor-based economy to a modern economy based on the quantum of knowledge. As a result the society's consumption function switches from being constant to increasing with the quantum of knowledge accumulation process.

Key words: economic growth, quantum of knowledge, common labor

DO TRABALHO COMUM AO QUANTUM DE CONHECIMENTO: O MODELO DE RAMSEY

RESUMO Neste artigo, o famoso modelo de crescimento de Ramsey é reinterpretado de acordo com a Teoria do Quantum de Conhecimento. Desse modo, um novo modelo de crescimento econômico de longo prazo emerge. Este modelo explica a passagem de uma economia baseada em trabalho comum para uma moderna, baseada em quantum de conhecimento. Como resultado desta transformação, a função de consumo da sociedade deixa de ser constante e passa a acompanhar o processo de acumulação de quantum de conhecimento.

Palavras-chave: crescimento econômico, quantum de conhecimento, trabalho comum

1. INTRODUCTION

All theory depends on assumptions which are not quite true. That is what makes it a theory. The art of successful theorizing is to make the inevitable simplifying assumptions in such a way that the final results are not very sensitive.

SOLOW (1956)

The first economic growth model may be credited to Ramsey (1928). This also comprises the introduction of the technique used nowadays in modeling. Theoretically, the real improvement that may be credited to researchers in this field during all these years of research lies in the understanding of the growth-driven variable, or rather, the factor of production that causes economies to grow. In the past, the growth of the factors of production, labor, capital and land, was regarded as responsible for economic growth. Notwithstanding their importance, present models are shifting away from these factors. Some recent models, like the one to be developed in the forthcoming sections, do not rely at all on any of these factors as means of economic growth.

The employment of physical elements (capital, land and labor) in growth models imposes a limitation on the growth process. This limitation arises from the available amount of these elements and their productivity. Let's suppose there is a continuous expansion in the economy's output based purely on exploration of new land. Considering that land is a limited resource, after a certain period the economy would forcibly reach its limit in production. Growth, thereafter, must be based on labor. The new incoming labor ought either to be more productive than the one leaving the production process or to be outnumbered. Since all labor is identical, hence identical in productivity, productivity growth would only be possible through the continuous expansion of the number of workers. Considering that their productivity is identical, growth per capita would be zero, the common labor society. Thus, the economy would be in a steady state with no *per capita* growth. A continuous growth based on physical capital would be possible if its productivity were constant or continuously increasing. In other words, adding more capital to the production process would not cause a fall in marginal productivity. Hence, continuous growth based on physical capital requires the assumption of constant or increasing returns to scale. Considering that physical capital is a combination of physical elements, a finite point in time will be reached in which growth will not be possible, though such a point could take hundreds of years to come. In the meantime, all the resources available on earth would have been transformed into physical capital. Thus, growth based on physical elements does impose a limit on the growth process.

A further question may be allowed: Is a positive growing economy possible? The answer is yes. It is possible as long as its growth process is based on a factor that causes continuous increases in the overall productivity of the physical elements, thereby also making their productivity constant or continuously increasing. The next section discusses a feasible element.

2. THE QUANTUM OF KNOWLEDGE THEORY

Ramsey (1928) and Solow (1956) agreed on the element of growth. Both authors emphasized the great importance of physical capital accumulation as a growth engine. Following the same line, the R&D (research and development) models gave much importance to the physical capital innovation as a means of enhancing the economy's productivity (Romer, 1990; Grossman & Helpman, 1991).

Later on, Romer (1986) developed knowledge as an input. This idea was widely spread when it was transformed into human capital by Lucas (1988). Differences in human capital across time and countries became the centerpiece for explaining long-run growth in productivity and inequality among economies.

According to Dias & McDermott (2000), the above models take for granted the demand for R&D and human capital. In other words, the accumulation of these two inputs through time takes place by generating their own demand. However, this supply side condition doesn't seem to fit reality properly, especially when it is perceived that the economies are not taking advantage of the high return given by skill improvement[2] — human capital accumulation — or the high return estimated for the investment in R&D.[3] In our view, there is an important element that constraints the economies to grow at a higher pace and to take advantage of high rates of return. This element is called quantum of knowledge (Dias, 1992, 1993, 1994, 1995a, 1995b and 1996).

The quantum of knowledge theory states that there is a specific knowledge that controls demand for all the others. It is the entrepreneur's knowledge. The entrepreneur hires different workers and physical capital to form a team to produce goods of a certain specified quality. This is done in order to cater to consumer preferences and requires the production and innovation of human and physical capital. The profit opportunity visualized by the entrepreneur constitutes a specific knowledge on preferences and production processes and is the motivating element. Production knowledge includes the match between workers with different knowledge levels and equipment that requires different types of knowledge levels to be operated. Thus, the team formed by the entrepreneur, himself included, epitomizes a certain level of knowledge which we refer to as quantum of knowledge. In the last instance, it is this quantum of knowledge that matters to the economy. Moreover, improvement in the quantum of knowledge is equivalent to a more efficient combination of physical and human capital inputs which leads to continuous productivity enhancement.

The average of all quanta of knowledge in the economy or the average entrepreneurial knowledge is the single most important element of economic growth and, by implication, productivity enhancement. Its accumulation means an increase in the demand for human capital (skilled workers) and physical capital innovation (R&D). Consequently, in this paper we use this element to replace physical capital in the Ramsey (1928) model as a means of seeing the condition for an economy based on common labor to transform itself into one based on the quantum of knowledge.

3. BASIC ASSUMPTIONS OF THE RAMSEY MODEL

In developing his model to explain optimal savings, Ramsey (1928) introduced techniques still popular today in solving growth models. Welfare and capital accumulation functions have changed very little since his initial effort. Small changes consisted in replacing the saving variable in the welfare function by the consumption variable. Moreover, physical capital in the production function now contains a new concept. Thus, we will replace the physical capital in the Ramsey model for the average quantum of knowledge of society as has been suggested in the previous section.

We will analyze the model by listing Ramsey's assumptions:

A1: there is no population or labor growth;

A2: the parameters of the utility function are fixed;

A3: there is no new invention, change in technology, or improvement in organization;

A4: the distribution of labor and consumption is given;

A5: there is only one kind of labor and goods.

Let L stand for the number of workers, C for consumption and K for the total quantum of knowledge at any time t. The country's total output at any time is a function of K and L, or

$$Y = F(K, L). \tag{1}$$

Output may be consumed and saved (invested) according to the following equilibrium condition:

$$S + C = F(K, L) \text{ or } \frac{dK}{dt} = F(K, L) - C, \tag{2}$$

where dK/dt is the investment identical to savings, S, in the model.

As in Ramsey (1928: 549), returns to the quantum of knowledge and common labor are assumed to remain constant and independent. So that

$$F(K, L) = \frac{W_K}{p} K + \frac{W_L}{p} L, \tag{3}$$

where p is the price of the output Y, $W_K = \dfrac{\partial F(K, L)}{\partial K} > 0$ is the real return to the quantum of knowledge and $W_L = \dfrac{\partial F(K, L)}{\partial K} > 0$ is the basic real wage.

According to Ramsey, the basic real wage and the return to the quantum of knowledge are assumed to be constant. The following production function complies with the above requirements:

$$F(K, L) = aK + bL. \tag{4}$$

Employing the above equation, the accumulation of the quantum of knowledge per worker is the following:

$$\dot{k} = \frac{\dot{K}}{L} - \frac{\dot{L}}{L}k \tag{5}$$

where $\dot{k} = \dfrac{dk}{dt}$ and $\dot{L} = \dfrac{dL}{dt}$.

Simplifying, if we adopt assumption A1, the growth rate of common labor is zero, the second term in equation (5) vanishes. The first term in equation (5) is equation (2) divided by the number of workers, *L*. By substituting equation (4) into (2) and its result into equation (5), we have the following:

$$\dot{k} = b + ak - \boldsymbol{c}. \tag{6}$$

The above equation is the average quantum of knowledge accumulation function. In this equation, under the condition of zero initial quantum of knowledge, $k(0)=0$, the quantum of knowledge accumulation will start only if $\boldsymbol{c} < b$. In other words, the productivity of common labor must exceed consumption. The conditions $k(0) = 0$ and $b = \boldsymbol{c}$ refer to the primitive economy or the economy based on common labor. The quantum of knowledge accumulation will take place only if an exogenous positive endowment comes into the economy, or even if an effort is made in such way that $\boldsymbol{c} < b$. The condition for a modern economy exists when the accumulation of the quantum of knowledge takes place and starts increasing over time at a positive rate.

As may be perceived in the above paragraph, the passage from a primitive economy based on common labor, where no quantum of knowledge is present, to a modern one can only be made by using intentional policies.[4] The focus of policies must be the reduction of consumption in order to initiate a quantum of knowledge accumulation. This is equivalent to having an average reduction of consumption so that someone can accumulate entrepreneurial knowledge. Later on the entrepreneur will organize the remaining members in the production process in such way that their overall productivity will be enhanced. Thus, this process would make $\boldsymbol{c} < b$ at some point in time, causing the emergence of an initial stock of quantum of knowledge $k(0) > 0$. Thereafter, the endogenous growth of *k* would lead to an ever increasing productivity, $\dot{k} > 0$. In a modern economy, this is equivalent to adopting policies that replace intertemporal consumption by quantum of knowledge accumulation in order to enhance productivity. This is equivalent to saying that today's generation would have to sacrifice consumption in order to have a larger quantum of knowledge in the future. Intentional policies towards quantum of knowledge accumulation are thus required.

Now, a minimum consumption level, $\bar{c}$, is assumed to exist. It is incorporated in the above equation in the variable consumption, *c*. In other words, consumption has to be split into two parts: the minimum consumption necessary to for survival, $\bar{c}$, and the second part, or rather, all consumption above minimum, *c*, or:

$$\dot{k} = b + ak - \bar{c} - c, \tag{7}$$

where $c = \bar{c} + c$.

Let us assume the form of the welfare function as the sum of all future utilities discounted at a subjective rate, ρ. It is equivalent to[5]

$$W(c) = \int_0^{\infty} \frac{[\bar{c} + c^{1-\sigma}]}{1-\sigma} e^{-\rho t} dt \qquad \text{for} \quad \sigma \neq 1, \text{ and} \tag{8}$$

$$W(c) = \int_0^{\infty} \left[ln(c) + \bar{c} \right] e^{-\rho t} dt \qquad \text{for} \quad \sigma \neq 1,$$

where c is the consumption above $\bar{c}$; $\bar{c}$ is the minimum consumption; ρ is the subjective discount rate; and σ is the welfare function parameter.

Given the above definition of the welfare function and given the condition that the aim is to maximize it at time $t = 0$, the elected representative consumer maximizes the following Hamiltonian function:

$$H = \left(\frac{\left[\bar{c} + c^{1-\sigma} \right]}{1-\sigma} + \lambda b + ak - \bar{c} - c \right) e^{\rho t} \tag{9}$$

The necessary conditions are

$$\frac{\partial H}{\partial c} = Hc = 0, \; c^{-\sigma} = \lambda; \tag{10}$$

$$\dot{\lambda} = \rho\lambda - \frac{\partial H}{\partial k}, \; \dot{\lambda} = (\rho - a)\lambda; \tag{11}$$

The transversality condition is

$$\mathrm{Lim}_{t\to\infty}\, \lambda(t)\ k(t) e^{-\rho t} = 0. \tag{12}$$

A sustained equilibrium growth path in line with modern linear models such as those by Barro (1990), Rebelo (1991) and Dias (1995) requires that $\bar{c} = b$. This assumption appeared implicitly in Ramsey (1928: 556) in his definition of savings: $S = C - bL$. Thus the worker saving is $s = c - b$. Hence, in his conception, the non-saving condition is $c = b$. Therefore, b represents the basic wage. If we assume this condition, the function of the quantum of knowledge accumulation will be:

$$\dot{k} = ak - c. \tag{13}$$

Although this hypothesis would make our job easier, since it constitutes the main hypothesis of the linear model, we preferred not to use it. We derive an equilibrium condition based on a balanced growth condition. An equilibrium growth path for this model requires that all variables grow at the same rate. More specifically, consumption and quantum of knowledge must grow evenly.

$$\gamma = \frac{\dot{c}}{c} = \frac{\dot{k}}{k}, \tag{14}$$

where γ is the balanced growth rate of the economy. The further restriction imposed is the transversality condition. The result of the sum of the growth rates of the shadow price and the quantum of knowledge must be inferior to ρ, as below

$$\frac{\dot{\lambda}}{\lambda} + \frac{\dot{k}}{k} < \rho \tag{15}$$

Now, recall equation (11). Dividing it by λ, the equation becomes the growth rate of λ. The growth rate of the quantum of knowledge is arrived at by dividing equation (13) by k. But, according to equation (14), the growth rate of the quantum of knowledge and consumption must be the same, γ. Therefore, instead of the quantum of knowledge rate, the growth rate of consumption will be used to derive condition (15). The growth rate of consumption is obtained by differentiating equation (10) with respect to time and substituting equation (11) into it. This gives us the following result:

$$\frac{\dot{c}}{c} = \frac{1}{\sigma}(a - \rho) = \gamma \tag{16}$$

By comparing equation (16) with (11), we have the growth rate of λ:

$$\frac{\dot{\lambda}}{\lambda} = -\sigma\gamma \tag{17}$$

In equation (16), the term $\frac{1}{\sigma}$ is the elasticity of substitution between consumption at any two points in time. The term $-\sigma$ in equation (17) is the elasticity of the marginal utility.[6]

Now, substituting equations (16) and (17) into (15), we obtain the following condition for the satisfaction of the transversality condition:

$$(1 - \sigma)\gamma < \rho \tag{18}$$

Given that $\rho > 0$, for all $\sigma \geq 1$ transversality is satisfied. However, our problem consists in showing that for $0 < \sigma < 1$ the transversality condition is satisfied. To show the implication of having too small an σ, we recall the condition expressed by equation (14). Substituting equations (7) and (16) into expression (14) and solving it, the following is obtained:

$$c(t) = b - \bar{c} + \left[a - \left(\frac{a - \rho}{\sigma}\right)\right] k(t) \tag{19}$$

As expected, the consumption over time, $c(t)$, is linearly related to the average quantum of knowledge, $k(t)$. The $b - \bar{c}$ is consumption based on common labor, while the terms in brackets are due to the accumulation of the quantum of knowledge. So that for consumption over time to be positive, two conditions must be met. The first one requires that $a > \rho$, or the productivity of the average quantum of knowledge must exceed the subjective discount rate; the second one requires that $\frac{1}{\sigma}$, the elasticity of substitution between consumption at any two points, either not be too big or σ be so small that it causes the part in brackets in equation (19) to be negative. This limit must be obeyed, otherwise present consumption will be negative. Thus, the positive consumption condition imposes a minimum value for σ.

The lower bound limit for σ guarantees the satisfaction of the transversality condition. This solution differs from the one derived by Ramsey (1928: 556). There he looks for a steady state in which all variables are growing at zero rates. This would be equivalent to $a = \rho$.

The interesting result of this model is the consumption function in equation (19). It shows the big incentive for the quantum of knowledge accumulation process to take place. The consumption function switches from being constant to increasing steadily with the quantum of knowledge accumulation process. Therefore, the gains are far greater as compared to the sacrifice to be made at some point in time.

4. CONCLUSION

According to the quantum of knowledge theory, the reinterpretation of Ramsey's model explains the condition by which an economy based on common labor can move towards a modern one based on human capital. In the case where the initial endowment of the quantum of knowledge is near zero, there must be an initial effort to reduce consumption below the productivity of common labor. Saving caused by reduction in consumption must be employed entirely in the accumulation of a quantum of knowledge. Thus, an intentional policy towards accumulating the quantum of knowledge must exist. As a consequence the consumption function moves from being constant to increasing steadily with the process of quantum of knowledge accumulation.

NOTES

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2. For a detailed discussion see Lloyd-Ellis (1999).
3. Jones & Williams (1998) estimated that the social return to R&D ranges from 27% to 100%.
4. Lucas (1998) developed a model in which the possible explanation for productivity to exceed consumption is either property right on the land or the introduction of private property at a given point in time.

5. See Barro (1990), Dias (1995) and Barro and Sala-I-Martin (1995) for a complete derivation of this problem.
6. See Blanchard and Fisher (1988) for more details.

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