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The development of the verbal counting system: from non-verbal to verbal tallies

O desenvolvimento do sistema de contagem verbal: de contagens não-verbais a verbais

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ABSTRACT

Humans are born with not one but two systems that create representations with numerical content. Unlike verbal counting, neither of them defines numerical symbols in terms of their positions in an ordered list. Together with other considerations, this led many to believe that, when children learn how to use counting to define number word meanings, they solve what seems like an impossible problem: they learn numerical principles that cannot be defined in terms of the numerical representations available to them at the time. We propose that there may be more continuity between what children learn when they learn how to use counting to define number word meanings and some of the principles available to them prior to language learning than previous researchers have thought. Specifically, we propose that children learn how to use counting to define number word meanings by thinking that counting is a tallying system just like one of the two core number systems – namely, parallel individuation (which we refer to as "mental tallies"). That is, when they learn the verbal counting system, children learn to define the meaning of expressions like *"n X's*" where *n* is a number word (e.g., "five") and *X* is a noun phrase (e.g., "old chairs that come from Sweden") as *a collection of Xs that match one to one with a count that ends with "n."* We argue that this view of the acquisition of the meaning of number words and counting provides a better account of the way non-verbal representations of number are integrated with verbal counting and of children's knowledge of the meaning of number words both before and after they learn how verbal counting represents numbers than any other view proposed before.

KEYWORDS: Numbers. Counting. Cognitive development. Language learning.

RESUMO

Os humanos nascem não com um, mas dois sistemas que criam representações com conteúdo numérico. Ao contrário da contagem verbal, nenhum deles define símbolos numéricos em termos de suas posições em uma lista ordenada. Juntamente com outras considerações, isso levou muitos a acreditar que as crianças, ao aprender a usar a contagem para definir o significado das palavras numéricas, resolvem o que parece ser um problema impossível: aprendem princípios numéricos que não podem ser definidos em termos das representações numéricas disponíveis para elas nesse período. Propomos que, diferentemente do que pesquisadores anteriores pensavam, pode haver mais continuidade entre o que as crianças aprendem, ao aprender a usar a contagem para definir os significados das palavras numéricas, e alguns dos princípios disponíveis para elas antes do aprendizado da linguagem. Especificamente, propomos que as crianças aprendem a usar a contagem para definir os significados das palavras numéricas, considerando que a contagem é um sistema de contagem, assim como um dos dois sistemas numéricos centrais – ou seja, individuação paralela (à qual nos referimos como "contagens mentais"). Ou seja, quando aprendem o sistema de contagem verbal, as crianças aprendem a definir o significado de expressões como "n X's", onde n é uma palavra numérica (por exemplo, "cinco") e X é um sintagma nominal (por exemplo, "cadeiras velhas que vêm da Suécia") como uma coleção de Xs que pareiam um a um com uma contagem que termina com "n". Argumentamos que, comparativamente a qualquer outra proposta anterior, essa visão da aquisição do significado das palavras numéricas e da contagem fornece uma melhor explicação do modo como as representações não verbais do número são integradas à contagem verbal e do conhecimento das crianças sobre o significado das palavras numéricas antes e depois de aprenderem como a contagem verbal representa números.

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One, two, three, four, five...The whole numbers[2]. These apparently mundane thoughts are perhaps some of our most abstract and productive. If they can be thought of as a property of collections, then they are a property of an intriguing kind. Indeed, when we say of something that it is one, we abstract away all the properties of the thing, except for the fact that it is a member of a particular class. Thus, a ball is one ball regardless of its size, weight, texture, or color. A tone is one tone regardless of its pitch, its volume or its duration. And, although tones and balls have nothing in common a tone is one in precisely the same sense that a ball is one. By defining operations like adding and multiplying over the whole numbers we obtain a system of arithmetic. Arithmetic inherits the abstractness of the whole numbers. Thirteen and fifteen make twenty-eight regardless of whether one is adding steps, bank branches, weddings, or phases of the moon. Thus, arithmetic has become fundamental for a multitude of human activities. We use the very same principles of arithmetic to define the rules of our favorite sports and games, to balance a checkbook or a corporate account, and to make tragic calculations such as the remaining number of standing soldiers after an attack.

What is the developmental history of the representations of the whole numbers? Are they learned or are they innate? If they are learned, what are the representations from which we build them and what are the learning mechanisms that allow us to learn them? To be able to answer these questions, we first present two criteria that define what we take to be evidence that an organism has representations of whole numbers. We refer to the first criterion as the "exact number criterion." As per this criterion, an organism has representations of whole numbers if it provides evidence that a difference of one is necessary and sufficient for it to assign different numbers to two collections. The same criterion applies to a single collection that changes over time, that is, the organism must provide evidence that it assigns different representations to a collection if and only if one or more elements are added to it (and this change is not compensated by the removal of an equal number of elements) or if one or more elements are removed from it (and this change is not compensated by the addition of an equal number of elements).

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[2] Many students of the development of what we refer to as representations of the whole numbers – i.e., representations of the exact number of entities in a collection – refer to them as representations of the *natural* numbers. In mathematics, the natural numbers are thought of as the objects that satisfy the theorems of Number Theory. The theorems are Number Theorem are not usually thought of as being about the properties of collections of actual physical entities. For example, Fermat's Last Theorem, i.e., that there are no natural numbers *a, b* and *c* that satisfy the equation $a^n + b^n = c^n$ for $n \geq 2$, is not thought to be a truth about the physical world. It is a truth about the natural numbers *as such*. More generally, the theorems of Number Theory are taken as truths that are independent of whatever the properties of the physical entities in the universe turn out to be. To be sure, whether the truths of Number Theory really are about the natural numbers as such in the sense that they are independent of the properties of the actual physical world is a matter of great debate in the philosophy of mathematics (e.g., Shapiro, 1997, and others). Nevertheless, we think that it is best to talk about the representations of the number of entities in actual physical collections as representations of "whole numbers" so as not to give the impression that we are talking about the development of representations of the objects that satisfy the truths of Number Theory.

The second criterion captures a characteristic of number that is implied by the first: the organism has representations of number only if these represent a property of collections that abstracts away from the properties of its individual members. For example, it makes perfect sense to say that, every year, the same number of students sign up for my cognitive psychology class despite the fact that the students who take my class change every year. In fact, thanks to this characteristic, one can compare the number of any two collections even if they are comprised of elements that have nothing in common – e.g., homilies and bones. Therefore, as per this criterion, to be granted representations of whole numbers, an organism must be able to judge whether two collections have the same cardinality or whether one has more or less members even if the individuals in the two collections are of different kinds. We refer to this as the "abstract property" criterion.

Our criteria are nearly identical to those that have been used to grant representations of whole numbers to infants or children for the last two or three decades, with, perhaps, one important difference. We have been careful to make sure that our criteria only require organisms to have representations of what is also known as the "cardinality" of collections. They deliberately do not require the organism to have any other knowledge of the whole numbers. This last part is especially important for our purposes because whether the acquisition of whole number representations appears to be intertwined with the acquisition of knowledge of relations between whole numbers will be one of the main questions addressed in this paper. That is, although, in the end, some beliefs or pieces of knowledge about relations between whole numbers may be constitutive of our representations of the whole numbers (as proposed by (CAREY, 2009; GALLISTEL, 1990; PIAGET; SZEMINSKA, 1941) we prefer to leave this issue as a question that is open to empirical investigation.

1. Are there innate representations of the whole numbers?

The approximate number system (ANS; also known as the "analog magnitude system"). One way to investigate whether there are innate representations of the whole numbers is to probe whether infants with little experience with collections can determine whether two or more collections have the same number, even if the collections are not all comprised of elements with the same properties. This has been done by habituating infants to collections that share the same number of elements but that are otherwise different from each other. For example, in a now classic study, six-month-olds were presented with various collections until they habituated to them (XU; SPELKE, 2000). All collections were composed of circles. However, they differed from each other with respect to the spatial arrangement of the circles, and the areas of the individual circles. The collections also differed with respect to their total area (i.e., the sum of the individual areas of the circles that comprised them). One group of infants was habituated to collections of eight circles and another to collections of sixteen. The average of the total areas of the collections presented during habituation were the same for both groups. Once they were habituated, both groups were presented with the same test stimuli. Half of the test stimuli were collections of eight circles, and the other half were collections of sixteen. All test stimuli had the same density. Both groups of infants showed that they could discriminate eight circles from sixteen circles on the basis of number; i.e., both paid more attention to the number they had not been habituated to. Given the design of the study, the most plausible explanation of the infants' response is that they compared the collections to each other on the basis of number.

Using the same experimental logic – i.e., controlling for all non-numerical properties of the stimuli by equating some of them across groups and by equating the others across test stimuli – a different study (LIPTON; SPELKE, 2003) obtained evidence that six-month-olds can also compare sequences of sounds on the basis of number, even if the sounds are of different kinds (e.g., trumpets, phone rings, drums, or duck quacks). The fact that infants can compare both circles and sounds on the basis of number suggests that their representations of number are abstract – i.e., that the same representations apply to all collections of the same number regardless of the properties of the elements that comprise them. This was spectacularly confirmed by a study where newborns were simultaneously presented with sequences of syllables and collections of objects (IZARD et al., 2009). The newborns paid more attention to the collections of objects when their number matched the number of syllables in a sequence than when it did not.

These studies suggest that humans are equipped with innate, abstract non-verbal representations of number. However, as per our "exact number" criterion, these non-verbal representations of number are not representations of *whole number*s. For example, although eight and twelve, and sixteen and twenty-four differ by more than one, six-month-olds do not notice the numerical difference between them (XU; SPELKE; GODDARD, 2005). Likewise, newborns notice numerical mismatches between objects and syllables only when the mismatch is large, i.e., they distinguish between four and twelve, and six and eighteen, but not between four and eight (IZARD et al., 2009). Nine-month-olds can make finer numerical discriminations than younger infants; they can discriminate eight from twelve. However, they cannot discriminate eight from ten (LIPTON; SPELKE, 2003).

In fact, humans can use these non-verbal representations throughout their lifetime, and, at all ages, numerical comparisons based on these non-verbal representations follows Weber's Law; i.e., it is limited by the ratio of the numbers of the collections (BARTH; KANWISHER; SPELKE, 2003; HALBERDA; FEIGENSON, 2008). For example, the studies of numerical discrimination in 6-month-olds suggest that they cannot discriminate collections on the basis of number unless the ratio of the numbers is equal to or greater than 2:1. Over time, the threshold for numerical discrimination improves (HALBERDA; FEIGENSON, 2008), but it never reaches a point where humans are capable of discriminating between any two collections that differ by exactly one. There is always a limit beyond which these non-verbal representations cannot be used to compare collections on the basis of number. For example, it seems highly unlikely that anyone of any age could use these representations to discriminate collections of eighteen from collections of nineteen on the basis of number (BARTH; KANWISHER; SPELKE, 2003; HALBERDA; FEIGENSON, 2008). Therefore, these innate, abstract non-verbal representations of number are not exact, but rather are noisy or approximate. For that reason, this system of non-verbal numerical representation is now commonly referred to as the Approximate Number System, or ANS.

Weber's Law implies that the noise in ANS representations is proportional to the size of the number represented. Take six-month-olds for example. When they are habituated to collections of eight, presenting them with collections of eight more elements is enough for them to notice the numerical difference. However, when they are habituated to collections of sixteen, presenting them with collections of eight more elements is not enough for them to notice the numerical difference. Rather, they must be presented with collections of sixteen more elements. Thus, when the habituated number is eight, the just noticeable numerical difference is eight. When they are presented with sixteen, the difference must be twice as large to be noticeable. This suggests that the representation of sixteen is twice as noisy as the representation of eight. Weber's Law applies across a broad numerical range (BARTH; KANWISHER; SPELKE, 2003; HALBERDA; FEIGENSON, 2008). Thus, the general rule is that the noise – or better the standard deviation – of the magnitude of the ANS representations generated to represent a number *ab* is *a* times greater than the standard deviation in the representation of *b*. In other words, the ratio of the standard deviation of the magnitude of the representations generated by the ANS to the number represented is constant. This is known as "scalar variability."

Parallel individuation as "mental tallying." Infants, children, and adults are able to compare collections on the basis of number as long as the ratio of the numbers of the collections is sufficiently large. However, when it comes to infants, something odd happens when they are presented with collections of four or less. They seem to lose their ability to discriminate on the basis of number. For example, we have seen that six-month-olds can discriminate collections of from other collections on the basis of number as long as the larger number is at least twice as large as the smaller one. Yet, six-month-olds who were first habituated to collections of two (or four) and were then presented with collections of four (or two) did not notice the change in number, despite the fact that the collections were presented in exactly the same way as the collections whose numbers six-month-olds could discriminate – e.g., collections of eight and sixteen (XU, 2003). In fact, when they are habituated to collections of four or fewer elements, infants respond to changes in the total area of the elements that comprise the collections (FEIGENSON; CAREY; SPELKE, 2002), a type of change they do not respond to when they are habituated to collections of four or more elements.

What happens when infants are presented with at least one collection of fewer than four elements? Why do they seem to inexplicably lose their ability to represent the approximate number of things in the collections they see? The current best explanation is that, when they are presented with four or fewer objects, infants spontaneously represent each object individually instead of representing the total number of objects they see. To be sure, the explanation is *not* that that the ANS cannot create representations of approximately one, two and three. Indeed, it has been shown that the ANS can and does represent the approximate number of entities in collections of one to three, but only under unusual circumstances, such as when the objects in arrays of two or three are very close to each other, or when the resources of attention and working memory cannot be dedicated to the task of representing a collection because they have been dedicated to another task (BURR; TURI; ANOBILE, 2010; HYDE; SPELKE, 2009, 2011). Thus, the current best explanation is that, *in typical conditions*, infants spontaneously use a different system when they are presented with fewer than four objects, namely "parallel individuation" (CAREY, 2009; HYDE, 2011; ULLER et al., 1999). Moreover, this proposal assumes that, for reasons that have yet to be clearly established, infants do not use this system to discriminate between collections on the basis of number in habituation paradigms (see FEIGENSON, 2005, for a possible explanation of this failure).

Parallel individuation refers to our capacity to attend to several objects in parallel so as to have access to perceptual information about each of these objects in real time (e.g., their motion and location in space, their color, their texture, etc.; KAHNEMAN et al., 1992; PYLYSHYN; STORM, 1988). This is sometimes known as "object-based attention" (SCHOLL, 2001). It also refers to our capacity to hold representations of several objects in parallel in working memory (VOGEL; WOODMAN; LUCK, 2001). Strikingly, the maximum number of objects adults can attend to and hold in working memory simultaneously corresponds quite closely to the boundary between the numbers that infants represent with the ANS in habituation paradigms and the numbers they do not respond to – i.e., about three or four. It is this correspondence that has led many to propose that, like adults, when infants are presented with four or fewer objects, they do not represent approximately how many there are, but rather represent each object separately (CAREY, 2009; FEIGENSON; CAREY, 2003; HYDE, 2011; ULLER et al., 1999; XU, 2003).

Infants' performance on the "manual search task" (FEIGENSON; CAREY, 2003, 2005) provides strong evidence that they have a capacity-limited system that represents objects. The task involves having infants retrieve toys hidden in a box after watching someone hide them in it one by one. Crucially, the experimenter hides the toys in the back of the box so that infants cannot feel them when they reach inside. Therefore, infants' search has to be based on their memory of the contents of the box. After infants have been given a fixed amount of time to search the box, the experimenter pulls out one of the remaining toys. As long as no more than three toys are hidden in the box, infants' searching behavior suggests that they remember what was hidden in the box, and that they can update their representation in memory when toys are removed. For example, if three toys are initially hidden, and two are then removed, infants keep searching inside the box for the third toy. Once they have been allowed to have the third, they spend much less time searching the box. However, if four toys are hidden in the box, infants seem to be unable to remember its contents. They do not spend more time searching the box when it contains toys than when it is empty, even if only one toy was removed from the box so that it still contains three of the four toys that had been hidden (FEIGENSON; CAREY, 2005).

The current best explanation of these results is that infants solve the manual search task by creating a separate representation of each hidden toy in their working memory. Like adults' working memory (COWAN, 2010; VOGEL; WOODMAN; LUCK, 2001), infants' working memory has a capacity limit: it can only hold on to up three distinct representations at a time. Therefore, infants successfully solve the task when up to three toys are hidden, but fail when four are hidden. The validity of this explanation is bolstered by the elimination of two plausible alternative explanations. First, infants do not solve the manual search task by representing the total size of the hidden toys (i.e., they do not represent the sum of the sizes of the individual toys). If two small cars are hidden in the box, and one large car whose size is equal to the sum of the sizes of the cars that were originally hidden in the box is removed from it, infants reach once more for another car, and then stop reaching in the box (FEIGENSON; CAREY, 2003). Therefore, their decision to search the box or not is not based on the size of the objects hidden in it. Second, infants do not solve the search task by using the ANS to represent the approximate number of toys in the box. To use the ANS to solve the task, infants would have to compare the number of objects taken out to the number hidden initially and stop reaching when they cannot distinguish the former from the latter. If we suppose that this is the computation infants carry out to solve the task, the search data would mean that infants can distinguish two from three, but not one from four. This is a direct contradiction of the signature of numerical discrimination based on the ANS – namely, Weber's Law. Therefore, infants do not use the ANS to solve the search task, at least not when four or fewer objects are involved (also see ULLER et al., 1999 for evidence that infants represent each individual object separately rather than the total number when they are presented with two objects in a different task). This leaves the idea that infants can maintain up to three separate representations of individual objects in working memory as the best explanation of the manual search data.

Are the representations of individual objects created with parallel individuation representations of whole numbers? That is, are the representations created with parallel individuation equivalent to representations of one, two and three? It may seem as though they are not because they fall short of the "abstract property" criterion. Indeed, so far, we have said that parallel individuation represents objects. It may thus seem as though the representations created by parallel individuation cannot possibly function as representations of a property of collections that abstracts away from the objects that comprise them. However, we have not said *how* parallel individuation represents objects. To understand how parallel individuation represents objects, we must take a step back and think carefully about the manual search task.

One characteristic of the task is particularly important: the toys that are initially hidden in the box are always identical copies of the same kind – e.g., red plastic balls of exactly the same size. This implies that, whenever two or more toys are hidden in the box, infants cannot solve the task by assigning an index (i.e., a representation that functions like a "proper name" for each toy) to each toy and by then tracking each toy over space and time. To understand how this follows, consider a trial where two red balls have been hidden in the box. Call them Red-1 and Red-2. The experimenter pulls out one of the balls. Is it Red-1 or Red-2? There is no way of knowing because the two balls are identical to each other. Therefore, infants cannot solve the task by thinking: "Red-1 and Red-2 are in the box. The experimenter just pulled out Red-1. Therefore, Red-2 is still in the box." In other words, infants cannot solve the task by creating representations that each refer to a ball as a *particular* individual (e.g., Red-1 or Red-2). The representations they use must thus be abstract enough to fit any of the toys that were hidden, but also precise enough to capture the exact number of toys in the box. What sort of representation fits this function?

Drawing inspiration from previous proposals of how a representational system based on parallel individuation can represent number (e.g., FEIGENSON; CAREY, 2003; LE CORRE; CAREY, 2008), we propose that infants solve the manual search task by representing individuals with mental tallies of sorts. Consider how someone can easily keep track of hidden, identical objects by following the rules of tallying. For each object that one sees being hidden, one draws one and only one mark, e.g., "|". Thus, if two balls were hidden, one would end up with "||." Then, to know whether all the balls have been removed from the box, one matches each ball that is removed to one and only one of the marks. Crucially, matching is not based on the identity of the balls. Rather, each mark can be matched to either of the balls. Therefore, tallying does not depend on being able to recognize which of the balls has been pulled out (e.g., Red-1 or Red-2). When each mark in the tally has been matched to one and only one ball, one concludes that all the balls have been removed.

Our idea is that infants solve the manual search task by creating symbols that function just like the marks in a tally and by holding them in memory. One and only one symbol is created (or "activated") for each hidden object. Once all of the objects have been hidden, all the symbols created are held in memory. When an object is removed from the box, one and only one of the symbols is matched to it. When all the symbols in memory have been matched to an object, the infant concludes that all the objects have been removed from the box. Unlike physical tallies, mental tallies are limited by working memory capacity. Moreover, they are limited by one's ability to match individual mental marks to individual objects. Thus, if one cannot match marks to objects by attending and tagging each object in a collection serially, the mental tallies that one can create will be limited by the number of objects that one can attend to in parallel (i.e., about 3 or 4 in adults, PYLYSHYN; STORM, 1988). We suspect that this is the case for infants – i.e., that they cannot attend to all the objects in a collection by moving their attention from one object to another stopping when they have attended to every object. This is why their ability to keep track of objects by using mental tallying is limited by the capacity of their working memory, and by the number of objects they can attend to and tag in parallel, namely three.

Cheung and Le Corre (2018) obtained evidence that, by age two, the capacity-limited system used to track one collection of objects in the manual search task is also used to compare the cardinalities of *distinct* collections. Cheung and Le Corre presented two-, three-, and four-year-olds with pairs of collections of rectangles and asked them to indicate which of the two contained more rectangles. In two experiments, children viewed the rectangles as long as they wished, but were instructed not to count them. In a third experiment, the rectangles were presented too quickly to be counted. Thus, in all three experiments, children had to rely on non-verbal representations to solve the task. Crucially, they compared some pairs where both collections fell within the capacity of the capacity-limited system – namely, two vs. three – and some where the cardinality of the collections could only be represented with the ANS – namely, six vs. nine, ten vs. fifteen, and twelve vs. eighteen. The ratio of the pairs of cardinalities was always equal to 2:3. Therefore, if children represented and compared the cardinalities of all collections with the ANS, they should have been equally accurate on all comparisons. Contrary to this prediction, in all three of their experiments, Cheung and Le Corre found that two-, three-, and four-year-olds were slightly but consistently more accurate when they compared two and three than when they compared larger numbers. However, as expected, they were equally accurate on all comparisons of cardinalities that could only be represented with the ANS. Moreover, Cheung and Le Corre obtained evidence that better performance on comparisons of two and three was not due to the fact that these were the only cardinalities that children could name rapidly without counting. Indeed, even children who had not learned the meaning of any number word beyond the number word for one also performed better when they compared two and three than when they compared larger cardinalities. Thus, the best explanation of Cheung and Le Corre's results is that they did not use the ANS to compare two and three but instead did so via a different representational system, most likely parallel individuation.

Our mental tallying model of parallel individuation can also explain how children compare collections of two and three elements and determine which contains more elements. The process begins with the tallying of either one of the collections; i.e., each element in one of the collections is assigned one and only one symbol. Let's call the first collection to be tallied the "tallied collection." Then, the tally is held in working memory, and each of the symbols in it is matched to one and only one object in the other collection. Let's call this collection the "matched collection." If one or more of the symbols in the tally cannot be matched to an object of the matched collection, the tallied collection contains more elements than the matched one. On the other hand, if one or more of the objects in the matched collection cannot be matched to a symbol in the tally, then the tallied collection contains fewer elements than the matched collection. Thus, the comparison of cardinalities via mental tallying requires that infants match individual mental symbols with individual objects, and that they hold the mental tally in working memory. Therefore, their ability to compare cardinalities via mental tallying should be limited by the number of individuals they can attend to in parallel, and by the capacity of their working memory. It follows that children could have used mental tallying to compare two and three but not to compare larger cardinalities. This is consistent with Cheung and Le Corre's finding that preschoolers performed differently when they compared two and three than when they compared any of the pairs of collections that contained more than three objects.

So far, we have assumed that the symbols in the mental tallies created by infants and children are abstract in the sense that, while each symbol applies to one and only one individual per collection, it can apply to any kind of individual. If this is true, then infants and/or young children should be able to use mental tallying to compare collections that are composed of individuals of different kinds – e.g., frogs and cars. Two studies suggest that infants' mental tallies are indeed abstract in this sense. Feigenson (2005) asked there are any conditions under which six-month-old infants can distinguish one from two in a habituation paradigm. In the studies that preceded Feigenson's study of habituation to small cardinalities (XU, 2003; XU; SPELKE, 2000), the objects were all identical to each other. In contrast, Feigenson presented objects that were different from each other to make it easier for infants to notice that the objects were distinct from each other and to encode them as such. Most importantly, unlike what had been done in previous studies, the objects presented *during* habituation were markedly different from the objects presented *after* habituation – i.e., at test. That is, all the objects were rectangular blocks that had a small face and that stood on their short edge. Aside from that, they were markedly different from each other. For example, one of the objects presented during the two-object habituation condition was covered with horizontal stripes all over its body, from top to bottom. The other object was painted in a single uniform color and was not covered with any pattern or texture, but its head was covered with pins. The objects presented at test did not have any of these properties. Instead, their bodies were covered with zig-zags and/or small furry balls. Crucially, the sum of the areas of the visible (frontal) side of the objects in every collection that was presented during habituation was different from the total area of visible side of the objects in every collection presented at test. Therefore, if the infants had encoded the areas of the visible of the objects they saw, they should have dishabituated equally to every collection presented at test, regardless of their cardinalities. Instead, once they had been habituated to a given number of objects (e.g., 2), infants paid more attention to collections with a new number of objects (e.g., 3) than to collections with the number of objects they had been habituated to. Since the objects presented during habituation were different from those presented at test, this suggests that infants' representations of individual objects are abstract enough to allow them to compare objects that are different from each other on the basis of number.

Second, in a study by Féron, Gentaz and Streri (2006), four-month-old infants first held two or three distinct objects (e.g., a ring, a cube and a sphere) that they could not see in their hand, one at a time, for thirty seconds each. Then they looked at two or three objects on a screen. Critically, the objects presented on the screen were quite different from the objects that were explored manually; they consisted of squares with a hole in the middle and with a sphere bulging from one of their corners. Despite the fact that the objects explored manually were different from the objects presented visually, infants compared them on the basis of cardinality; they looked longer at the screen when the number of objects they saw corresponded to the number of objects they had held.

In sum, between four and six months of age, infants can compare collections of a certain kind of object to collections of another kind of object (e.g., a striped rectangular block vs. two rectangular blocks covered with zig-zags and furry balls, or a cube and a sphere vs. two irregular shapes) on the basis of number[3]. Moreover, their representations of the objects are not tied to a particular perceptual

[3] There is also evidence that 5-month-olds (KOBAYASHI et al., 2004) and 6-month-olds (KOBAYASHI; HIRAKI; HASEGAWA, 2005) can establish representations of individual physical objects on the basis of their visual appearance and on the basis of the sound they make when they hit the ground. These studies shows that infants can create representations of objects on the basis of visual and auditory *inputs*, they do not show that they have representations that are abstract

modality; they can be created visually or haptically. Unfortunately, neither of the aforementioned studies compared infants' performance on comparisons of the cardinality of collections that do not exceed the limit of a capacity-limited system like the mental tallying system proposed above to their performance on comparisons of collections whose cardinality can only be represented with the ANS. They only included collections of one to three objects. Therefore, it is not possible to draw definitive conclusions about the type of representation used by the infants in these studies. However, all of the evidence presented thus far suggests that, in most conditions, infants do not use the ANS when they are presented with collections of fewer than four elements. Therefore, we take these studies to support the view that, by six months of age and perhaps earlier, infants represent the elements of collections with abstract mental tallies, and that they cannot use mental tallies to solve tasks that involve representing more than three elements at a time due to limits on their attention and working memory.

Are tallies, mental or physical, representations of whole numbers? Are "|," "| |," and "| | |" representations of one, two, and three? They do meet our two criteria. They are exact. In all the studies of representations of one, two and three entities reviewed thus far, a difference of one is sufficient to trigger a difference in behavior – whether it is reaching in the box again when one of three balls is still in it, or looking longer at two than at three novel squarish shapes when one has just explored a ring and a cube haptically. Moreover, as was just reviewed, there is evidence that they are abstract in the sense that they can be used to compare collections composed of elements of different kinds – e.g., a ring and a cube vs. two novel squarish shapes. Moreover, infants and children use mental tallies to carry out basic numerical computations, namely same/different and more/less. Thus, on our view, mental tallies do function as representations of one, two and three.

However, mental tallies do not function in the same way as number words like "one", "two", and "three." The latter represent the cardinalities one, two, and three and do not represent anything else; in expressions like "two balls" and "two rings", the symbol of cardinality ("two") is separate from the symbols for the collections ("balls" and "rings"). Mental tallies like "|," "| |," and "| | |" do not distinguish between collections and cardinalities because they are collections themselves. Thus, although they do function as representations of cardinality that support numerical computations, they do not represent cardinality *as a property of collections*. However, in our opinion, the question whether symbols must represent cardinality as a property of collections to count as symbols for the whole numbers might not have a clear answer. Thus, debates about whether mental tallies are symbols for whole numbers might not be very productive. What is likely to be much more productive is recognizing that mental tallies and number words represent cardinality in *different ways*, for this might turn out to be critical for understanding how children learn the meanings of number words and create number concepts.

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enough to *apply* to both collections of sounds and collections of objects. In other words, the nature of the referents to which representations apply (e.g., objects or sounds) should not be confused with the nature of the perceptual evidence on the basis of which representations are created. Both of KOBAYASHI and colleagues' studies speak to the latter only. They do not provide evidence that mental tallies are abstract enough to apply to different kinds of objects.

2. Interlude: From the core number systems to verbal counting.

The developmental history of representations of whole numbers begins with two representational systems: the ANS and mental tallying. Both systems are likely innate or develop very early, on the basis of very little experience with individual events and things. They are thus often referred to as "core number systems" (CAREY, 2009; FEIGENSON; DEHAENE; SPELKE, 2004). The development of the core number systems is not the end of the story of the development of symbols for whole numbers. Indeed, the meanings of the whole number symbols used by all or nearly all adults who live in numerate cultures – i.e., the number words – cannot be expressed with these innate systems. The representations created with the ANS are most likely abstract. However, they are approximate. They cannot be used to generate an indefinitely long sequence of representations of numbers in which each number is equal the number that precedes it plus one. Mental tallies are exact and possibly abstract, and can be used to compute numerical equality and inequality. In this sense, they function as representations of whole numbers. However, mental tallies cannot represent collections that contain any more than three elements. Moreover, even if mental tallies can be said to function as representations one, two, and three, the way they represent these cardinalities is arguably different from the way they are represented by number words. Number words represent cardinality as a property that is separate from the collections that embody them. Mental tallies do not separate collections from their cardinality for they are collections of sorts themselves.

There is another important discontinuity between the core number systems and the way adults represent the whole numbers. Adult number word meanings are defined by verbal counting. When one counts a collection, the number words must be recited in a fixed sequence, and each number word applies to one and only one of the elements of the collection that is being counted so that the last number word to have been uttered designates the cardinality of the collection formed by the elements that have been counted (GELMAN; GALLISTEL, 1986). Verbal counting thus defines the meanings of the number words by marrying what is arguably one of the most mathematically fundamental properties of the set of whole numbers – i.e., that it consists of an ordered sequence – with the relation that defines whether two or more collections have the same cardinality – i.e., whether their elements correspond one-to-one.

Neither of the core systems creates representations by applying symbols following one-to-one correspondence in a fixed order. In both infants and adults, the time required to create ANS representations of the approximate cardinality of a collection is independent of the number of elements in the collection (BARTH; KANWISHER; SPELKE, 2003; WOOD; SPELKE, 2005). Therefore, ANS representations are most likely not created by going through a sequence of symbols in a fixed order while matching each one to one and only one entity. The creation of mental tallies follows one of the principles of verbal counting – namely, one-to-one correspondence. However, order plays no role in defining their meaning. Take the tally "| | |." Nothing about this tally defines one of the marks as the first, another as the second, and another as the third. In other words, the tally represents a collection of three regardless of the order in which the marks are matched to its elements. In fact, the tally would still function as a representation of three individuals even if all three marks were applied to distinct objects simultaneously. In sharp contrast, the French number word "cinq" means five if and only if it is the fifth number word in the French count list. Therefore, the ANS, mental tallies, and verbal counting are different representational systems because they create representations of numbers by following qualitatively different principles.

The way children learn how to use verbal counting to define number word meanings is consistent with the claim that verbal counting and the numerical representations that are available to children prior to language learning do not create representations of number in the same way. If the principles of counting were innate or at least available prior to language learning, learning how verbal counting defines number word meanings should only require identifying a list of symbols in one's language that follows these principles. In other words, children should know the meaning of every number word in their verbal count list shortly after they begin to count correctly.

Multiple studies in multiple languages and cultures suggest that this is not how it happens (see SARNECKA, 2015, for a review). Instead, for a long period, children's knowledge of number word meanings is independent of their counting ability. Children learn the exact meanings of the number words for one to four one at a time, in increasing numerical order, with long periods separating the time when they first show knowledge of the meaning of one of these number words and the time when they begin to show knowledge of the next one. Crucially, children can count collections of ten objects or more long before they learn the meanings of three or four. Thus, during this period, children's knowledge of the meaning of individual number words (e.g., the meaning of "one", and the meaning of "two" in English) lags behind their counting ability (i.e., how high they can count correctly). Depending on how often they hear competent speakers in their immediate community count and use number words (LEVINE et al., 2010), this period can last anywhere from months to years (e.g., ALMOAMMER et al., 2013; PIANTADOSI; JARA-ETTINGER; GIBSON, 2014; WYNN, 1992). Some time after they learn the meaning of the number word for three (or, sometimes, after they learn the meaning of the number word for four), children finally show evidence that they have learned how to use counting to define the number words meanings; namely they learn the meanings of the remaining number words in their count list all at once (SARNECKA; LEE, 2009). Henceforth, following the convention in the literature, we will refer to children who have learned how to use counting to define number word meanings as "CP-knowers" (where "CP" stands for the "cardinal principle" – i.e., the principle whereby the last number word of a correct count stands for the cardinality of the counted collection).

Let us illustrate the contrast between children who have learned the cardinal principle and children who have not with examples of how children perform on Give-a-Number task – a task which has become something of a gold standard for assessing children's knowledge of number word meanings and their understanding of counting (CAREY, 2009; LE CORRE et al., 2006; SARNECKA, 2015; WYNN, 1992). On this task, children are asked to give up to ten toys to an experimenter or a puppet out of a bunch of a dozen or so. Children who have not learned the meaning of number words for numbers beyond four rarely count to assemble collections of toys. If they are explicitly asked to count the toys, they can do so, and frequently do so correctly. However, they do not know how to use their count to determine how many objects they have given. For example, if their count reveals that they have given the wrong number of toys (e.g., they gave six instead of three but count all six objects correctly when asked to do so), they rarely change the number of toys they have given, even if they are explicitly told they have given the wrong number, and are asked to fix their answer.

In sharp contrast, children who have learned the meaning of number words for numbers beyond four frequently respond to requests for different numbers of toys by counting out toys without any prompting from the experimenter, especially when they are asked to give four or more. When these children are asked to count the toys they gave to make sure they gave the correct number and their count reveals that they did not give the correct number, they almost always fix their answer correctly. Crucially, children who reliably use counting to assemble collections of the correct number of toys on the Give-a-Number task can do so for any number in their count list. In other words, there is some evidence that children who know the exact meaning of the number word for "five" also know the exact meaning of all the other number words in their count list, most likely because they can use counting to define the meaning of any number word, as long as the word is part of their count list. In sum, sometime after children have learned the meaning of the number word for three or four, children seem to shift from learning number words one at a time, independently of counting, to using counting to define the meanings of all the number words in their count list (LE CORRE et al., 2006; SARNECKA; LEE, 2009).

All of the aforementioned evidence suggests that there is a conceptual gap between (1) the way counting defines number words meanings (2) the principles whereby mental tallies and the ANS create representations with numerical content. If this is correct, then all children who are exposed to verbal counting in their language somehow construct a representational system – i.e., verbal counting – that cannot be defined in terms of the representations with numerical content available to them prior to language learning.

This presents something of a puzzle. On the one hand, it obviously cannot be that children construct the verbal counting system by simply mapping the number words onto core number representations. On the other hand, it cannot be that children learn the principles of counting out of thin air. They must start with something. Although the creation of mappings between number words and representations created with core number systems cannot be the *endpoint* of the process of learning how to represent numbers with verbal counting, it is perhaps the most obvious candidate *starting* point for two reasons. First, the core systems may be the only systems with at least some numerical content that are available to children prior to language learning. Second, multiple studies have shown that, by age five, children have in fact created some form of mapping between at least some of the number words in their count list and the core systems (LE CORRE & CAREY, 2007;

LIPTON & SPELKE, 2005; SULLIVAN & BARNER, 2014; also see CORDES et al., 2001; IZARD; DEHAENE, 2008; SULLIVAN; BARNER, 2013; WHALEN; GALLISTEL; GELMAN, 1999 for evidence of mappings between number words and the ANS in adults). Therefore, the question is not whether number words are ever mapped onto representations created with the core number systems. Rather, the question is when these mappings are formed. More specifically, the question is whether children map at least some number words onto representations created with the core number systems prior to learning how to use counting to define these, and whether they use the information provided by core representations as a scaffold to construct the verbal counting system. In what follows, we review studies that bring evidence to bear on these questions.

3. How children construct number word meanings and the verbal counting system.

The evidence that supports the learning trajectory of the meanings of number words and of verbal counting is exceptionally solid. Children show essentially the same knowledge of number words and of verbal counting regardless of the demands of the task they are tested with (LE CORRE et al., 2006; WYNN, 1990, 1992). Moreover, the very same learning trajectory has been observed in at least nine different languages by different groups of researchers (ALMOAMMER et al., 2013; BARNER et al., 2009; LE CORRE et al., 2016; PIANTADOSI; JARA-ETTINGER; GIBSON, 2014; SARNECKA et al., 2007; VILLARROEL; MIÑÓN; NUÑO, 2011). Therefore, there is broad agreement that children learn exact meanings for the number words for one to four prior to learning how to define number word meanings with counting, and that they most likely learn the meanings of these number words by mapping them onto representations created with core number systems. However, the evidence described thus far is not sufficient to determine what core number system (or systems) are used to learn number word meanings. While mental tallies may be the default system for representing collections of up to three (or four in adults, see BURR; TURI; ANOBILE, 2010, for example), the ANS can and does represent these numbers too (BURR; TURI; ANOBILE, 2010; HYDE, 2011). Thus, in principle, children could use either system to learn meanings for one to four, and either system (or both of them) could be part of the scaffolds that children use to construct the verbal counting system.

To determine whether children use mental tallies, the ANS or both systems to learn number word meanings prior to learning how to define them with verbal counting, researchers have asked whether the number words learned prior to the acquisition of the verbal counting system show any of the processing "signatures" or limits of the core number systems. Mental tallies represent collections exactly but cannot represent more than four individuals. Representations created with the ANS have no known upper limit but they are imprecise. More specifically, the imprecision or variability in ANS representations of numbers scales with the number they represent – i.e., the ratio of the standard deviation of ANS values that correspond to a given objective number to the mean of these values is the same for all numbers. This ratio is known as the "coefficient of variation."

Therefore, the nature of the core systems that support number word learning prior to the acquisition of the verbal counting system can be revealed by answering the following questions. What is the range of number word meanings that children learn as part of the process of the construction of the verbal counting system? Does it include approximate meanings for number words that can only be represented with the ANS – i.e., numbers larger than four – or is it limited to learning meanings for the number words for numbers that fall within the range of mental tallies – i.e., one to four? Whereas the latter would indicate that mental tallies are the only core number system involved in the construction of the verbal counting system, the former would suggest that the ANS is involved either by itself (as proposed by DEHAENE, 2011, for example) or in conjunction with mental tallies (as proposed by SPELKE & TSIVKIN, 2001). Furthermore, when children begin to show evidence that they have mapped number words for numbers greater than four onto the ANS, is the variability in their use of number words scalar all the way down to one? That is, is the coefficient of variation of their verbal estimates of the number of items in collections the same for numbers that can be represented with mental tallies and/or the ANS – i.e., one to four – and for numbers that can only be represented with the ANS – i.e., numbers greater than four? Or are the coefficients of variation of children's use of the number words for one to four different from the coefficients of variation of their use of number words greater than four? Whereas the latter would suggest that the number words for one to four are mapped onto mental tallies, the former would indicate that all the number words in children's count list are mapped onto the ANS only.

It may seem that we have already said (and thus already know) that children learn meanings only for number words that that denote numbers within the range of the mental tallying system – i.e., one to three or four. However, this is not quite what the learning trajectory shows. It shows that these are the only *exact* meanings that children learn prior to acquiring the verbal counting system. It is based on analyses of number word knowledge that grant knowledge of the meaning of a number word if and only if children apply it to a single exact number. For example, as per the criteria typically used to assess children's knowledge of number word meanings on the Give a Number task, English learners are granted knowledge of "ten" if and only if they give an incorrect number at most once out of three requests for "ten", and they never or almost never give ten toys when they are asked for other numbers. Similar criteria have been used with other tasks (LE CORRE et al., 2006; WYNN, 1992).

These criteria are not fit for determining whether children learn meanings based on mappings to the ANS, especially when it comes to number words for larger numbers. ANS representations are not exact; they are noisy, variable. Thus, when children and adults use mappings to the ANS to estimate how many objects they see or how many sounds they hear without counting, on average, their estimates of each target number increases linearly as a function of number, but they are not exact (CORDES et al., 2001; IZARD; DEHAENE, 2008; SULLIVAN; BARNER, 2013; WHALEN; GALLISTEL; GELMAN, 1999). For example, they do not always say "ten" as their estimate of collections of ten and they apply "ten" to collections that do not have "ten" objects. Moreover, the imprecision of ANS representations increases as number increases – i.e., the variability of ANS representations is scalar.

Therefore, it could be that children use the ANS to learn number word meanings prior to acquiring the verbal counting system, but that, due to the scalar variability of representations created with the ANS, the meanings they have learned for number words for numbers beyond four are too approximate to meet exact criteria like the ones that have been used to evaluate children's number word knowledge with the Give-a-Number task (see GALLISTEL; GELMAN, 1992, for example).

To find out what core number representations are mapped onto number words as part of the scaffolds that children use to learn how to use counting to define number word meanings researchers used tasks that are quite similar to the tasks that have been used to discover the learning trajectory described above, but with slightly different instructions, and with different measures of performance (CHEUNG; SLUSSER; SHUSTERMAN, 2016; GUNDERSON; SPAEPEN; LEVINE, 2015; LE CORRE; CAREY, 2007; ODIC; LE CORRE; HALBERDA, 2015; WAGNER; JOHNSON, 2011). As in previous studies of number word learning, the tasks required children to create a collection of objects in response to a verbal request for particular number, or to report how many things they saw in collections of different numbers. Unlike previous studies, children were sometimes explicitly encouraged to guess even if they were not sure of their answer (LE CORRE; CAREY, 2007). Most importantly, none of the studies assessed children's knowledge of number word meanings by measuring the accuracy of their answers. Rather, they measured whether the slope of the best least squares linear fit of children's average estimates was significantly greater than 0. Therefore, to be granted knowledge of mappings to core number systems, children did not have to perform perfectly. They simply had to produce larger verbal estimates for larger numbers. For example, a child whose average estimates for five and ten were six and seven respectively could be considered to have mapped – albeit imperfectly – some number words onto ANS representations of five and ten. For the sake of simplicity, children who produce number words denoting larger numbers for larger numbers (i.e., whose estimates were best fit by a curve with a slope that was greater than 0) will be said to be "able to estimate."

Of course, children were expected to be able to estimate up to four prior to learning the cardinal principle. The question at stake was whether this new way of measuring mappings to core number systems would reveal that children *also* learn to estimate numbers that can be represented with the ANS only – i.e., numbers larger than four – as part of the process of constructing the verbal counting system. As far as we know, there are five published studies that used this measure to assess what mappings are formed as part of the construction of the verbal counting system (CHEUNG et al., 2016; GUNDERSON et al., 2015; LE CORRE & CAREY, 2007; ODIC et al., 2015; WAGNER & JOHNSON, 2011). All included subset-knowers. Some also included CP- knowers (CHEUNG; SLUSSER; SHUSTERMAN, 2016; LE CORRE; CAREY, 2007; ODIC; LE CORRE; HALBERDA, 2015). The studies that included CP-knowers prevented counting by presenting collections too quickly for young children to be able to count them (i.e., for 1 second or less) to ensure that their estimates reflected the range of number words they had mapped onto core number representations only, and that they could not be contaminated by estimates based on counting.

Although each of these studies had its own idiosyncrasies (e.g., different studies used different tasks to measure estimation), the pattern of results across all of the experiments that have been reported (many of the studies included two experiments, some with multiple tasks) can be summarized as follows. About two thirds of the experiments suggest that children map number words that denote numbers larger than four only sometime *after* they become CP-knowers (CHEUNG; SLUSSER; SHUSTERMAN, 2016; GUNDERSON; SPAEPEN; LEVINE, 2015; LE CORRE; CAREY, 2007; ODIC; LE CORRE; HALBERDA, 2015). That is, most find that subset-knowers and many CP-knowers cannot estimate numbers beyond four. CP-knowers who cannot estimate numbers beyond four became known as "CP non-mappers"; those who can became known as "CP-mappers." The remaining third of the experiments found children who could produce slightly but significantly higher estimates for numbers in the vicinity of ten than for five and/or six before they became CP-knowers (GUNDERSON; SPAEPEN; LEVINE, 2015; ODIC; LE CORRE; HALBERDA, 2015; WAGNER; JOHNSON, 2011).[4]

In sum, these studies suggest that the contribution of mappings between number words and the ANS to the construction of the verbal counting system is limited at best. A study where CP-knowers were trained to map "ten" onto the ANS representation of ten provides especially strong evidence that the contribution of mappings between the ANS and number words is quite limited indeed (CAREY et al., 2017). The mapping was taught by presenting children with collections of ten pictures of animals. Each collection of ten was paired with a collection of five, seven, fifteen or thirty pictures of animals. Children were asked to point to the collection of "ten" animals (e.g., pigs) and were given feedback on each training trial. To test whether they had learned to map "ten" to the ANS representation of ten, children were tested once more with collections of animals they had not seen during training. They were not given feedback on these test trials. Despite the fact that they had experienced multiple pairings of the number word "ten" with collections of ten, and that the items

[4] Two proposals have been put forward to explain the inconsistent pattern of results across these studies. One is that the creation of mappings between number words and the ANS and the construction of the verbal counting system are two independent learning processes (see GUNDERSON et al., 2015). Thus, some children begin to map ANS representations of numbers greater than four onto number words before they become CP-knowers. Others do so later. On this view, the differences across experiments are due to sampling error; i.e., some experiments simply include more children who began to map ANS representations of numbers greater than four onto number words prior to becoming CP-knowers than others. The other proposal is that studies that assess children's knowledge of mappings between number words and the ANS with number word comprehension tasks are more sensitive than studies that require the production of verbal estimates (ODIC et al., 2015). Unfortunately, neither of these proposals accounts for all of the evidence. Variation in the composition of the samples across studies cannot be the explanation because, in some studies, the very same children showed evidence of having mapped ANS representations of numbers greater than four on one task, but also failed to show any such evidence on another, despite the fact that they were tested on both tasks on the very same day (GUNDERSON et al.; 2015; ODIC et al.; 2015). Differences between comprehension and production do not seem to be the explanation either. Of the experiments that used comprehension tasks, some found evidence for weak mappings to ANS representations of numbers greater than four in children who had not learned the cardinal principle (ODIC et al., 2015; WAGNER; JOHNSON, 2011), but some did not (GUNDERSON et al., Studies 1 and 2). Moreover, one experiment obtained evidence of weak mappings with a number word production task (GUNDERSON et al., Study 2). Therefore, at this point, the reasons why some experiments provide evidence that children map ANS representations of numbers greater than four onto number words prior to or when they become CP-knowers while others suggest that they do so sometime after they have become CP-knowers remain elusive.

presented at test were not so different from the items they been trained with (i.e., they were all animals), CP-knowers could not successfully pick the collection of "ten" when it was contrasted with seven, fifteen or thirty new animals. However, they could pick the collection of "ten" when it was contrasted with five new animals. In fact, they succeeded on these five vs. ten comparisons from the very first *training* trial, *before* being taught that "ten" applies to ten and not to five. This provides strong evidence that children do not map "ten" onto ANS representations of ten to construct the verbal counting system. However, they may learn something of the meaning of at least one number word beyond four, namely the number word for five, by mapping it onto the ANS.

The results of the training study suggest that subset-knowers' fleeting ability to distinguish five or six from numbers in the vicinity of ten on verbal estimation tasks is not due to the fact that they have mapped ANS representations of about five *and* ANS representations of about ten onto number words. Instead, it only reflects the fact that they have mapped the number word for five onto some core number representation – most likely the ANS. On this view, the reason why subset-knowers use number words that denote numbers greater than five when they are presented with numbers larger than five is *not* they have mapped these number words onto the ANS. Rather, they do so because they know that the number word for five does not apply to numbers greater than about five or six, and they thus resort to using number words they have not mapped onto any core number representations (i.e., number words that denote numbers greater than five) when they are presented with collections of more than five or six.

In sum, all of the evidence reviewed thus far suggests that, although children can count beyond five before becoming CP-knowers, they do not map number words for any numbers beyond five onto core number representations to construct the verbal counting system. Since the ANS is the only system that represents five, this could be taken to suggest that children map the number words for all numbers for one to five onto this system. It would the simplest, most parsimonious explanation. However, analyses of the coefficient of variation of CP-mappers' verbal estimates of one to ten strongly suggest that this is not the case. Whereas various studies estimate that the coefficient of variation for estimates of numbers between six and ten is equal to about 0.2 to 0.25 (LE CORRE & CAREY, 2007; ODIC et al., 2015), the coefficient of variation of CP-mappers' estimates of 1 to 4 is equal to or nearly equal to 0 (LE CORRE & CAREY, 2007; also see WAGNER & JOHNSON, 2011). This strongly suggests that the number words for one to four are mapped onto mental tallies. This in turn implies that the number word for five is the only one that may be mapped onto the ANS (but see CAREY et al., 2017 for ideas about how the estimation of five could be based on mental tallies). Although the mapping of the number word for five may play a role in the construction of the verbal counting system, in what follows we review hypotheses according to which mental tallies is the only core number system that provides part of the scaffolds that children use to construct the verbal counting system.

4. Do children use the mappings to mental tallies to construct the verbal counting system? If so, how?

Children map the number words for one to four onto mental tallies prior to learning how to use verbal counting to define number word meanings. Now, as we know all too well, evidence that A always occurs before B does not mean that A causes B, or that one must go through A in order to get to B. Thus, it could be that children learn meanings for one to four (and five) prior to becoming CP-knowers because these are the only meanings that they can learn prior to becoming CP-knowers, not because they *have* to learn these meanings to learn how to use counting to define number word meanings. In other words, in and of itself, the fact that children map the number words for one to four onto mental tallies prior to becoming CP-knowers is not sufficient to conclude that children use these mappings to construct the verbal counting system.

To find out whether children use mental tallies to construct the verbal counting system, one has to imagine how they could use mental tallies to formulate hypotheses about how to use counting to define number word meanings and then test whether CP-knowers do define number word meanings that way. According to an influential proposal (CAREY, 2004, 2009), children use the mappings to mental tallies (which Carey refers to as "enriched parallel individuation") to define the meaning of every number word in their count list in terms of the meaning of the one that precedes it. They do so by noticing a relation between differences in the number of marks in the mental tallies associated with the number words for one to four and the order of these number words in the counting sequence. That is, they notice that the mental tally associated with the number word for four – i.e., "││││" – contains exactly one more mark than the tally associated with the number word for three – i.e., "│││", and that the same relation holds between the number word for three and the number word for two, and between the number for two and the first number word in the count list – i.e., the number word for one. They also notice that the number word for two immediately follows the number word for one, that the number word for three immediately follows the number word for two, and that the number word for four immediately follows the number word for three. Thus, they learn that, in the range of the number words for one to four, whenever a number word *n* immediately follows a number word *m* then the mental tally associated with *n* contains one more mark than the one associated with *m*. Informally, they learn that "next" means "add 1." They then generalize this mapping between "next in the count list" and "add 1" to all number words in their count list, and now know how to use counting to define number word meanings.

Carey's proposal has strong intuitive appeal. How else could counting define number word meanings but through their relative positions? For example, isn't the meaning of "nine" defined by the fact that it comes right after "eight"? And, what is counting but a potentially endless process of adding one to a running total? For all the intuitive appeal of Carey's idea, a study by Davidson; Eng; Barner (2012) provides strong evidence that "next in the count list" means "add 1" is not what children learn when they become CP-knowers. As mentioned above, there is good evidence that what distinguishes CP-knowers from subset-knowers (children who have learned the exact meaning of a limited subset of the number words in their count list) is that the former have learned a general rule for defining the exact meaning of every number word in their count list (LE CORRE et al., 2006; SARNECKA; LEE, 2009). Therefore, if becoming a CP-knower means learning that "next" means "add 1", then, given any number word in their count list, all CP-knowers should know that the number it denotes is equal to the number denoted by the number word that immediately precedes it plus one. This means that any CP-knower who is told how many objects are hidden in a box and who then sees one more object added to the box should know that the number word that denotes how many things are in the box is the one that follows the number word that denotes the number of objects that were in the box prior to the addition (as long as both number words are part of their count list).

To test this, Davidson, Eng and Barner (2012) presented CP-knowers with problems of the same form as the problem just described. They hid objects in a box, told the children how many they had hidden, and then added one or two more. They then asked them whether they final number was equal to the hidden number plus one or the hidden number plus two. For example, if five had been hidden, children had to decide whether there were "six" or "seven" objects in the box after the addition. Participants were divided into three sub-groups based on how high they could count: low counters (who could count up to some number between ten and nineteen but no higher), medium counters (who could count up to some number between twenty and twenty-nine but no higher), and high counters (who count up to thirty or higher). Low counters were tested on problems that started with four or five objects, medium counters were tested on the same problems as low counters and on problems that started with fourteen or fifteen objects, and high counters were tested on the same problems as middle counters and on problems that started with twenty-four or twenty-five objects. Thus, all children were tested with number words that were well within their count list. Yet, about half of each sub-group of children failed to answer correctly on at least one pair of numbers. Specifically, more than half of the low counters and of the medium counters did not know that "five" is one more than "four" and that "six" is one more than "five", half of the medium counters and of the high counters did not know that "fifteen" is one more than "fourteen" and that "sixteen" is one more than "seventeen", and half of the high counters did not know that "twenty-five" is one more than "twenty-four" and that "twenty-six" is one more than "twenty-five." Therefore, even children with the best counting skills – i.e., the high counters – failed to answer at least some of the problems correctly. This is strong evidence that becoming a CP-knower does not involve learning that "next in the count list" means "add 1." Moreover, Davidson et al. found that high counters were significantly more accurate on the problems that started with "four" or "five" than on the problems that started with higher numbers. This strengthens the evidence further, for it suggests that, contrary to what would be expected if children learned that "next" means "add 1" when they become CP-knowers, they do not deduce these addition facts all at once from a general rule that applies to all the number words in their count list but rather learn each fact separately.

Thus, children do not learn that "next in the count list" means "add 1" when they become CP-knowers. But might they at least learn something about the meaning of the order of the number words? For example, long before they learn how to use counting to define number words meanings, children can use mental tallies to determine which of two collections contains more elements (CHEUNG & LE CORRE, 2018). Therefore, children could use the mappings between number words and mental tallies to notice that number words that occur later in the count list denote larger numbers than number words that occur earlier in the list. Children could know this without knowing the exact arithmetical relations between number words. For example, they could know that "ten" is more than "eight" without knowing that it is exactly two more than eight. In fact, knowing that "ten" is more than "eight" might be a prerequisite, or at least might help children learn the exact arithmetic relation between the two number words. However, Le Corre (2014) provides evidence that becoming a CP-knower does not involve learning that number words that occur later in the count list denote larger numbers than those that occur earlier in the list. Specifically, he identified a group of English-speaking CP non-mappers – i.e., CP-knowers who could not verbally estimate numbers between six and ten – and found that they did not know that "ten" is more than "eight", and that they did not know that it is more than "six" either. However, they did know that "three" is more than "two", and that "eight" is more than "one", showing that they did not fail the comparisons of "six" and "ten" and "eight" and "ten" because they did not understand the questions.

The combined results of Davidson et al. (2012) and Le Corre (2014) suggest that children do not become CP-knowers by using mental tallies to learn that "next" means "add 1" nor do they use them to learn that words that occur later in the count list denote larger numbers than words that occupy earlier positions. In other words, becoming a CP-knower does not involve learning anything about the meaning of the order of the number words in the count list. This raises two theoretically important questions. First, does this mean that children do not use mental tallies to learn how counting defines number word meanings? Is there any other general rule for defining number word meanings from counting that could be induced from the mappings between the number words for one to four and mental tallies? Second, how can it be that, at least for a time, children know how to use to counting to know which number word applies to a collection and that they actually use it frequently to solve cardinality tasks like Give-a-Number (e.g., when asked to create a collection of "ten", they do so by counting ten objects correctly), but that they do not know that "six" is one more than "five", and that "ten" is more than "eight" and "six"?

A possible answer to both of these questions is that children use mappings to mental tallies to learn that counting itself is a form of tallying. Children can count collections correctly long before they learn how to use counting to define number word meanings, especially if the collections contain few objects. Once they have mapped the number words for one to three or four onto mental tallies, they could notice that, whenever they count two, three or four objects, the last number word of their count is always the very number word that is mapped onto the mental tally that represents the objects in the collection. They might also note that the coincidence between the last word of a count and mental tallying holds when and only when they apply one and only one number word to each object and when they recite the count list in the conventional order. They could thus induce the general rule that whenever one labels every object in a collection with one and only one number word, and says the number words in the conventional order, one creates a tally of the collection, and that the last number word of the count can be used as a symbol of the tally that matches the collection. They would thus learn that all expressions of the form *n Xs* where *n* is a number word and *X* is a noun phrase (e.g., *n* blue socks or *n* short pencils) means *a collection of Xs that matches a correct count to "n."*

This proposal explains all of the results of all the studies of the nature of the number word meanings acquired by children when they become CP-knowers reviewed thus far. Children who have learned that "five horses" means *a collection of horses that matches a correct conventional count to "five"* and that "six horses" means *a collection of horses that matches a correct conventional count to "six"* have learned what it takes to give "five horses" or "six horses" to another person, even if they have not learned anything else about the meanings of these words; they simply need to count horses correctly and stop when they get to "five" or "six." However, they do not necessarily know that that "six" is more than "five" and much less that "six" is exactly one more than "five", for the meaning they have learned is not a mental representation of the number of elements in the collections that match counts that end with these words. *Any collection that matches a correct conventional count to "five"* is not a mental representation of the *number* five, any more than *any collection that matches a correct conventional count to "six"* is a mental representation of the *number* six. They are procedures for creating collections that turn out to have the property of being five or of being six. Therefore, children cannot tell that "six" is (one) more than "five" by simply retrieving the meaning they have learned for these words from their memory. And, of course, these meanings do not support estimation without counting, for they are defined in terms of counting. Thus, children who have learned this rule will not be able to estimate unless they map their number words onto mental representations of number that do not depend on counting, e.g., representations created with the ANS.

Although these meanings are not mental representations of numbers, they could grant children the knowledge that whenever a collection A and a collection B are labelled with the same number word *n*, the elements in the two collections match the same tally – i.e., a count that ends with *n* – and that, therefore, the As match the Bs one to one. For example, suppose that the proposal under consideration is correct, and that a young English-speaking CP-knower is told that "eight" of her friends are coming to her birthday party and that there are "eight" bags of party favors. In her mind, this means that every friend who is coming to her party can get one bag of party favors because her friends and the bags both match the same tally – i.e., a count to "eight". However, further suppose that, in accord with what Le Corre (2014) has observed, she still has not learned the relative sizes of the cardinalities of the collections that match "seven" and "eight." Then, if she were told that there are "seven" bags of party favors she would know that her guests and the bags do not match one to one because they do not match the same tally. However, she would not know whether there are too few bags, or whether there are too many because she does not know whether collections that match counts to "eight" contain more elements than collections that match counts to "seven" or whether they contain fewer. Thus, if she were asked whether every friend who is coming to her party will get a bag of party favors, she would probably say that she does not know, or she would randomly say "yes" or "no."

Soto-Alba and Le Corre (2019) asked whether the general rule that children learn to define number word meanings in terms of counting grants them knowledge that whenever a collection A and a collection B are labelled with the same number word, every A can be paired with one B. They first screened a large group of three- to five-year-olds for their knowledge of number word meanings with the Give-a-Number task. About one-third of the children they evaluated were subset-knowers, and two-thirds were CP-knowers. The fact that the age range included a fair number of subset-knowers suggests that the many of the children who were CP-knowers had become so in the recent past, thus making it at least possible that whatever knowledge this group of children would show is knowledge that children acquire when they learn how to define number word meanings with counting. They presented all children with problems of the same form as the problem of the party guests and the bags of party favors discussed above. The A's were always children and the B's were different types of "prizes" (cars, lollipops, rabbits, etc.…). The experimenter told the participants how many children there were and placed that very number of small plastic dolls in front of them to represent the children. The prizes were hidden in an opaque box. Participants were allowed to look in the box but only after they had answered the experimenter's question. Therefore, they had to rely on their understanding of the meaning of number words to determine whether each child could get one prize.[5]

On every trial, participants were told how many children and how prizes there were, and were asked whether each child could get one prize. Half of the trials presented to subset-knowers included at least one number word *within* the range of number words they had learned; i.e., on these trials, they were presented with two children and one or two prizes. On the other trials, both number words were *beyond* the range of number words that subset-knowers can learn; i.e., they included six boys and five or six prizes. CP-knowers were tested on three children vs. two or three prizes, and nine children vs. eight or nine prizes. CP-knowers were also tested on a third type of trial where the number of children was equal to the highest number word they could count to (as assessed with a counting task), and the number of prizes was either equal to the number of children or to the number of children minus one.

Recall that children do not learn the relative sizes of the numbers denoted by number words beyond four (or five) until sometime after they become CP-knowers (LE CORRE, 2015). Therefore,

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[5] The objective of Soto-Alba and Le Corre's study was not to determine whether CP-knowers understand that sentences of the form "Each A has one B" (where A and B are nouns) mean that each A has *exactly* one B. Rather, their objective was to determine whether CP-knowers can use what they have learned about number words to infer whether the elements in a collection correspond one-to-one to the elements of another. To ensure that the study assessed children's knowledge of number words and not their knowledge of the meaning of "Each A has one B", children were first tested on a control task that assessed whether they understood the meaning of sentences of the form "Each A has one B." Only children who passed the control task were included in the study."

in this age range, children who know the relation between number words one-to-one correspondence should always answer correctly when the number of boys is the same as the number of prizes. However, they are not expected to always answer correctly when the number words are beyond four, and there are fewer prizes than boys. Rather, in this this case, they are expected to guess since they still might not have learned the relative sizes of the numbers denoted by number words beyond four (e.g., there may be some CP-knowers who do not know that "eight" is less than "nine"). Therefore, it is enough for children to say that there are enough for prizes for each child to get one *significantly more frequently* when the numbers are equal than when they are not for them to show that they know the relation between number words meanings and one-to-one correspondence. To put it in very simple terms, it is enough for them to provide different answers when the number words are the same and when they different.

Soto Alba and Le Corre obtained two key results. First, subset-knowers did not distinguish trials with identical number words from trials with different number words when the number words were beyond the range of number words they had learned. However, they did distinguish these two types of trials when the number words were "one" and "two". In other words, when there were two boys and two prizes, these children always said that each boy could get a prize when there were two prizes, but said so significantly less frequently when there was only one prize. Second, unlike what has been observed with tests of CP-knowers' knowledge of that "next in the count list" means "add 1" (BARNER et al., 2012), the CP-knowers in Soto Alba and Le Corre's study distinguished trials with identical number words from trials with different numbers for *all* number words they were tested on, all the way up to the highest number word in their count list. This suggests that the general rule that children learn when they become CP-knowers could indeed be that every number word corresponds to a tally. These results strongly support the proposal that the general rule that children learn to define number word meanings with counting is that every number word corresponds to a tally, and that the mappings to mental tallies that they learn prior to learning this general rule are the basis for learning it.

Conclusion

We propose that there may be more continuity between what children learn when they learn how to use counting to define number word meanings and some of the principles available to them prior to language learning than previous influential proposals have assumed (e.g., CAREY, 2004, 2009; SPELKE, 2003). In particular, we suggest that children learn how to use counting to define number word meanings by thinking that, just like one of the two core number systems – i.e., mental tallies – counting is a tallying system. That is, the general rule that children learn when they become CP-knowers is that expressions like *"n X's*" where *n* is a number word and X is a noun phrase mean *a collection of Xs that match one to one with a count that ends in n.* This proposal explains why young CP-knowers do not know that "next" means "add 1" (in fact, it explains why many do not even know that "six" is one more than "five"), why many of them do not know that "ten" is more than "eight" and "six", and why it seems that all young CP-knowers nonetheless know that when the same number word applies to two collections the two collections are in one-to-one correspondence that have been proposed to explain how children how to use counting to define number word meanings. As far as we know, no other proposal explains all of these results. In fact, our proposal may reach even further. Some 30,000 years ago, humans already represented the number of objects in large collections by tallying (EVERETT, 2017). Thus, it may be that mental tallies are not only the source of children's first interpretation of the meaning of verbal counting, but were also the origin of the very first number symbols in human history.

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